Using gretl for Principles of Econometrics, 3rd Edition
Version 1.0[1]

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[1] Visit `http://www.LearnEconometrics.com/gretl.html` for the latest version of this book.

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Preface

This manual is about using the software package called **gretl** to do various econometric tasks required in a typical two course undergraduate or masters level econometrics sequence. It is written specifically to be used with *Principles of Econometrics, 3rd edition* by Hill, Griffiths, and Lim, although it could be used with many other introductory texts. The data for all of the examples used herein are available as a package from my website at `http://www.learneconometrics.com/gretl.html`. If you are unfamiliar with **gretl** and are interested in using it in class, Mixon Jr. and Smith (2006) have written a brief review of **gretl** and how it can be used in an undergraduate course that you may persuade you to give it a try.

The chapters are arranged in the order that they appear in *Principles of Econometrics*. Each chapter contains a brief description of the basic models to be estimated and then gives you the specific instructions or **gretl** code to reproduce all of the examples in the book. Where appropriate, I've added a bit of pedagogical material that complements what you'll find in the text. I've tried to keep this to a minimum since this is not supposed to serve as a substitute for your text book. The best part about this manual is that it, like **gretl**, is free. It is being distributed in Adobe's pdf format and I will make corrections to the text as I find errors.

To estimate a few of the models in *POE* I've had to resort to another free software called ***R***. As **gretl** develops I suspect that this small reliance on R will diminish. In any event, **gretl** contains a utility that makes using R quite easy. You'll find an appendix in this book that will get you started.

Gretl also gives users an ability to write his or her own functions, which greatly expands the usefulness of the application. In Chapters 14 and 16 functions are used to estimate a few of the models contained in *POE*. What's more, functions can be shared and imported easily through **gretl**, especially if you are connected to the internet. If **gretl** doesn't do what you want it to now, stay tuned. It soon may. If recent activity is any indication, I am confident that the the **gretl** team will continue to improve this already very useful application. I hope that this manual is similarly useful to those using *Principles of Econometrics*.

I want to thank the **gretl** team of Allin Cottrell and Riccardo "Jack" Lucchettii for putting so much effort into **gretl**. It is a wonderful program for **teaching** and **doing** econometrics. It has many capabilities beyond the ones I discuss in this book and other functions are added regularly. Also, Jack has kindly provided me with suggestions and programs that have made this much better than it would have been otherwise. Any errors, of course, are mine alone.

Finally, I want to thank my good friend and colleague Carter Hill for suggesting I write this and Oklahoma State University for continuing to pay me while I work on it.

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Contents

List of Figures

Chapter 1

Introduction

In this chapter you will be introduced to some of the basic features of **gretl**. You'll learn how to install it, how to get around the various windows in **gretl**, and how to import data. At the end of the chapter, you'll be introduced to **gretl**'s powerful language.

1.1 What is Gretl?

Gretl is an acronym for Gnu Regression, Econometrics and Time-series Library. It is a software package for doing econometrics that is easy to use and reasonably powerful. **Gretl** is distributed as free software that can be downloaded from `http://gretl.sourceforge.net` and installed on your personal computer. Unlike software sold by commercial vendors (SAS, Eviews, Shazam to name a few) you can redistribute and/or modify **gretl** under the terms of the GNU General Public License (GPL) as published by the Free Software Foundation.

Gretl comes with many sample data files and a database of US macroeconomic time series. From the **gretl** web site, you have access to more sample data sets from many of the leading textbooks in econometrics, including ours *Principles of Econometrics* by Hill et al. (2008). **Gretl** can be used to compute least-squares, weighted least squares, nonlinear least squares, instrumental variables least squares, logit, probit, tobit and a number of time series estimators. **Gretl** uses a separate Gnu program called ***gnuplot*** to generate graphs and is capable of generating output in LaTeX format. As of this writing **gretl** is under development so you can probably expect some bugs, but in my experience it is pretty stable to use with my Windows XP systems.

1.1.1 Installing Gretl

To install **gretl** on your system, you will need to download the appropriate executable file for the computer platform you are using. For Microsoft Windows users the appropriate site is `http://gretl.sourceforge.net/win32/`. One of the nice things about **gretl** is that Macintosh and Linux versions are also available. If you are using some other computer system, you can obtain the source code and compile it on whatever platform you'd like. This is not something you can do with any commercial software package that I've seen.

Gretl depends on some other (free) programs to perform some of its magic. If you install **gretl** on your Mac or Windows based machine using the appropriate executable file provided on **gretl**'s download page then everything you need to make **gretl** work should be installed as part of the package. If, on the other hand, you are going to build your own **gretl** using the source files, you may need to install some of the supporting packages yourself. I assume that if you are savvy enough to compile your own version of **gretl** then you probably know what to do. For most, just install the self-extracting executable, `gretl_install.exe`, available at the download site. **Gretl** comes with an Adobe pdf manual that will guide you through installation and introduce you to the interface. I suggest that you start with it, paying particular attention to Chapters 1 and 2 which discuss installation in more detail and some basics on how to use the interface.

Since this manual is based on the examples from *Principles of Econometrics, 3rd edition* (*POE*) by Hill et al. (2008), you should also download and install the accompanying data files that go with this book. The file is available at `http://www.learneconometrics.com/gretl/poesetup.exe`.

This is a self-extracting windows file that will install the *POE* data sets onto the `c:\userdata\gretl\data` directory of your computer's harddrive. If you have installed **gretl** in any place other than `c:\userdata\gretl` then you are given the opportunity to specify a new location in which to install the program during setup.

1.1.2 Gretl Basics

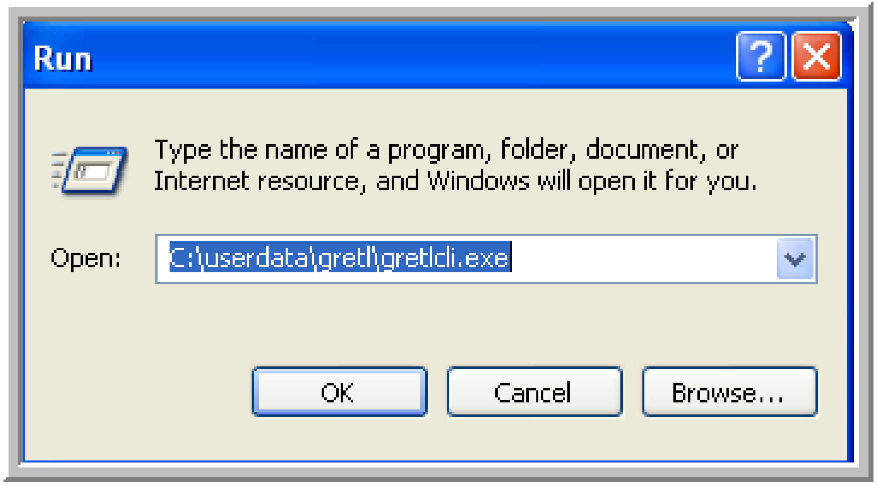
There are several different ways to work in **gretl**. Until you learn to use **gretl**'s rather simple and intuitive language syntax, the easiest way to use the program is through its built in graphical user interface (GUI). The graphical interface should be familiar to most of you. Basically, you use your computer's mouse to open dialog boxes. Fill in the desired options and execute the commands by clicking on the OK button. Those of you who grew up using MS Windows or the Macintosh will find this way of working quite easy. **Gretl** is using your input from the dialogs, delivered by mouse clicks and a few keystrokes, to generate computer code that is executed in the background.

Gretl offers a command line interface as well. In this mode you type in valid **gretl** commands either singly from the **console** or in batches using **scripts**. Once you learn the commands, this is surely the easiest way to work. If you forget the commands, then return to the dialogs and let the graphical interface generate them for you.

There is a command line version of **gretl** that skips the dialogs altogether. The command line version is launched by executing `gretlcli` in a dos command window. In Windows choose `Start>Run` to open the dialog shown in figure 1.1. In the box, use `Browse` button to locate the directory in which **gretl** is installed. Click OK and the command line version shown in figure 1.2 opens. This version of the program is probably the most useful for Linux users wishing to run **gretl** from a terminal window. We won't be using it in this manual.

A better way to execute single **gretl** commands is through the **gretl console**. In normal practice, the console is a lot easier to use than the

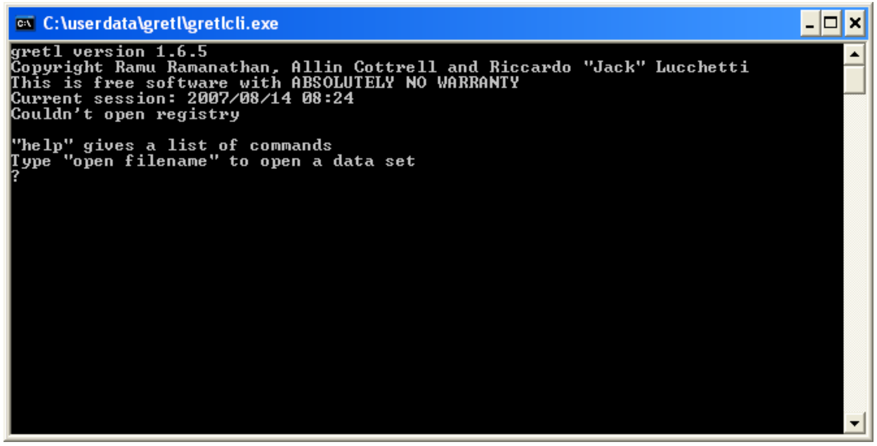
Figure 1.1: Opening the command line interface version of **gretl** using `Start>Run`



Type the name of a program, folder, document, or Internet resource, and Windows will open it for you.

Open: C:\userdata\gretl\gretlcli.exe

Figure 1.2: The command line version of **gretl**



```
gretl version 1.6.5
Copyright Ramu Ramanathan, Allin Cottrell and Riccardo "Jack" Lucchetti
This is free software with ABSOLUTELY NO WARRANTY
Current session: 2007/08/14 08:24
Couldn’t open registry

"help" gives a list of commands
Type "open filename" to open a data set
?
```

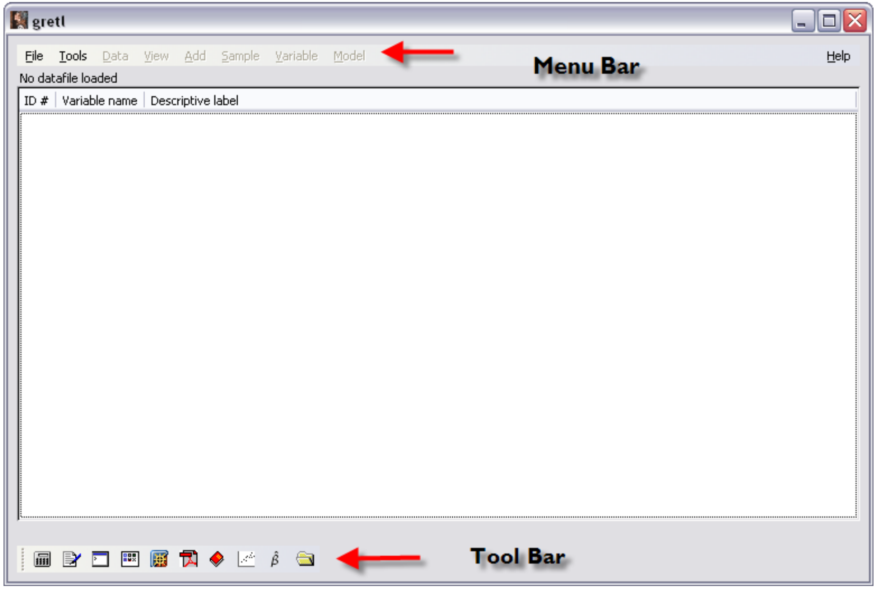
gretlcli. It offers some editing features and immediate access to other ways of using **gretl** that aren't available in the straight command line version of the program. The console and its use is discussed in section 1.3.1.

If you want to execute a series of commands, you do this using **scripts**. One of the great things about **gretl** is that it accumulates commands executed singly from the console into a **command log** that can be run in its entirety at another time. This topic can be found in section 1.3.2. So, if you have completed an analysis that involves many sequential steps, the script can be open and run in one step to get the desired result.

You can use the script environment to conduct Monte Carlo studies in econometrics. Monte Carlo studies use computer simulation (sometimes referred to as experiments) to study the properties of a particular technique. This is especially useful when the mathematical properties of your technique are particularly difficult to ascertain. In the exercises below, you will learn a little about doing these kinds of experiments in econometrics.

In Figure 1.3 below is the main window in **gretl**.

Figure 1.3: The main window for gretl's GUI



Across the top of the window you find the menu bar. From here you import and manipulate data, analyze data, and manage output. At the bottom of the window is the **gretl** toolbar. This contains a number of useful utilities that can be launched from within **gretl**. Among other things, you can get to the **gretl** web site from here, open the pdf version of the manual, or open the MS Windows calculator (very handy!). More will be said about these functions later.

1.1.3 Common Conventions

In the beginning, I will illustrate the examples using a number of figures (an excessive number to be sure). These figures are screen captures of **gretl**'s windows as they appear when summoned from the pull-down menus. As you become familiar with **gretl** the frequency of these figures will diminish and I will direct you to the proper commands that can be executed in the console or as a script using words only. More complex series of commands may require you to use the **gretl** script facilities which basically allow you to write simple programs in their entirety, store them in a file, and then execute all of the commands in a single batch. The convention used will be to refer to menu items as `A>B>C` which indicates that you are to click on option A on the menu bar, then select B from the pull-down menu and further select option C from B's pull-down menu. All of this is fairly standard practice, but if you don't know what this means, ask your instructor now.

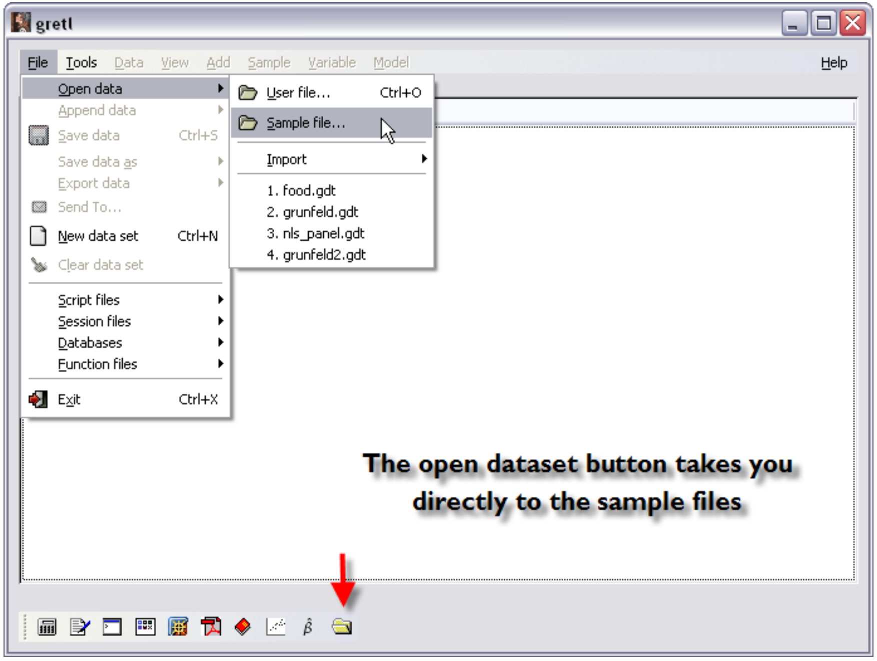
1.2 Importing Data

Obtaining data in econometrics and getting it into a format that can be used by your software can be challenging. There are dozens of different pieces of software and many use proprietary data formats that make transferring data between applications difficult. You'll notice that the authors of your book have provided data in several formats for your convenience. In this chapter, we will explore some of the data handling features of **gretl** and show you (1) how to access the data sets that accompany your textbook (2) how to bring one of those data sets into **gretl** (3) how to list the variables in the data set and (4) how to modify and save your data. **Gretl** offers great functionality in this regard. Through **gretl** you have access to a very large number of high quality data sets from other textbooks as well as from sources in industry and government. Furthermore, once opened in **gretl** these data sets can be exported to a number of other software formats.

First, we will load the food expenditure data used in Chapter 2 of *POE*. The data set contains two variables named x and y. The variable y is weekly expenditures on food in a household and x is weekly income measured in $100 increments.

Open the main **gretl** window and click on `File>Open data>sample file` as shown in Figure 1.4.

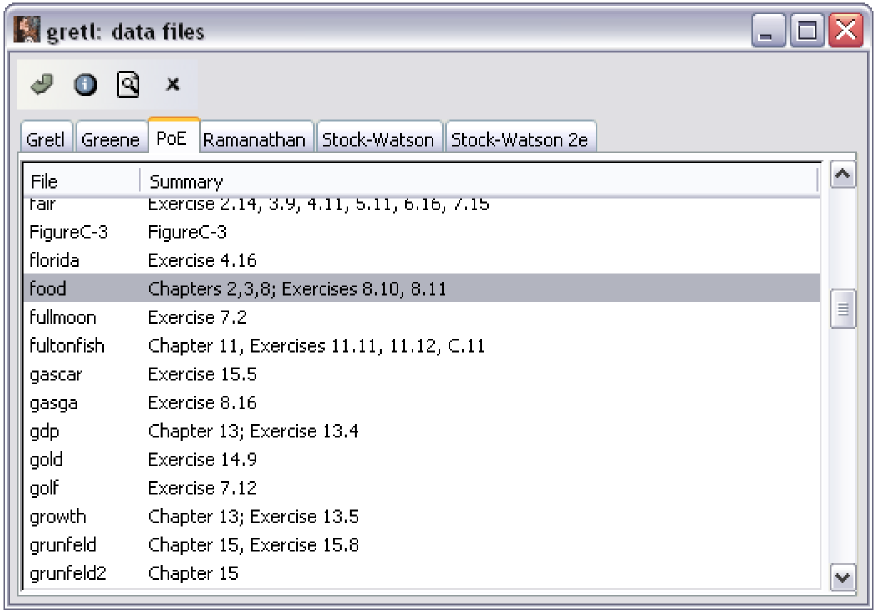
Figure 1.4: Opening sample data files from gretl's main window



The open dataset button takes you directly to the sample files

Alternately, you could click on the open dataset button on the toolbar. The button looks like a folder and is on the far right-hand side of the toolbar. This will open another window (Figure 1.5) that contains tabs for each of the data compilations that you have installed in the `gretl/data` directory of your program. If you installed the data sets that accompany this book using the self extracting windows program then a tab will appear like the one shown in Figure 1.5.

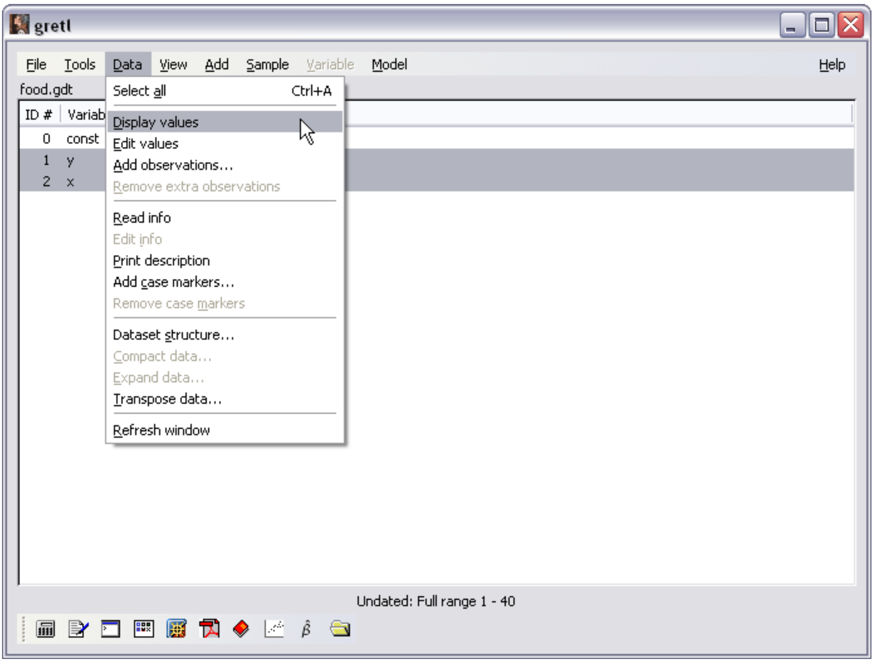
Figure 1.5: This is **gretl**'s data files window. Notice that in addition to POE, data sets from Ramanathan (2002), Greene (2003), Stock and Watson (2006) are installed on my system.



File	Summary
fair	Exercise 2.14, 3.9, 4.11, 5.11, 6.16, 7.15
FigureC-3	FigureC-3
florida	Exercise 4.16
food	Chapters 2,3,8; Exercises 8.10, 8.11
fullmoon	Exercise 7.2
fultonfish	Chapter 11, Exercises 11.11, 11.12, C.11
gascar	Exercise 15.5
gasga	Exercise 8.16
gdp	Chapter 13; Exercise 13.4
gold	Exercise 14.9
golf	Exercise 7.12
growth	Chapter 13; Exercise 13.5
grunfeld	Chapter 15, Exercise 15.8
grunfeld2	Chapter 15

Click on the **POE** tab and scroll down to find the data set called 'food', highlight it using the cursor, and open it using the 'open' button at the top of the window. This will bring the variables of the food expenditure dataset into **gretl**. At this point, select `Data` on the menu bar and then `Display values` as shown in Figure 1.6.

Figure 1.6: Use the `Data>Display values>all variables` to list the data set.



From the this pull-down menu a lot can be accomplished. You can edit, add observations, and impose a **structure** of your dataset. The structure of your dataset is important. You can choose between time series, cross sections, or panel data structures. The options **Gretl** gives you depend on this structure. For instance, if your data are structured as a time series, **gretl** will allow you to take lags and differences of the variables. Certain procedures that can be used for time series analysis will only be available to you if your dataset has been structured structured for it. If a **gretl** command is not available from the defined dataset structure, then it will be greyed out in the pull-down menus.

Notice in Figure 1.4 that **gretl** gives you the opportunity to **import** data. Expanding this (`File>Open data>Import`) gives you access to several other formats, including ASCII, CSV, EXCEL and others. Also, from the `File` pull-down menu you can export a data set to another format. If you click on `File>Databases>On database server` (Figure 1.4) you will be taken to a web site (provided your computer is connected to the internet) that contains a number of high quality data sets. You can pull any of these data sets into **gretl** in the same manner as that described above for the *POE* data sets. If you are required to write a term paper in one of your classes, these data sets may provide you with all the data that you need.

1.3 Using the gretl Language

The **gretl** GUI is certainly easy to use. However, you can get results even faster by using **gretl**'s language. The language can be used from the **console** or by collecting several lines of programming code into a file and executing them all at once in a **script**.

An important fact to keep in mind when using **gretl** is that its language is **case sensitive**. This means that lower case and capital letters have different meanings in **gretl**. The practical implication of this is that you need to be very careful when using the language. Since **gretl** considers x to be different from X, it is easy to make programming errors. If **gretl** gives you a programming error statement that you can't quite decipher, make sure that the variable or command you are using is in the proper case.

1.3.1 Console

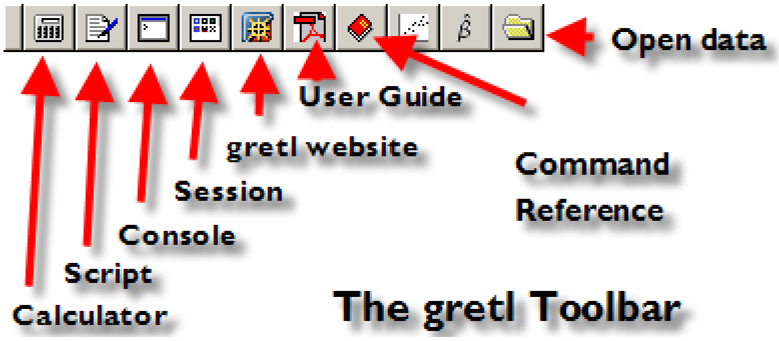
Gretl's console provides you a way to execute programs interactively. A console window opens and from the prompt (?) you can execute **gretl** commands one line at a time. You can open the **gretl console** from the `Tools` pull-down menu or by a left mouse click on the "**Gretl** console" button on the toolbar. This button is the third one on the left side of the toolbar in Figure 1.3. From the console you execute commands, one by one by typing **gretl** code after the command prompt. Each command that you type in is held in memory so that you can accumulate what amounts to a "command history." To reuse a command, simply use the up arrow key to scroll through the commands you've typed in until you get to the one you want. You can edit the command to fix any syntax errors or to make any changes you desire before hitting the enter key to execute the statement.

From the command prompt, '?' you can type in commands from the **gretl** language. For instance, to estimate the food expenditure model in section 2.4 using least squares type

```
? ols y const x
```

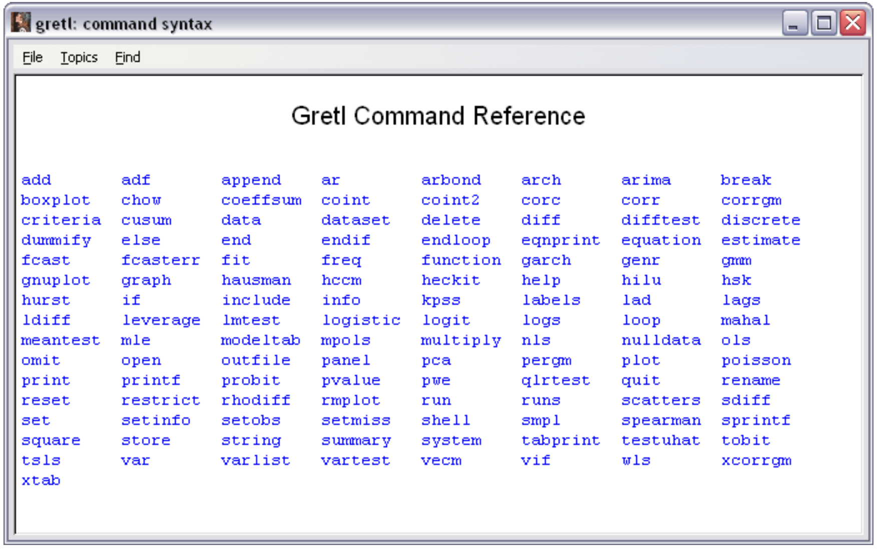
The results will be output to the console window. You can use the window's scroll bar on the right hand side to scroll up if you need to.

Remember, (almost) anything that can be done with the pull-down menus can also be done through the console. Of course, using the console requires you to use the correct language syntax, which can be found in the **gretl** command reference. The command reference can be accessed from the toolbar by clicking the button that looks like a red book. It's the fourth one from the right hand side of the toolbar:



. It is also accessible from the menu bar through Help. The option marked `plain text F1` actually brings up all of the commands in a hypertext format. Clicking on anything in blue will take you to the desired information for that command. Obviously, the keyboard shortcut F1 will also bring up the command reference (Figure 1.7). Notice that you can also search for commands by topic from the command syntax window. Select Topics and choose the desired category from the list. This can be helpful because it narrows the list to those things that you actually want (e.g., `Topics>Estimation>ols`).

Figure 1.7: The command reference can be accessed in a number of ways: The 'red book' icon on the toolbar, `Help>Command reference>plain text` from the pull-down menu, or keyboard shortcut F1.



Gretl Command Reference

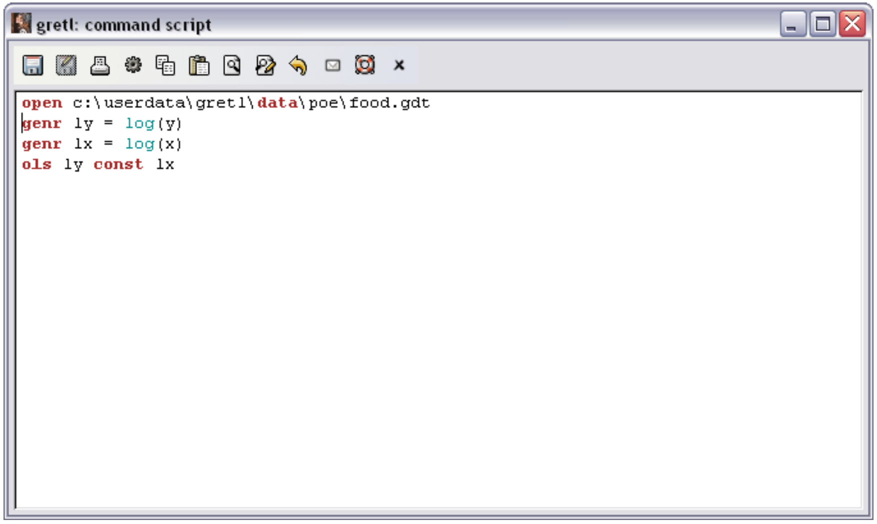
```
add       adf       append    ar        arbond    arch      arima     break
boxplot   chow      coeffsum  coint     coint2    corc      corr      corrgm
criteria  cusum     data      dataset   delete    diff      difftest  discrete
dummify   else      end       endif     endloop   eqnprint  equation  estimate
fcast     fcasterr  fit       freq      function  garch     genr      gmm
gnuplot   graph     hausman   hccm      heckit    help      hilu      hsk
hurst     if        include   info      kpss      labels    lad       lags
ldiff     leverage  lmtest    logistic  logit     logs      loop      mahal
meantest  mle       modeltab  mpols     multiply  nls       nulldata  ols
omit      open      outfile   panel     pca       pergm     plot      poisson
print     printf    probit    pvalue    rmplot    pwe       quit      rename
reset     restrict  rhodiff   rmplot    run       runs      scatters  sdiff
set       setinfo   setobs    setmiss   shell     smpl      spearman  sprintf
square    store     string    summary   system    tabprint  testuhat  tobit
tsls      var       varlist   vartest   vecm      vif       wls       xcorrgm
xtab
```

1.3.2 Scripts

Gretl commands can be collected and put into a file that can be executed at once and saved to be used again. This is accomplished by opening a new **command script** from the file menu. The command `File>Script files>New script` from the pull-down menu opens the command script editor shown in Figure 1.8. Type the commands you want to execute in the box using one line for each command. If you have a very long command that exceeds one line, use the backslash (\) as a **continuation command**. Then, to save the file, use the "save" button at the top of the box (first one from the left). If this is a new file, you'll be prompted to provide a name for it.

To run the program, click your mouse on the "gear" button. In the figure shown, the `food.gdt` **gretl** data file is opened. The `genr` commands are used to take the logarithm of y and x, and the `ols` command discussed in section 2.4 is used to estimate a simple linear regression model that has $ln(y)$ as its dependent variable and $ln(x)$ as the independent variable. Note, the model also includes constant.

Figure 1.8: The Command Script editor is used to collect a series of commands into what **gretl** calls a **script**. The script can be executed as a block, saved, and rerun at a later time.



```
open c:\userdata\gretl\data\poe\food.gdt
genr ly = log(y)
genr lx = log(x)
ols ly const lx
```

A new script file can also be opened from the toolbar by mouse clicking on the “new script” button or by using the keyboard command, Ctrl+N.[1]

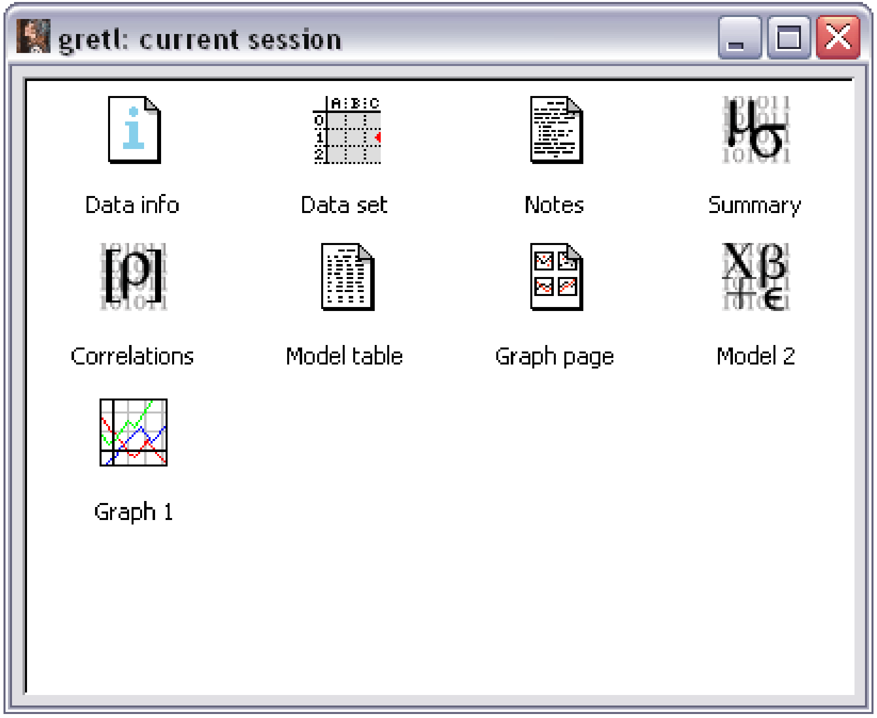
One of the handy features of the command script window is how the help function operates. At the top of the window there is an icon that looks like a lifesaver . Click on the help button and the cursor changes into a question mark. Move the question mark over the command you want help with and click. Voila! You either get an error message or you are taken to the topic from the command reference. Slick!

1.3.3 Sessions

Gretl also has a “session” concept that allows you to save models, graphs, and data files into a common “iconic” space. The session window appears below in Figure 1.9. Objects are represented as icons and these objects can be saved for later use. When you save your session, the objects

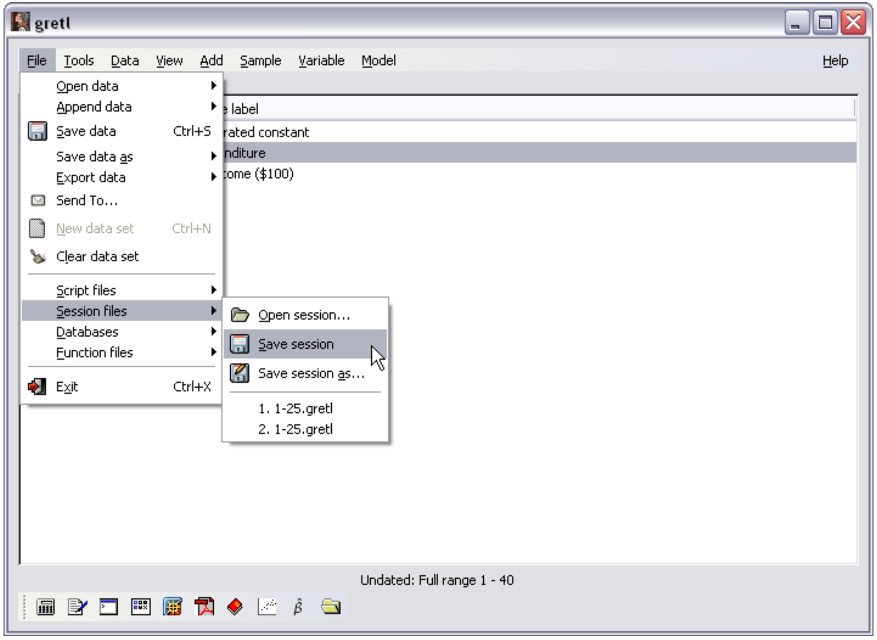
[1] “Ctrl+N” means press the “Ctrl” key and, while holding it down, press “N”.

Figure 1.9: The session window



you have added should be available again when you re-open the session. To add a model to your session, use the `File>Save to session as icon` option from the model's pull-down menu. To add a graph, right click on the graph and choose the option `save to session as icon`. If you want to save the results in your session, don't forget to do so; right click on the session window and choose `Save session` or from the main **gretl** window, select `File>Session files>Save session` as shown below in Figure 1.10.

Figure 1.10: Saving a session



Gretl also collects all of the commands you've executed via the GUI in the icon labeled 'session.' This makes it very easy to use the GUI to execute unfamiliar commands and then use the code generated by **gretl** to put into a script.

Once a model or graph is added, its icon will appear in the **session icon view** window. Double-clicking on the icon displays the object, while right-clicking brings up a menu which lets you display or delete the object. You can browse the dataset, look at summary statistics and correlations, and save and revisit estimation results (Models) and graphs.

The model table is a way of combining several estimated models into a single table. This is very useful for model comparison. From page 16 of the **gretl** manual ((Cottrell and Lucchetti, 2007)):

> In econometric research it is common to estimate several models with a common dependent variable the models contain different independent variables or are estimated using different estimators. In this situation it is convenient to present the regression results in the form of a table, where each column contains the results (coefficient estimates and standard errors) for a given model, and each row contains the estimates for a given variable across the models.
>
> In the Icon view window **gretl** provides a means of constructing such a table (and copying it in plain text, LATEX or Rich Text Format). Here is how to do it:
>
> 1. Estimate a model which you wish to include in the table, and in the model display window, under the File menu, select `Save to session as icon` or `Save as icon and close`.
> 2. Repeat step 1 for the other models to be included in the table (up to a total of six models).
> 3. When you are done estimating the models, open the icon view of your **gretl** session, by selecting `Icon view` under the View menu in the main **gretl** window, or by clicking the `session icon view` icon on the **gretl** toolbar.
> 4. In the Icon view, there is an icon labeled `Model table`. Decide which model you wish to appear in the left-most column of the model table and add it to the table, either by dragging its icon onto the Model table icon, or by right-clicking on the model icon and selecting `Add to model table` from the pop-up menu.
> 5. Repeat step 4 for the other models you wish to include in the table. The second model selected will appear in the second column from the left, and so on.
> 6. When you are finished composing the model table, display it by double-clicking on its icon. Under the Edit menu in the window which appears, you have the option of copying the table to the clipboard in various formats.

7. If the ordering of the models in the table is not what you wanted, right-click on the model table icon and select `Clear table`. Then go back to step 4 above and try again.

In section 6.5 you'll find an example that uses the model table and a Figure (6.13) that illustrates the result.

Chapter 2

Simple Linear Regression

In this chapter you are introduced to the simple linear regression model, which is estimated using the principle of least squares.

2.1 Simple Linear Regression Model

The simple linear regression model is

$$y_t = \beta_1 + \beta_2 x_t + e_t \quad t = 1, 2, \ldots, T \tag{2.1}$$

where y_t is your dependent variable, x_t is the independent variable, e_t is random error, and β_1 and β_2 are the parameters you want to estimate. The errors of the model, e_t, have an average value of zero for each value of x_t; each has the same variance, σ^2, and are uncorrelated with one another. The independent variable, x_t, has to take on at least two different values in your dataset. If not, you won't be able to estimate a slope! The error assumptions can be summarized as $e_t|x_t$ *iid* $N(0, \sigma^2)$. The expression *iid* stands for *independently and identically distributed* and means that the errors are statistically independent from one another (and therefor uncorrelated) and that each has the same probability distribution. Taking a random sample from a single population accomplishes this.

2.2 Retrieve the Data

The first step is to load the food expenditure and income data into **gretl**. The data file is included in your **gretl** sample files–provided that you have installed the *Principles of Econometrics* data supplement that is available from our website. See section 1.1.1 for details.

Figure 2.1: Food Expenditure data is loaded from food.gdt using `File>Open data>sample file` and choosing the food dataset from the sample files that accompany *POE*.

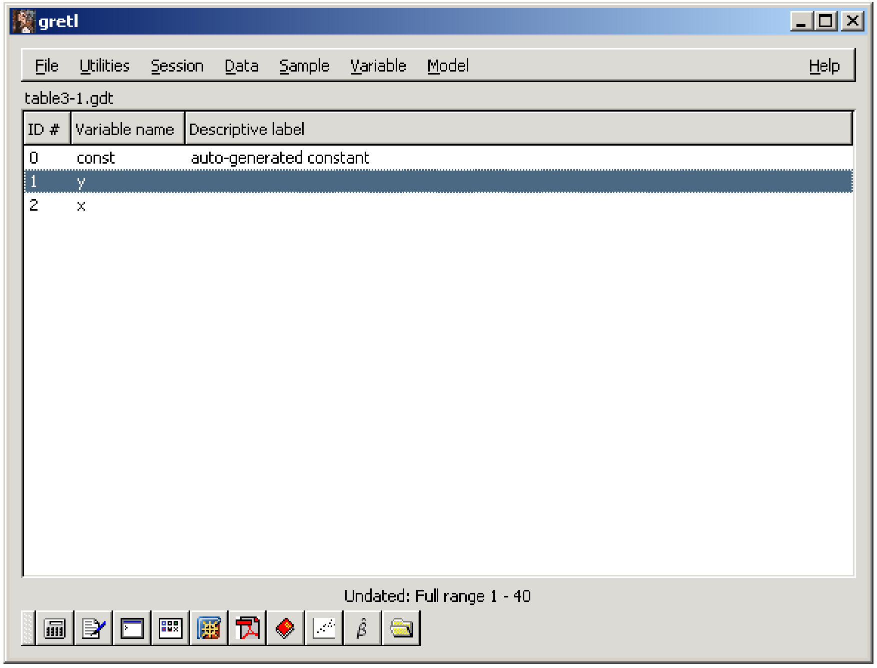


table3-1.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	y	
2	x	

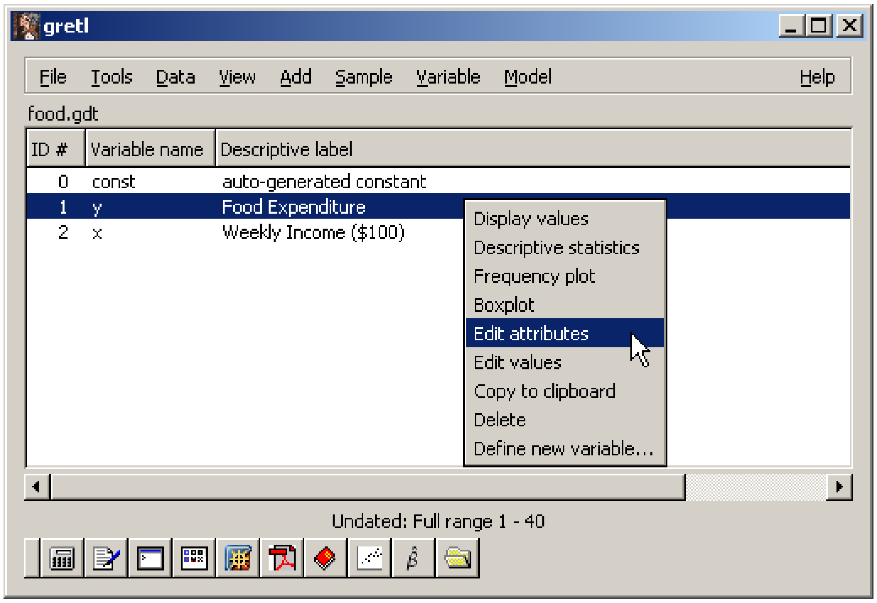
Undated: Full range 1 - 40

Load the data from the data file *food.gdt*. Recall, this is accomplished by the commands `File>Open data>sample file` from the menu bar.[1] Choose *food* from the list. When you bring the file containing the data into **gretl** your window will look like the one in Figure 2.1. Notice that in the `Descriptive label` column contains some information about the variables in the program's memory. For some of the datasets included with this book, it may be blank. These descriptions, when they exist, are used by the graphing

[1] Alternately, you could click on the open data button on the toolbar. It's the one that looks like a folder on the far right-hand side.

program to label your output and to help you keep track of variables that are available for use. Before you graph your output or generate results for a report or paper you may want to label your variables to make the output easier to understand. This can be accomplished by editing the attributes of the variables.

Figure 2.2: Highlight the desired variable and right-click to bring up the pull-down menu shown here.



food.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	y	Food Expenditure
2	x	Weekly Income ($100)

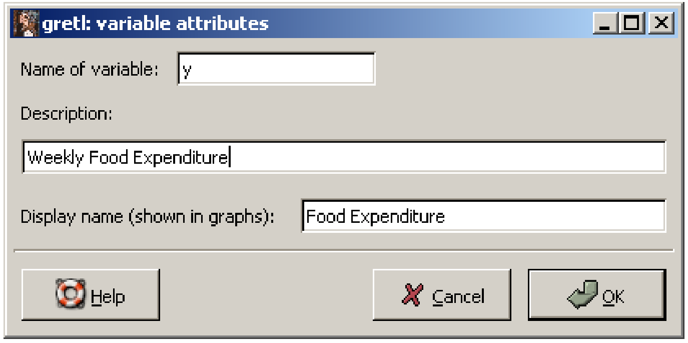
Undated: Full range 1 - 40

To do this, first highlight the variable whose attributes you want to edit, right-click and the menu shown in (see Figure 2.2) appears. Select Edit attributes to open a dialog box (Figure 2.3) where you can change the variable's name, assign variable descriptions and display names. Describe and label the variable y as 'Food Expenditure' and x as 'Weekly Income ($100).'

You can also bring up the edit attributes dialog from the main window by selecting `Variable>Edit attributes`. Finally, the `setinfo` command can be used to set the Description and the label used in graphs. For *andy.gdt* the commands to add such information to the data are:

```
setinfo S -d "Monthly Sales revenue ($1000)" \
        -n "Monthly Sales ($1000)"
```

Figure 2.3: Variable edit dialog box

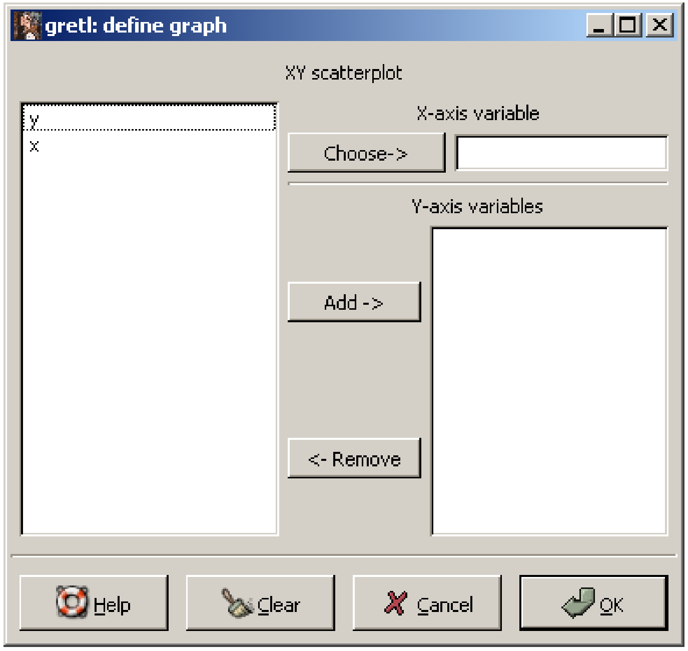


Name of variable: y

Description: Weekly Food Expenditure

Display name (shown in graphs): Food Expenditure

Figure 2.4: Use the dialog to plot of the Food Expenditure (y) against Weekly Income (x)



```
setinfo P -d "$ Price" -n "Price"
setinfo A -d "Monthy Advertising Expenditure ($1000)" \
        -n "Monthly Advertising ($1000)"
labels
```

The `-d` flag is given followed by a string in double quotes. It is used to set the descriptive label. The `-n` flag is used similarly to set the variable's name in graphs. Notice that in the first and last uses of `setinfo` in the example that I have issued the continuation command (\) since these commands are too long to fit on a single line. If you issue the `labels` command, **gretl** will respond by printing the descriptions to the screen.

2.3 Graph the Data

To generate a graph of the Food Expenditure data that resembles the one in Figure 2.6 of *POE*, you can use the button on the **gretl** toolbar (third button from the right). Clicking this button brings up a dialog to plot the two variables against one another. Figure 2.4 shows this dialog where x is placed on the x-axis and y on the y-axis. The result appears in Figure 2.5. Notice that the labels applied above now appear on the axes of the graph.

Figure 2.5 plots Food Expenditures on the y axis and Weekly Income on the x. **Gretl**, by default, also plots the fitted regression line. The benefits of assigning labels to the variables becomes more obvious. Both X- and Y-axes are informatively labeled and the graph title is changed as well. More on this later.

2.4 Estimate the Food Expenditure Relationship

Now you are ready to use **gretl** to estimate the parameters of the Food Expenditure equation.

$$y_t = \beta_1 + \beta_2 x_t + e_t \tag{2.2}$$

From the menu bar, select `Model>Ordinary Least Squares` from the pull-down menu (see Figure 2.6) to open the dialog box shown in Figure 2.7.

Figure 2.5: XY plot of the Food Expenditure data

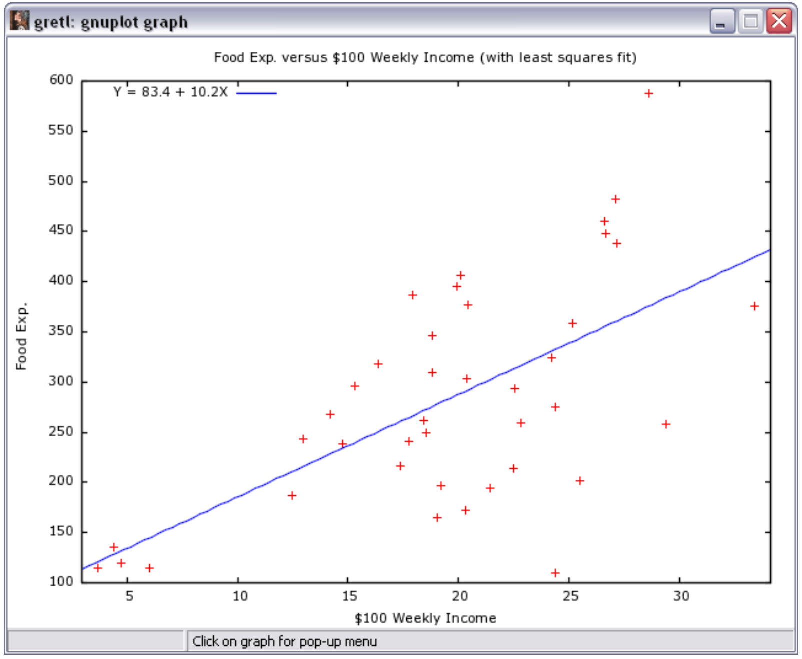
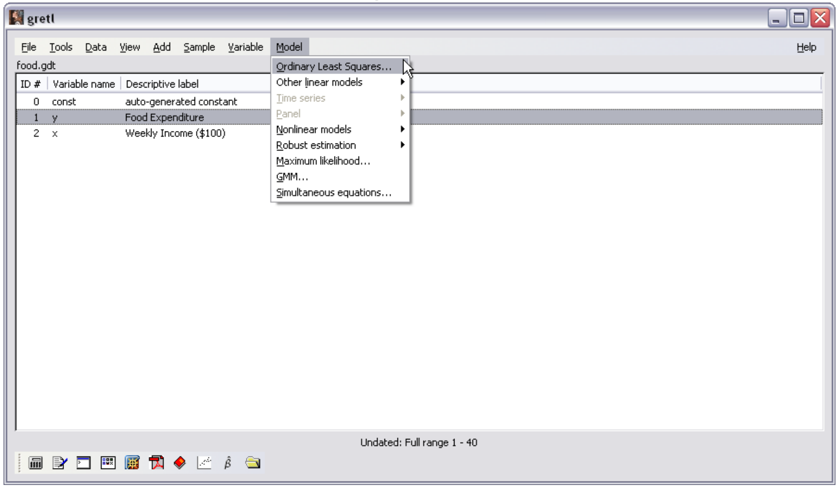


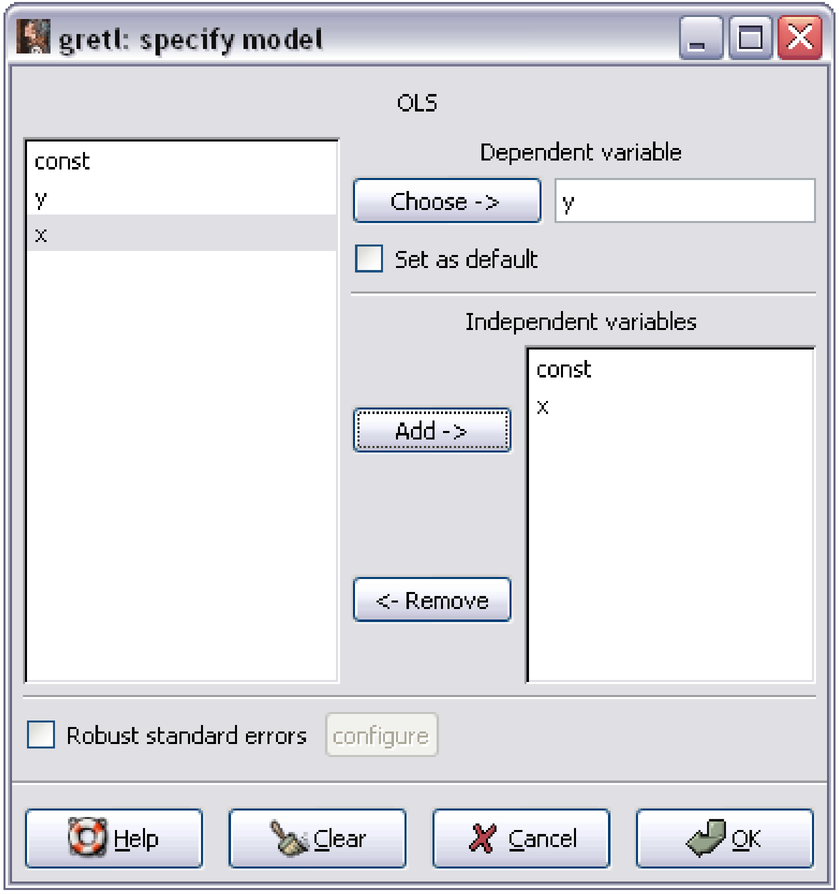
Figure 2.6: From the menu bar, select `Model>Ordinary Least Squares to open the least squares dialog box`



ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	y	Food Expenditure
2	x	Weekly Income ($100)

Undated: Full range 1 - 40

Figure 2.7: The Specify Model dialog box opens when you select `Model>Ordinary least squares`



gretl: specify model

OLS

const
y
x

Dependent variable: y

□ Set as default

Independent variables:

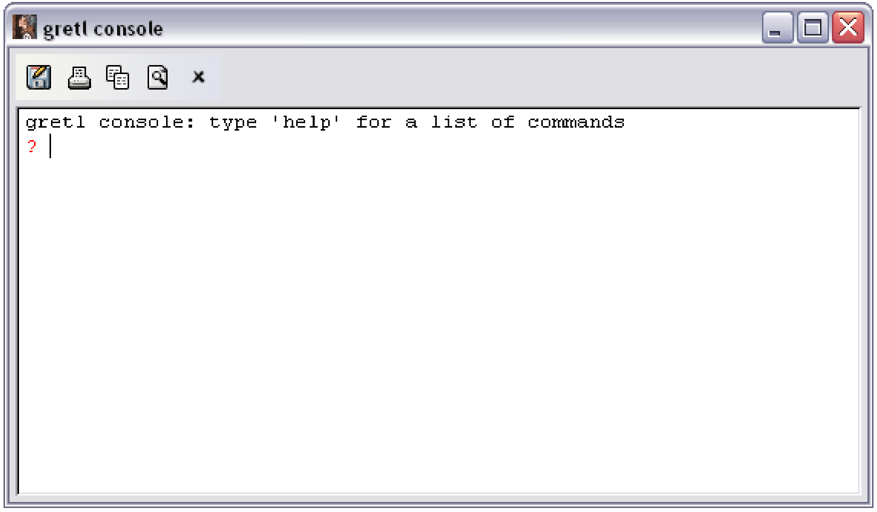
const
x

□ Robust standard errors

From this dialog you'll need to tell **gretl** which variable to use as the dependent variable and which is the independent variable. Notice that by default, **gretl** assumes that you want to estimate an intercept (β_1) and includes a constant as an independent variable by placing the variable `const` in the list by default. To include x as an independent variable, highlight it with the cursor and click the '`Add->`' button.

The **gretl** console (see section 1.3.1) provides an easy way to run a regression. The **gretl** console is opened by clicking the console button on the toolbar, ▭. The resulting console window is shown in Figure 2.8.

Figure 2.8: The Gretl console window. From this window you can type in **gretl** commands directly and perform analyses very quickly–if you know the proper **gretl** commands.



```
gretl console: type 'help' for a list of commands
?
```

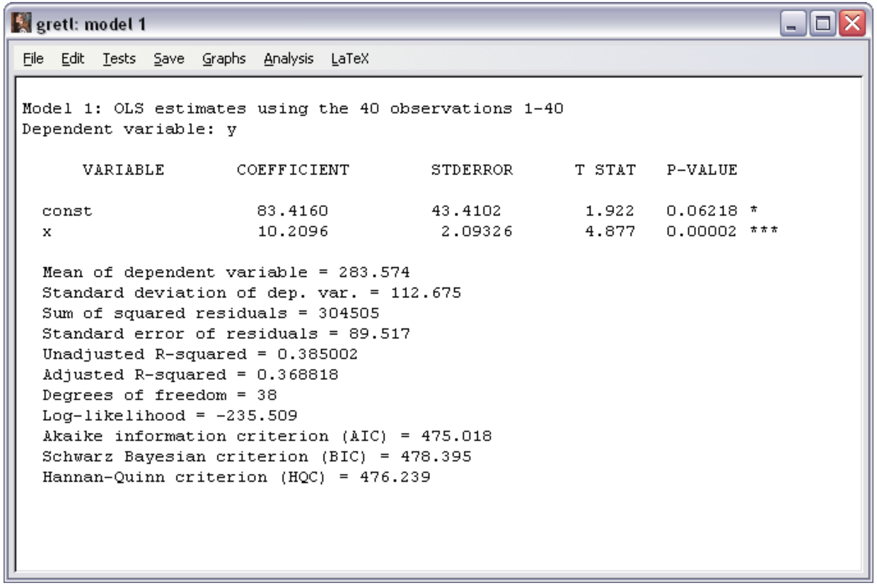
At the question mark in the console simply type

```
ols y const x
```

to estimate your regression function. The syntax is very simple, `ols` tells **gretl** that you want to estimate a linear function using ordinary least squares. The first variable listed will be your dependent variable and any that follow, the independent variables. These names must match the ones used in your data set. Since ours in the food expenditure example are named, y and x, respectively, these are the names used here. Don't forget to estimate an intercept by adding a constant (const) to the list of regressors. Also, don't forget that **gretl** is case sensitive so that x and X are different entities.

This yields window shown in Figure 2.9 below. The results are summarized in Table 2.1.

Figure 2.9: The model window appears with the regression results. From here you can conduct subsequent operations (graphs, tests, analysis, etc.) on the estimated model.



```
Model 1: OLS estimates using the 40 observations 1-40
Dependent variable: y

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                83.4160          43.4102        1.922   0.06218 *
  x                    10.2096           2.09326       4.877   0.00002 ***

  Mean of dependent variable = 283.574
  Standard deviation of dep. var. = 112.675
  Sum of squared residuals = 304505
  Standard error of residuals = 89.517
  Unadjusted R-squared = 0.385002
  Adjusted R-squared = 0.368818
  Degrees of freedom = 38
  Log-likelihood = -235.509
  Akaike information criterion (AIC) = 475.018
  Schwarz Bayesian criterion (BIC) = 478.395
  Hannan-Quinn criterion (HQC) = 476.239
```

An equivalent way to present results, especially in very small models like the simple linear regression, is to use equation form. In this format, the **gretl** results are:

$$\widehat{\mathrm{y}} = \underset{(1.922)}{83.4160} + \underset{(4.877)}{10.2096}\,\mathrm{x}$$

$$T = 40 \quad \bar{R}^2 = 0.3688 \quad F(1,38) = 23.789 \quad \hat{\sigma} = 89.517$$

$$\text{(}t\text{-statistics in parentheses)}$$

Finally, notice in the main **gretl** window (Figure 2.1) that the first column has a heading called *ID #*. An ID # is assigned to each variable in memory

Table 2.1: OLS estimates using the 40 observations 1–40.

Dependent variable: y

Variable	Coefficient	Std. Error	t-statistic	p-value
const	83.4160	43.4102	1.9216	0.0622
x	10.2096	2.09326	4.8774	0.0000

Mean of dependent variable	283.574
S.D. of dependent variable	112.675
Sum of squared residuals	304505.
Standard error of residuals ($\hat{\sigma}$)	89.5170
Unadjusted R^2	0.385002
Adjusted $\bar{R}^2$	0.368818
Degrees of freedom	38
Akaike information criterion	475.018
Schwarz Bayesian criterion	478.395

and you can use these ID #s instead of variable names in your programs. For instance, the following two lines yield identical results:

```
ols y const x
ols 1 0 2
```

One (1) is the ID number for y and two (2) is the ID number of x. The constant has ID zero (0). If you tend to use long and descriptive variable names (recommended, by the way), using the ID number can save you a lot of typing (and some mistakes).

2.4.1 Elasticity

Elasticity is an important concept in economics. It measures how responsiveness one variable is to changes in another. Mathematically, the concept of elasticity is fairly simple:

$$\varepsilon = \frac{\text{percentage change in } y}{\text{percentage change in } x} = \frac{\Delta y/y}{\Delta x/x} \tag{2.3}$$

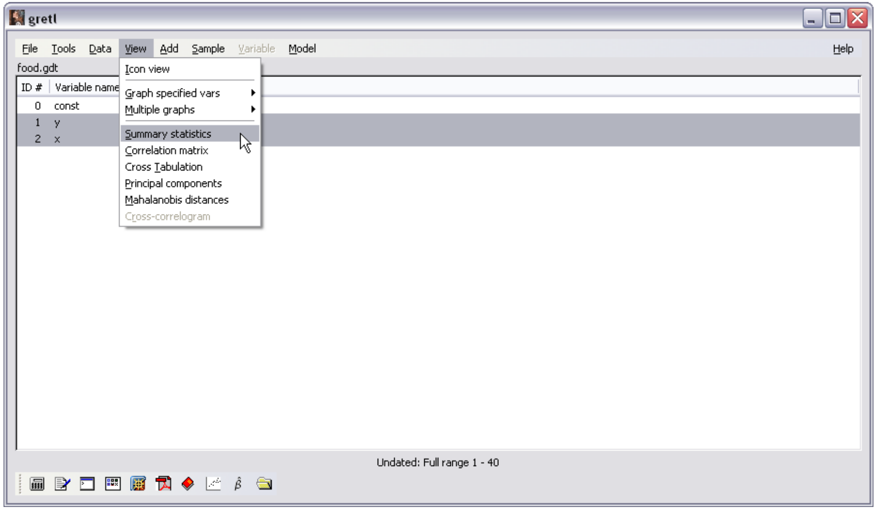
In terms of the regression function, we are interested in the elasticity of average food expenditures with respect to changes in income:

$$\varepsilon = \frac{\Delta E(y)/E(y)}{\Delta x/x} = \beta_2 \frac{x}{E(y)}. \tag{2.4}$$

$E(y)$ and x are usually replaced by their sample means and β_2 by its estimate. The mean of x and y can be obtained by using the cursor to highlight both variables, use the `View>Summary statistics` from the menu bar as shown in Figure 2.10, and the computation can be done by hand. However, you can make this even easier by using the **gretl** language to do all of the computations–no calculator needed! Simply open up a new script and type in:

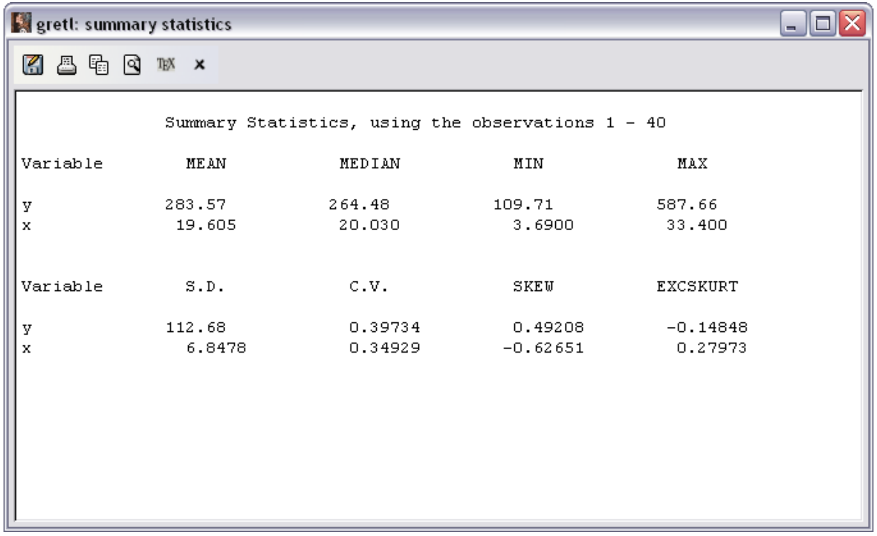
```
genr elast=$coeff(x)*mean(x)/mean(y)
```

Figure 2.10: Using the GUI to obtain summary statistics. Highlight the desired variables and use `View>Summary statistics` from the pull-down menu.



This yields the output shown in the next figure 2.11:

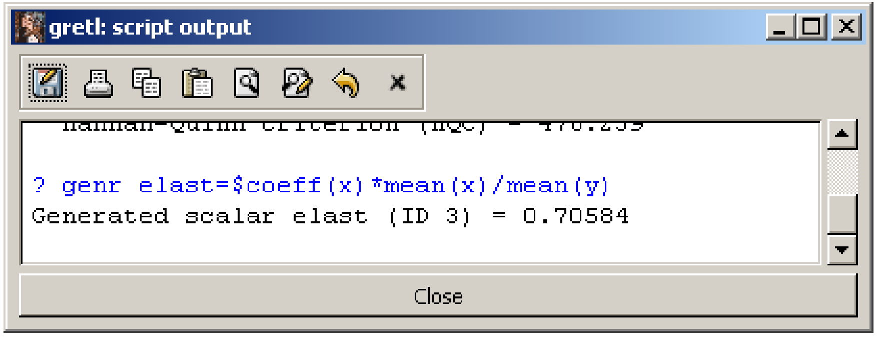
Figure 2.11: Summary statistics



```
               Summary Statistics, using the observations 1 - 40

Variable          MEAN            MEDIAN            MIN             MAX

y              283.57          264.48          109.71          587.66
x               19.605          20.030           3.6900          33.400


Variable          S.D.            C.V.              SKEW            EXCSKURT

y              112.68            0.39734          0.49208         -0.14848
x                6.8478          0.34929         -0.62651          0.27973
```

Following a least squares regression, **Gretl** stores the least squares estimates of the constant and the slope in variables called `$coeff(const)` and `$coeff(x)`, respectively. In addition, it uses `mean(x)` and `mean(y)`to compute the mean of the variables x and y. The result from this computation appears below in Figure 2.12.

Figure 2.12: Results from the script to compute an elasticity based on a linear regression.



```
  Hannan-Quinn criterion (HQC) = 470.2[illegible]

? genr elast=$coeff(x)*mean(x)/mean(y)
Generated scalar elast (ID 3) = 0.70584
```

2.4.2 Prediction

Similarly, **gretl** can be used to produce predictions. The predicted food expenditure of an average household having weekly income of $2000 is:

$$\hat{y}_t = 83.42 + 10.21x_t = 83.42 + 10.21(20) = 287.61 \tag{2.5}$$

Remember, income is measured in $100, so 20 in the above expression represents 20*$100=$2,000. The **gretl** script is:

```
genr yhat = $coeff(const) + $coeff(x)*20
```

which yields the desired result.

2.4.3 Estimating Variance

In section 2.7 of your textbook, you are given expressions for the variances of the least squares estimators of the intercept and slope as well as their covariance. These estimators require that you estimate the overall variance of the model's errors, σ^2. **Gretl** does not explicitly report the estimator, $\hat{\sigma}^2$, but rather, its square root, $\hat{\sigma}$. **Gretl** calls this "Standard error of residuals" which you can see from the output is 89.517. Thus, $89.517^2 = 8013.29$. **Gretl** also reports the sum of squared residuals, equal to 304505, from which you can calculate the estimate. Dividing the sum of squared residuals by the estimator's degrees of freedom yields $\hat{\sigma}^2 = 304505/38 = 8013.29$.

The estimated variances and covariance of the least squares estimator can be obtained once the model is estimated by least squares by selecting the `Analysis>Coefficient covariance matrix` command from the pull-down menu of the **model** window as shown in Figure 2.13. The result is:

```
Covariance matrix of regression coefficients

         const             x
       1884.44      -85.9032  const
                     4.38175  x
```

So, estimated variances of the least squares estimator of the intercept and slope are 1884.44 and 4.38175, respectively. The least squares standard errors are simply the square roots of these numbers. The estimated covariance between the slope and intercept -85.9032.

You can also obtain the variance-covariance matrix by specifying the `--vcv` option when estimating a regression model. For the food expenditure example use:

```
ols y const x --vcv
```

to estimate the model using least squares and to print the variance covariance matrix to the results window.

2.5 Repeated Sampling

Perhaps the best way to illustrate the sampling properties of least squares is through an experiment. In section 2.4.3 of your book you are presented with results from 10 different regressions (POE Table 2.2). These were obtained using the dataset *table2-2.gdt* which is included in the **gretl** datasets that accompany this manual. To reproduce the results in this table estimate 10 separate regressions

```
ols y1 const x
ols y2 const x
.
.
.
ols y10 const x
```

You can also generate your own random samples and conduct a Monte Carlo experiment using **gretl**. In this exercise you will generate 100 samples of data from the food expenditure data, estimate the slope and intercept parameters with each data set, and then study how the least squares estimator performed over those 100 different samples. What will become clear is this, the outcome from any single sample is a poor indicator of the true value of the parameters. Keep this humbling thought in mind whenever you estimate a model with what is invariably only 1 sample or instance of the true (but always unknown) data generation process.

We start with the food expenditure model:

$$y_t = \beta_1 + \beta_2 x_t + e_t \tag{2.6}$$

where y_t is total food expenditure for the given time period and x_t is income. Suppose further that we know how much income each of 40 households earns in a week. Additionally, we know that on average a household spends at least \$80 on food whether it has income or not and that an average household will spend ten cents of each new dollar of income on additional food. In terms of the regression this translates into parameter values of $\beta_1 = 80$ and $\beta_2 = 10$.

Our knowledge of any particular household is considerably less. We don't know how much it actually spends on food in any given week and, other than differences based on income, we don't know how its food expenditures might otherwise differ. Food expenditures are sure to vary for reasons other than differences in family income. Some families are larger than others, tastes and preferences differ, and some may travel more often or farther making food consumption more costly. For whatever reasons, it is impossible for us to know beforehand exactly how much any household will spend on food, even if we know how much income it earns. All of this uncertainty is captured by the error term in the model. For the sake of experimentation, suppose we also know that $e_t \sim N(0, 88^2)$.

With this knowledge, we can study the properties of the least squares estimator by generating samples of size 40 using the known data generation mechanism. We generate 100 samples using the known parameter values, estimate the model for each using least squares, and then use summary statistics to determine whether least squares, on average anyway, is either very accurate or precise. So in this instance, we know how much each household earns, how much the **average** household spends on food that is not related to income ($\beta_1 = 80$), and how much that expenditure rises **on average** as income rises. What we do not know is how any **particular** household's expenditures responds to income or how much is autonomous.

A single sample can be generated in the following way. The systematic component of food expenditure for the t^{th} household is $80 + 10 * x_t$. This differs from its actual food expenditure by a random amount that varies according to a normal distribution having zero mean and standard deviation equal to 88. So, we use computer generated random numbers to generate a random error, u_t, from that particular distribution. We repeat this for the remaining 39 individuals. The generates one Monte Carlo sample and it is then used to estimate the parameters of the model. The results are saved and then another Monte Carlo sample is generated and used to estimate the model and so on.

In this way, we can generate as many different samples of size 40 as we desire. Furthermore, since we know what the underlying parameters are for these samples, we can later see how close our estimators get to revealing these true values.

Now, computer generated sequences of random numbers are not actually random in the true sense of the word; they can be replicated exactly if you know the mathematical formula used to generate them and the 'key' that initiates the sequence. In most cases, these numbers *behave as if* they randomly generated by a physical process.

To conduct an experiment using least squares in **gretl** use the script found in Figure 2.14.

Let's look at what each line accomplishes. The first line

```
open c:\userdata\gretl\data\poe\food.gdt
```

opens the food expenditure data set that resides in the *poe* folder of the data directory. The next line, which is actually not necessary to do the experiments, estimates the model using the original data using `ols`. It is included here so that you can see how the results from the actual sample compare to those generated from the simulated samples. All of the remaining lines are used for the Monte Carlo.

In Monte Carlo experiments loops are used to estimate a model using many different samples that the experimenter generates and to collect the results. The loop construct in **gretl** begins with the command `loop NMC`

`--progressive` and ends with `endloop`. `NMC` in this case is the number of Monte Carlo samples you want to use and the option `--progressive` is a command that suppresses the individual output at each iteration from being printed and allows you to store the results in a file. So that you can reproduce the results below, I have also initiated the sequence of random numbers using a key, called the *seed*, with the command `set seed 3213798`. Basically, this ensures that the stream of pseudo random numbers will start at the same place each time you run your program. Try changing the value of the seed (3213798) or the number of Monte Carlo iterations (100) to see how your results are affected.

Within this loop construct, you tell **gretl** how to generate each sample and state how you want that sample to be used. The data generation is accomplished here as

```
genr u = 88*normal()
genr y1 = 80 + 10*x + u
```

The `genr` command is used to generate new variables. In the first line u is generated by multiplying a normal random variable by the desired standard deviation. Recall, that for any constant, c and random variable, X, $Var(cX) = c^2Var(X)$. The **gretl** command `normal()` produces a computer generated standard normal random variable. The next line adds this random element to the systematic portion of the model to generate a new sample for food expenditures (using the known values of income in x).

Next, the model is estimated using least squares. Then, the coefficients are stored internally in variables you create `b1` and `b2` (I called them b1 and b2, but you can name them as you like). These are then stored to a data set `coeff.gdt`.

After executing the script, **gretl** prints out some summary statistics to the screen. These appear below in Figure 2.15. Note that the average value of the intercept is about 78.1497. This is getting close to the truth. The average value of the slope is 10.0623, also reasonably close to the true value. If you were to repeat the experiments with larger numbers of Monte Carlo iterations, you will find that these averages get closer to the values of the parameters used to generate the data. This is what it means to be unbiased. Unbiasedness only has meaning within the context of repeated sampling. In your experiments, you generated many samples and averaged results over those samples to get closer to the truth. In actual practice, you do not have this luxury; you have one sample and the proximity of your estimates to the true values of the parameters is always unknown.

After executing the script, open the `coeff.gdt` data file that **gretl** has created and view the data. From the menu bar this is done using `File>Open data>user file` and selecting `coeff.gdt` from the list of available data sets. From the example this yields the output in Figure 2.16. Notice that even though the actual value of $\beta_1 = 80$ there is considerable variation in the estimates. In sample 18 it was estimated to be 138.0067 and in sample 1 it was -7.8842. Likewise, β_2 also varies around its true value of 10. Notice that for any given sample, an estimate is never equal to the true value of the parameter!

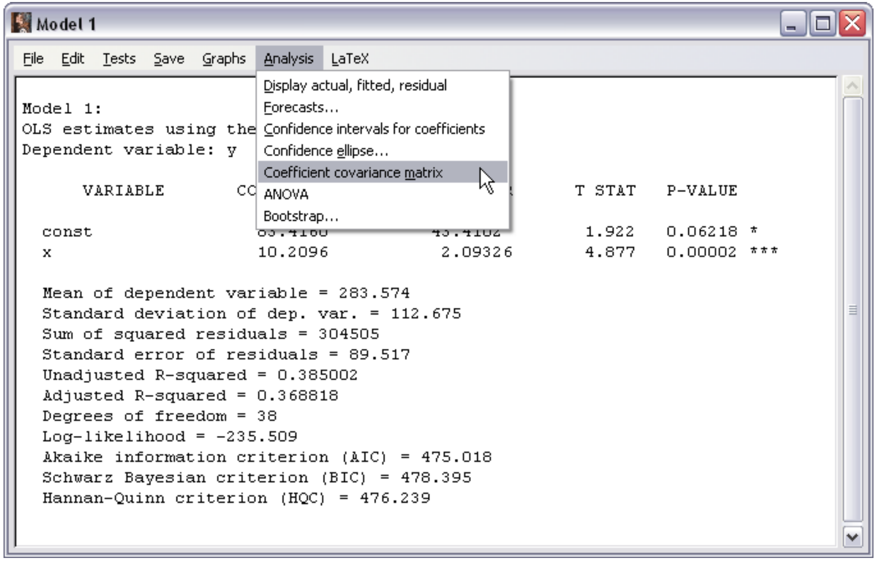
2.6 Script

The script for Chapter 2 is found below. These scripts can also be found at my website `http://www.learneconometrics.com/gretl`.

```
open c:\userdata\gretl\data\poe\food.gdt

#Least squares
ols y const x --vcv

#Summary Statistics
summary y x

#Plot the Data
gnuplot y x

#List the Data
print y x --byobs

#Elasticity
genr elast=$coeff(x)*mean(x)/mean(y)

#Prediction
genr yhat = $coeff(const) + $coeff(x)*20

#Table 2.2
open c:\userdata\gretl\data\poe\table2-2.gdt
ols y1 const x
ols y2 const x
ols y3 const x
ols y4 const x
ols y5 const x
ols y6 const x
ols y7 const x
ols y8 const x
ols y9 const x
ols y10 const x
```

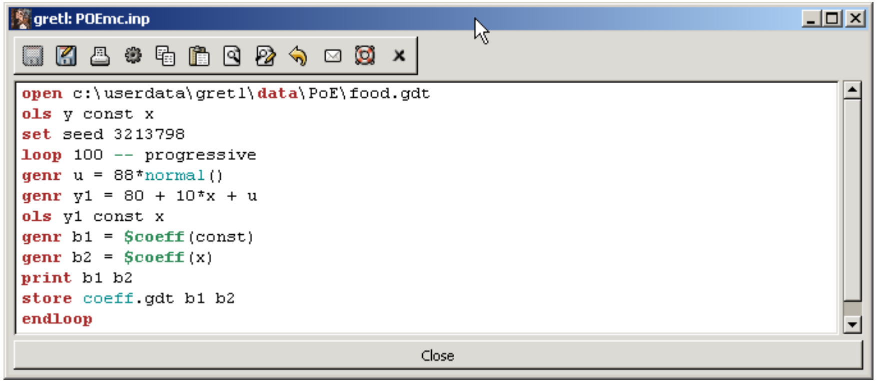
```
#Monte Carlo
open c:\userdata\gretl\data\poe\food.gdt

set seed 3213789
loop 100 -- progressive
genr u = 88*normal()
genr y1 = 80 + 10*x + u
ols y1 const x
genr b1 = $coeff(const)
genr b2 = $coeff(x)
print b1 b2
store coeff.gdt b1 b2
endloop
```

Figure 2.13: Obtain the matrix that contains the least squares estimates of variance and covariance from the pull-down menu of your estimated model.


Model 1
File
Edit
Tests
Save
Graphs
Analysis
LaTeX
Display actual, fitted, residual
Forecasts...
Confidence intervals for coefficients
Confidence ellipse...
Coefficient covariance matrix
ANOVA
Bootstrap...

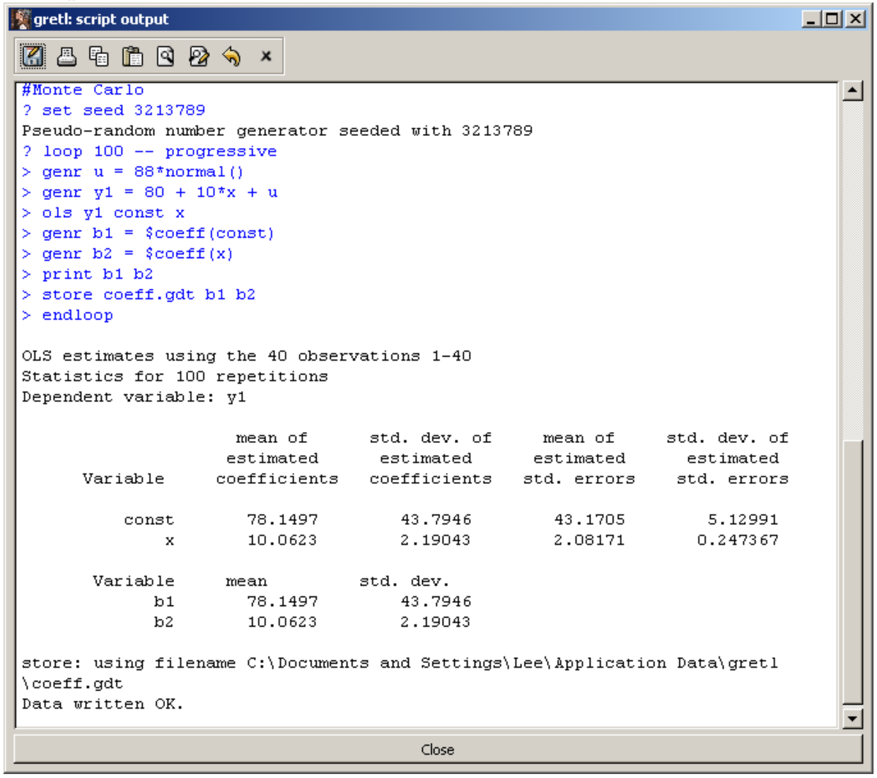
```
Model 1:
OLS estimates using the
Dependent variable: y

     VARIABLE       CC                            T STAT   P-VALUE

  const                 83.4160          43.4102       1.922   0.06218 *
  x                     10.2096           2.09326      4.877   0.00002 ***

  Mean of dependent variable = 283.574
  Standard deviation of dep. var. = 112.675
  Sum of squared residuals = 304505
  Standard error of residuals = 89.517
  Unadjusted R-squared = 0.385002
  Adjusted R-squared = 0.368818
  Degrees of freedom = 38
  Log-likelihood = -235.509
  Akaike information criterion (AIC) = 475.018
  Schwarz Bayesian criterion (BIC) = 478.395
  Hannan-Quinn criterion (HQC) = 476.239
```

Figure 2.14: In the **gretl** script window you can type in the following commands to execute a Monte Carlo study of least squares. Then to execute the program, click on the small gear icon.



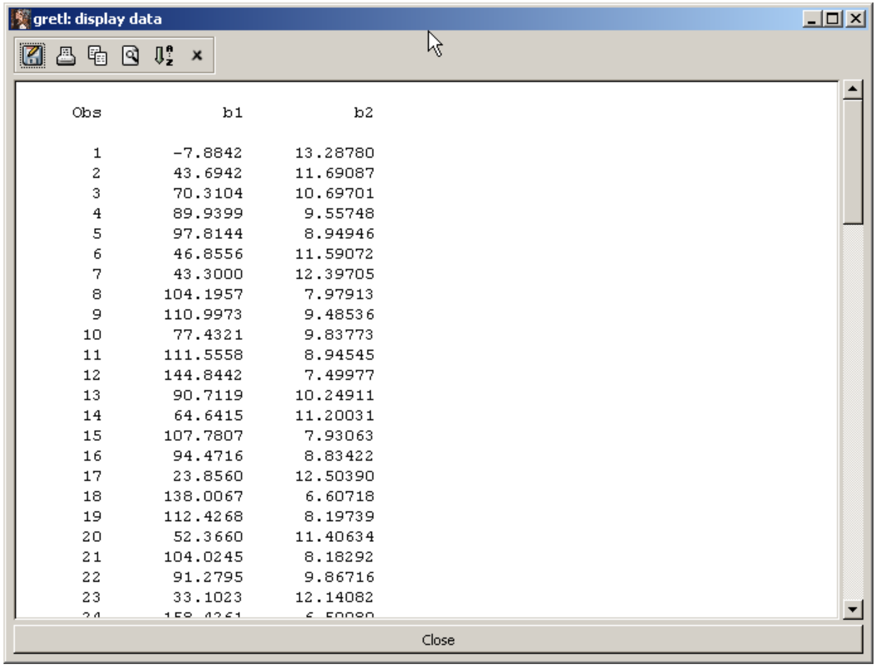
```
open c:\userdata\gretl\data\PoE\food.gdt
ols y const x
set seed 3213798
loop 100 -- progressive
genr u = 88*normal()
genr y1 = 80 + 10*x + u
ols y1 const x
genr b1 = $coeff(const)
genr b2 = $coeff(x)
print b1 b2
store coeff.gdt b1 b2
endloop
```

Figure 2.15: The summary results from 100 random samples of the Monte Carlo experiment.



```
#Monte Carlo
? set seed 3213789
Pseudo-random number generator seeded with 3213789
? loop 100 -- progressive
> genr u = 88*normal()
> genr y1 = 80 + 10*x + u
> ols y1 const x
> genr b1 = $coeff(const)
> genr b2 = $coeff(x)
> print b1 b2
> store coeff.gdt b1 b2
> endloop

OLS estimates using the 40 observations 1-40
Statistics for 100 repetitions
Dependent variable: y1

                     mean of      std. dev. of     mean of     std. dev. of
                    estimated      estimated      estimated     estimated
      Variable     coefficients   coefficients   std. errors   std. errors

          const       78.1497        43.7946        43.1705       5.12991
              x       10.0623        2.19043        2.08171      0.247367

       Variable     mean          std. dev.
             b1       78.1497        43.7946
             b2       10.0623        2.19043

store: using filename C:\Documents and Settings\Lee\Application Data\gretl
\coeff.gdt
Data written OK.
```

Figure 2.16: The results from the first 23 sets of estimates from the 100 random samples of the Monte Carlo experiment.



```
     Obs            b1            b2

       1       -7.8842      13.28780
       2       43.6942      11.69087
       3       70.3104      10.69701
       4       89.9399       9.55748
       5       97.8144       8.94946
       6       46.8556      11.59072
       7       43.3000      12.39705
       8      104.1957       7.97913
       9      110.9973       9.48536
      10       77.4321       9.83773
      11      111.5558       8.94545
      12      144.8442       7.49977
      13       90.7119      10.24911
      14       64.6415      11.20031
      15      107.7807       7.93063
      16       94.4716       8.83422
      17       23.8560      12.50390
      18      138.0067       6.60718
      19      112.4268       8.19739
      20       52.3660      11.40634
      21      104.0245       8.18292
      22       91.2795       9.86716
      23       33.1023      12.14082
      24      158.4261       6.50980
```

Chapter 3

Interval Estimation and Hypothesis Testing

In this chapter, I will discuss how to generate confidence intervals and test hypotheses using **gretl**. **Gretl** includes several handy utilities that will help you obtain critical values and p-values from several important probability distributions. As usual, you can use the dialog boxes or **gretl**'s programming language to do this.

3.1 Confidence Intervals

It is important to know how precise your knowledge of the parameters is. One way of doing this is to look at the least squares parameter estimate along with a measure of its precision, i.e., its estimated standard error.

The confidence interval serves a similar purpose, though it is much more straightforward to interpret because it gives you upper and lower bounds between which the unknown parameter will lie with a given probability.[1]

[1] This is probability in the frequency sense. Some authors fuss over the exact interpretation of a confidence interval (unnecessarily I think). You are often given stern warnings not to interpret a confidence interval as containing the unknown parameter with the given

In **gretl** you can obtain confidence intervals either through a dialog or by manually building them using saved regression results. In the 'manual' method I will use the `genr` command to generate upper and lower bounds based on regression results that are saved in **gretl**'s memory, letting **gretl** do the arithmetic. You can either look up the appropriate critical value from a table or use the **gretl**'s `critical` function. I'll show you both.

Here is how it works. Consider equation (3.5) from your text

$$P[b_2 - t_c se(b_2) \leq \beta_2 \leq b_2 + t_c se(b_2)] = 1 - \alpha \tag{3.1}$$

Recall that b_2 is the least squares estimator of β_2, and that $se(b_2)$ is its estimated standard error. The constant t_c is the $\alpha/2$ critical value from the t-distribution and α is the total desired probability associated with the "rejection" area (the area outside of the confidence interval).

You'll need to know t_c, which can be obtained from a statistical table, the `Tools>Statistical tables` dialog contained in the program, or using the **gretl** command `critical`. First, try using the dialog box shown in Figure 3.1. Pick the tab for the t distribution and tell **gretl** how much weight to put into the right-tail of the probability distribution and how many degrees of freedom your t-statistic has, in our case, 38. Once you do, click on OK. You'll get the result shown in Figure 3.2. It shows that for the t(38) with right-tail probability of 0.025 and two-tailed probability of 0.05, the critical value is 2.02439.[2] Then generate the lower and upper bounds (using the **gretl** console) with the commands:

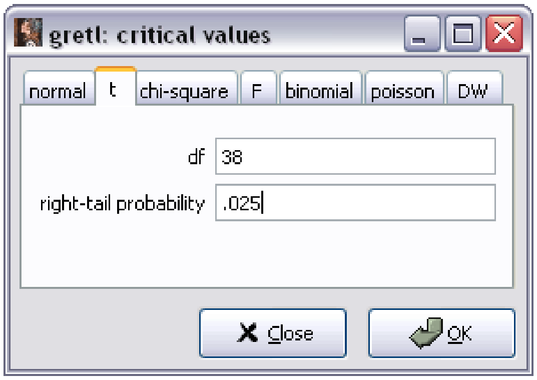
```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
genr lb = $coeff(x) - 2.024 * $stderr(x)
genr ub = $coeff(x) + 2.024 * $stderr(x)
print lb ub
```

The first line opens the data set. The second line (`ols`) minimizes the

probability. However, the frequency definition of probability refers to the long run relative frequency with which some event occurs. If this is what probability is, then saying that a parameter falls within an interval with given probability means that intervals so constructed will contain the parameter that proportion of the time.

[2] You can also get the α level critical values from the console by issuing the command `genr c= critical(t,38,`α`)`. Here α is the desired area in the right-tail of the t-distribution.

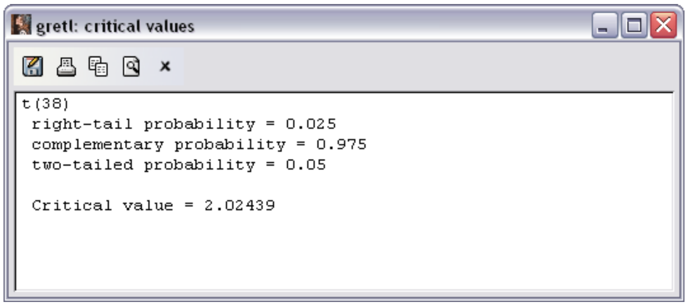
Figure 3.1: Obtaining critical values using the `Tools>Statistical tables` dialog box.



df 38

right-tail probability .025

Figure 3.2: The critical value obtained from `Tools>Statistical tables` dialog box.



```
t(38)
 right-tail probability = 0.025
 complementary probability = 0.975
 two-tailed probability = 0.05

 Critical value = 2.02439
```

sum of squared errors in a linear model that has y as the dependent variable with a constant and x as independent variables. The next two lines generate the lower and upper bounds for the 95% confidence interval for the slope parameter β_2. The last line prints the results of the computation.

The **gretl** language syntax needs a little explanation. When **gretl** makes a computation, it will save certain results like coefficient estimates, their standard errors, sum of squared errors and so on in memory. These results can then be used to compute other statistics, provided you know the variable name that **gretl** uses to store the computation. In the above example, **gretl** uses the least squares estimates and their estimated standard errors to compute confidence intervals. Following the `ols` command, least squares estimates are stored in `$coeff(`*`variable name`*`)`. So, since β_2 is estimated using the variable `x`, its coefficient estimate is saved in `$coeff(x)`. The corresponding standard error is saved in `$stderr(x)`. There are many other results saved and stored, each prefixed with the dollar sign $. Consult the **gretl** documentation for more examples and specific detail on what results can be saved and how to access them.

Equivalently, you could use **gretl**'s built in `critical` to obtain the desired critical value. The general syntax for the function depends on the desired probability distribution. This follows since different distributions contain different numbers of parameters (e.g., the t-distribution has a single degrees of freedom parameter while the standard normal has none!). This example uses the t-distribution and the script becomes:

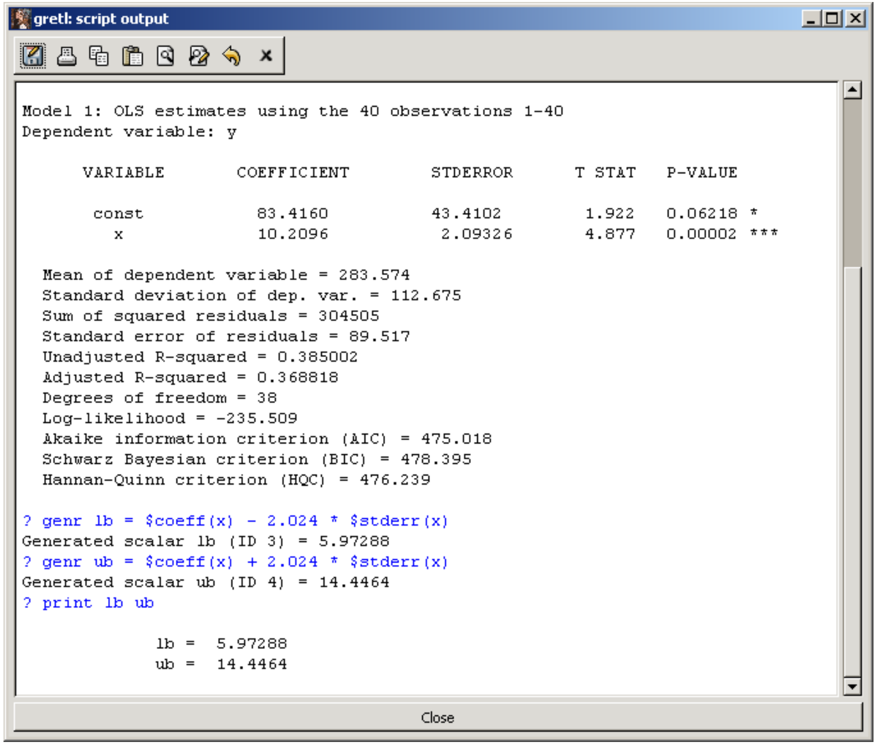
```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)
print lb ub
```

The syntax for the t-distribution is `critical(t,degrees of freedom,`$\alpha/2$`)`. The degrees of freedom from the preceding regression are saved as `$df` and for a $1-\alpha = 95\%$ confidence interval, $\alpha/2 = 0.025$.

The example found in section 3.1.3 of *POE* computes a 95% confidence interval for the income parameter in the food expenditure example. The **gretl** commands above were used to produce the output found in Figure

3.3.

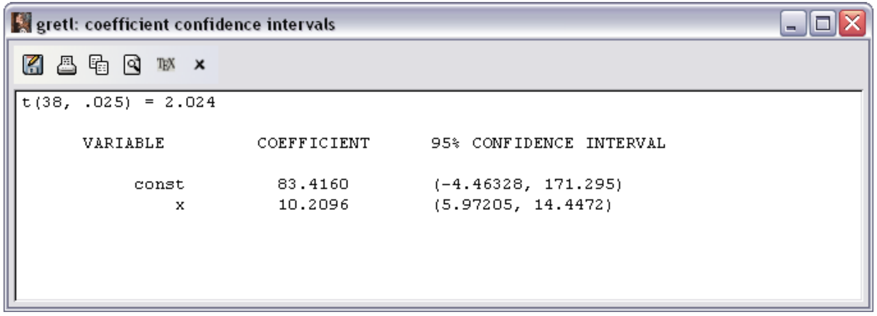
Figure 3.3: Obtaining 95% confidence interval for the food expenditure example.



```
Model 1: OLS estimates using the 40 observations 1-40
Dependent variable: y

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

       const           83.4160          43.4102        1.922   0.06218 *
         x             10.2096           2.09326       4.877   0.00002 ***

  Mean of dependent variable = 283.574
  Standard deviation of dep. var. = 112.675
  Sum of squared residuals = 304505
  Standard error of residuals = 89.517
  Unadjusted R-squared = 0.385002
  Adjusted R-squared = 0.368818
  Degrees of freedom = 38
  Log-likelihood = -235.509
  Akaike information criterion (AIC) = 475.018
  Schwarz Bayesian criterion (BIC) = 478.395
  Hannan-Quinn criterion (HQC) = 476.239

? genr lb = $coeff(x) - 2.024 * $stderr(x)
Generated scalar lb (ID 3) = 5.97288
? genr ub = $coeff(x) + 2.024 * $stderr(x)
Generated scalar ub (ID 4) = 14.4464
? print lb ub

             lb =   5.97288
             ub =   14.4464
```

To use the dialogs to get confidence intervals is easy as well. First estimate the model using least squares in the usual way. Choose `Model>Ordinary least squares` from the main pull-down menu, fill in the dependent and independent variables in the `ols` dialog box and click OK. The results appear in the model window. Now choose `Analysis>Confidence intervals for coefficients` from the model window's pull-down menu (seen in Figure 4.1). This generates the result shown in Figure 3.4.

Figure 3.4: Obtaining 95% confidence interval for the food expenditure example from the dialog.



```
t(38, .025) = 2.024

     VARIABLE         COEFFICIENT       95% CONFIDENCE INTERVAL

         const          83.4160         (-4.46328, 171.295)
             x          10.2096         (5.97205, 14.4472)
```

3.2 Monte Carlo Experiment

Once again, the consequences of repeated sampling can be explored using a simple Monte Carlo study. In this case, we will add the two statements that compute the lower and upper bounds to our previous program listed in Figure 2.14. We've also added a parameter, sig2, which is the estimated variance of the model's errors.

The new script looks like this:

```
open c:\userdata\gretl\data\poe\food.gdt
set seed 3213798
loop 100 -- progressive
genr u = 88*normal()
genr y1 = 80 + 10*x + u
ols y1 const x
genr b1 = $coeff(const)
genr b2 = $coeff(x)
genr s1 = $stderr(const)
genr s2 = $stderr(x)
# 2.024 is the .025 critical value from the t(38) distribution
genr c1L = b1 - 2.024*s1
genr c1R = b1 + 2.024*s1
genr c2L = b2 - 2.024*s2
genr c2R = b2 + 2.024*s2
genr sigma = $sigma
genr sig2 = sigma*sigma
print b1
print b2
store cicoeff.gdt b1 b2 s1 s2 sig2 c1L c1R c2L c2R
endloop
```

The results are stored in the gretl data set `cicoeff.gdt`. Opening this data set (`open C:\userdata\gretl\user\cicoeff.gdt`) and examining the data will reveal interval estimates that vary much like those in Tables 3.1 and 3.2 of your textbook. Also, notice that the estimated standard error of the residuals, $\hat{\sigma}$, is stored in `$sigma` and used to estimate the overall model variance.

3.3 Hypothesis Tests

Hypothesis testing allows us to confront any prior notions we may have about the model with what we actually observe. Thus, if before drawing a sample, I believe that autonomous weekly food expenditure is no less than $40, then once the sample is drawn I can determine via a hypothesis test whether experience is actually consistent with this belief.

In section 3.4 of your textbook the authors test several hypotheses about β_2. In 3.4.1a the null hypothesis is that $\beta_2 = 0$ against the alternative that it is positive (i.e., $\beta_2 > 0$). The test statistic is:

$$t = (b_2 - 0)/se(b_2) \sim t_{38} \tag{3.2}$$

provided that $\beta_2 = 0$ (the null hypothesis is true). Select $\alpha = 0.05$ which makes the critical value for the one sided alternative ($\beta_2 > 0$) equal to 1.686. The decision rule is to reject Ho in favor of the alternative if the computed value of your t statistic falls within the rejection region of your test; that is if it is larger than 1.686.

The information you need to compute t is contained in the least squares estimation results produced by **gretl**:

Model 1: OLS estimates using the 40 observations 1–40

Dependent variable: y

Variable	Coefficient	Std. Error	*t*-statistic	p-value
const	83.4160	43.4102	1.9216	0.0622
x	10.2096	2.09326	4.8774	0.0000

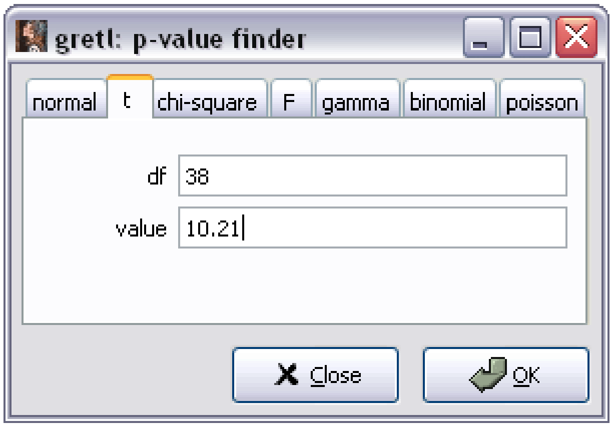
Mean of dependent variable	283.574
S.D. of dependent variable	112.675
Sum of squared residuals	304505.
Standard error of residuals ($\hat{\sigma}$)	89.5170
Unadjusted R^2	0.385002
Adjusted $\bar{R}^2$	0.368818
Degrees of freedom	38
Akaike information criterion	475.018
Schwarz Bayesian criterion	478.395

The computations

$$t = (b_2 - 0)/se(b_2) = (10.21 - 0)/2.09 = 4.88 \tag{3.3}$$

Since this value is within the rejection region, then there is enough evidence at the 5% level of significance to convince us that the null hypothesis is incorrect; the null hypothesis rejected at this level of significance. We can

Figure 3.5: The dialog box for obtaining p-values using the built in statistical tables in **gretl**.



df 38

value 10.21

use **gretl** to get the p-value for this test using the `Tools` pull-down menu. In this dialog, you have to fill in the degrees of freedom for your t-distribution (38), the value of b_2 (10.21), its value under the null hypothesis–something **gretl** refers to as 'mean' (0), and the estimated standard error from your printout (2.09). This will yield the information:

```
t(38): area to the right of 10.21 = 9.55024e-013
(two-tailed value = 1.91005e-012; complement = 1)
```

This indicates that the area in one tail is almost zero. The p-value is well below the usual level of significance, $\alpha = .05$, and the hypothesis is rejected.

Gretl also includes a programming command that will compute p-values from several distributions. The `pvalue` function works similarly to the `critical` function discussed in the preceding section. The syntax is:

```
genr p = pvalue(distribution,parameters,xval)
```

The `pvalue` function computes the area to the right of `xval` in the specified `distribution`. Choices include z for Gaussian, t for Student's t, X for chi-square, F for F, G for gamma, B for binomial or P for Poisson. The argument `parameters` refers to the distribution's **known** parameters, like its degrees of freedom. So, for this example try

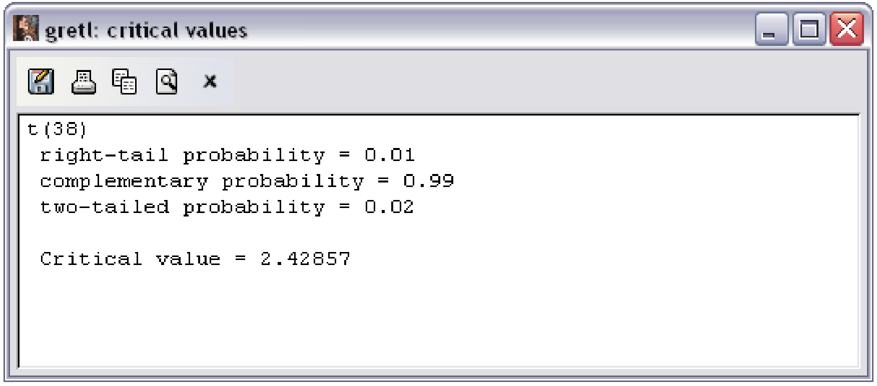
```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
genr t2 = ($coeff(x)-0)/$stderr(x)
genr p2 = pvalue(t,$df,t2)
```

In the next example, the authors of *POE* test the hypothesis that $\beta_2 = 5.5$ against the alternative that $\beta_2 > 5.5$. The computations

$$t = (b_2 - 5.5)/se(b_2) = (10.21 - 5.5)/2.09 = 2.25 \tag{3.4}$$

The significance level in this case is chosen to be 0.01 and the corresponding critical value can be found using a tool found in **Gretl** . The `Tools>Statistical tables` pull-down menu bring up the dialog found in Figure 3.1.

Figure 3.6: The results from the dialog box for obtaining critical values using the built in statistical tables in **gretl**.



```
t(38)
 right-tail probability = 0.01
 complementary probability = 0.99
 two-tailed probability = 0.02

 Critical value = 2.42857
```

This result is found in Figure 3.6. The .01 one-sided critical value is 2.429. Since 2.25 is less than this, we cannot reject the null hypothesis at the .01 level of significance.

In section 3.4.2 of *POE*, the authors conduct a one-sided test where the rejection region falls within the left tail of the t-distribution. The null hypothesis is $\beta_2 = 15$ and the alternative is $\beta_2 < 15$. The test statistic and distribution is

$$t = (b_2 - 15)/se(b_2) \sim t_{38} \tag{3.5}$$

provided that $\beta_2 = 15$. The computation is

$$t = (b_2 - 15)/se(b_2) = (10.21 - 15)/2.09 = -2.29 \tag{3.6}$$

Based on the desired level of significance, $\alpha = .05$, we would reject the null in favor of the one-sided alternative if $t < -1.686$. It is and therefore we conclude that the coefficient is less than 15 at this level of significance.

In section 3.4.3 examples of two-tailed tests are found. In the first example the economic hypothesis that households will spend \$7.50 of each additional \$100 of income on food. So, $H_0 : \beta_2 = 7.50$ and the alternative is $H_1 : \beta_2 \neq 7.50$. The statistic is $t = (b_2 - 7.5)/se(b_2) \sim t_{38}$ if H_0 is true which is computed $t = (b_2 - 7.5)/se(b_2) = (10.21 - 7.5)/2.09 = 1.29$. The two-sided, $\alpha = .05$ critical value is 2.024. This means that you reject H_0 if either $t < -2.024$ or if $t > 2.024$. The computed statistic is neither, and hence we do not reject the hypothesis that β_2 is \$7.50. There simply isn't enough information in the sample to convince us otherwise.

You can draw the same conclusions from using a confidence interval that you can from this two-sided t-test. The $100(1-\alpha)\%$ confidence interval for β_2 is

$$b_2 - t_c se(b_2) \leq \beta_2 \leq b_2 + t_c se(b_2) \tag{3.7}$$

In terms of the example

$$10.21 - 2.024(2.09) \leq \beta_2 \leq 10.21 + 2.024(2.09) \tag{3.8}$$

which as we saw earlier in the manual was $5.97 \leq \beta_2 \leq 14.45$. Since 7.5 falls within this interval, you could not reject the hypothesis that β_2 is different from 7.5 at the .05% level of significance.

In the next example a test of the overall significance of β_2 is conducted. As a matter of routine, you always want to test to see if your slope parameter is different from zero. If not, then the variable associated with it may not belong in your model. So, $H_0 : \beta_2 = 0$ and the alternative is $H_1 : \beta_2 \neq 0$. The statistic is $t = (b_2 - 0)/se(b_2) \sim t_{38}$, if H_0 is true, and this is computed $t = (b_2 - 0)/se(b_2) = (10.21 - 0)/2.09 = 4.88$. Once again, the two-sided, $\alpha = .05$ critical value is 2.024 and 4.88 falls squarely within the 5% rejection region of this test. These numbers should look familiar since this is the test that is conducted by default whenever you run a regression in Gretl.

As we saw earlier, **gretl** also makes obtaining one- or two-sided p-values for the test statistics you compute very easy. Simply use p-value finder dialog box available from the Tools pull-down menu (see Figure 3.6) to obtain one or two sided p-values.

3.4 Script for t-values and p-values

One thing we've shown in this chapter is that many of the results obtained using the pull-down menus (often referred to as the GUI) in **gretl** can be obtained using **gretl**'s language from the console or in a script. In fact, the **gretl**'s GUI is merely a front-end to its programming language.[3] In this chapter we used the `pvalue` and `critical` functions to get p-values or critical values of statistics. The following script accumulates what we've covered and completes the examples in the text.

[3] This is true in Stata as well.

```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
genr tratio1 = ($coeff(x) - 0)/ $stderr(x)

#One sided test (Ha:  b2 > zero)
genr c2 = critical(t,$df,.05)
genr p2 = pvalue(t,$df,tratio1)

#One sided test (Ha:  b2>5.5)
genr tratio2 = ($coeff(x) - 5.5)/ $stderr(x)
genr c2 = critical(t,$df,.05)
genr p2 = pvalue(t,$df,tratio2)

#One sided test (Ha:  b2<15)
genr tratio3 = ($coeff(x) - 15)/ $stderr(x)
genr c3 = -1*critical(t,$df,.05)
genr p3 = pvalue(t,$df,abs(tratio3))

#Two sided test (Ha:  b2 not equal 7.5)
genr tratio4 = ($coeff(x) - 7.5)/ $stderr(x)
genr c4 = critical(t,$df,.025)
genr p4 = 2*pvalue(t,$df,tratio4)

#Confidence interval
genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)

#Two sided test (Ha:  b2 not equal zero)
genr c1 = critical(t,$df,.025)
genr p1 = 2*pvalue(t,$df,tratio5)
```

The `pvalue` function in **gretl** measures the area of the probability distribution that lies to the right of the computed statistic. If the computed t-ratio is positive and your alternative is two-sided, multiply the result by 2 to measure the area to the left of its negative.

If your t-ratio is negative, **gretl** won't compute the area (and you wouldn't want it to, anyway). This is what happened for `tratio3` in the script and

I used the absolute value function, `abs( )`, to get its positive value. The area to the right of the positive value is equivalent to the area left of the negative value. Hence, the computation is correct.

Basically, proper use of the `pvalue` in tests of a single hypothesis requires a little thought. Too much thought, in my opinion. I would avoid it unless you are comfortable with its use. In other hypothesis testing contexts (e.g., χ^2 and F-tests) p-values are much easier to use correctly. I use them freely in those cases. With t-tests or z-tests (normal distribution), it is just easier conduct a test by comparing the computed value of your statistic to the correct critical value.

3.5 Script

```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
genr tratio1 = ($coeff(x) - 0)/ $stderr(x)

#One sided test (Ha:  b2 > zero)
genr c2 = critical(t,$df,.05)
genr p2 = pvalue(t,$df,tratio1)

#One sided test (Ha:  b2>5.5)
genr tratio2 = ($coeff(x) - 5.5)/ $stderr(x)
genr c2 = critical(t,$df,.01)
genr p2 = pvalue(t,$df,tratio2)

#One sided test (Ha:  b2<15)
genr tratio3 = ($coeff(x) - 15)/ $stderr(x)
genr c3 = -1*critical(t,$df,.05)
genr p3 = pvalue(t,$df,abs(tratio3))

#Two sided test (Ha:  b2 not equal 7.5)
genr tratio4 = ($coeff(x) - 7.5)/ $stderr(x)
genr c4 = critical(t,$df,.025)
genr p4 = 2*pvalue(t,$df,tratio4)

#Confidence interval
genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)

#Two sided test (Ha:  b2 not equal zero)
genr c1 = critical(t,$df,.025)
genr p1 = 2*pvalue(t,$df,tratio1)
```

Chapter 4

Prediction, Goodness-of-Fit, and Modeling Issues

Several extensions of the simple linear regression model are now considered. First, conditional predictions are generated using results saved by **gretl**. Then, a commonly used measure of the quality of the linear fit provided by the regression is discussed. We then take a brief detour to discuss how **gretl** can be used to provide professional looking output that can be used in your research.

The choice of functional form for a linear regression is important and the RESET test of the adequacy of your choice is examined. Finally, the residuals are tested for normality. Normality of the model's errors is a useful property in that, when it exists, it improves the the performance of least squares and the related tests and confidence intervals we've considered when sample sizes are small (finite).

4.1 Prediction in the Food Expenditure Model

Generating predicted values of food expenditure for a person with a given income is very simple in **gretl**. After estimating the model with least squares, you can use the `genr` to get predicted values. In the example, a household having $x_o = \$2000$ of weekly income is predicted to spend approximately \$287.61 on food. Recalling that income is measured in hundreds of dollars in the data, the **gretl** commands to compute this from the console are:

```
ols y const x
genr yhat0 = $coeff(const) + $coeff(x)*20
```

This yields $\hat{y}_0 = 287.609$.

Obtaining the 95% confidence interval is slightly harder in that there are no internal commands in **gretl** that will do this. The information needed is readily available, however. The formula is:

$$v\hat{a}r(f) = \hat{\sigma}^2 + \frac{\hat{\sigma}^2}{T} + (x_o - \bar{x})^2 v\hat{a}r(b_2) \qquad (4.1)$$

In section 2.4 we estimated $\hat{\sigma}^2 = 8013.29$ and $v\hat{a}r(b_2) = 4.3818$. The mean value of income is found by highlighting the variable x in the main **gretl** window and the selecting `View>Summary Statistics` from the pull-down menu. This yields $\bar{x} = 19.6047$.[1] The t_{38} 5% critical value is 2.0244 and the computation[2]

$$v\hat{a}r(f) = 8013.2941 + \frac{8013.2941}{40} + (20 - 19.6047)^2 * 4.3818 = 8214.31 \quad (4.2)$$

Then, the confidence interval is:

$$\hat{y}_0 \pm t_c se(f) = 287.6069 \pm 2.0244\sqrt{8214.31} = [104.132, 471.086] \qquad (4.3)$$

The complete script to produce the computed results in **gretl** is:

```
ols y const x
genr yhat0 = $coeff(const) + $coeff(x)*20
genr f=8013.2941+(8013.2941/40)+4.3818*(20-19.6047)**2
genr ub=yhat0+2.0244*sqrt(f)
genr lb=yhat0-2.0244*sqrt(f)
```

[1] Your result may vary a little depending on how many digits are carried out to the right of the decimal.

[2] You can compute this easily using the **gretl** console by typing in: `genr f=8013.2941+(8013.2941/40)+4.3818*(20-19.6047)**2`

At this point, you may be wondering if there is some way to use the internal functions of **gretl** to produce the same result? As we've seen, **gretl** saves many of the results we need internally and these can in turn be called into service in subsequent computations.

For instance, the sum of squared errors from the least squares regression is saved as `$ess`. The degrees of freedom and number of observations are saved as `$df` and `$nobs`, respectively. Also, you can use an internal **gretl** function to compute $\bar{x}$, `mean(x)` and the `critical` function discussed in the preceding chapter to get the desired critical value. Hence, the prediction interval can be automated and made more precise by using the following script.

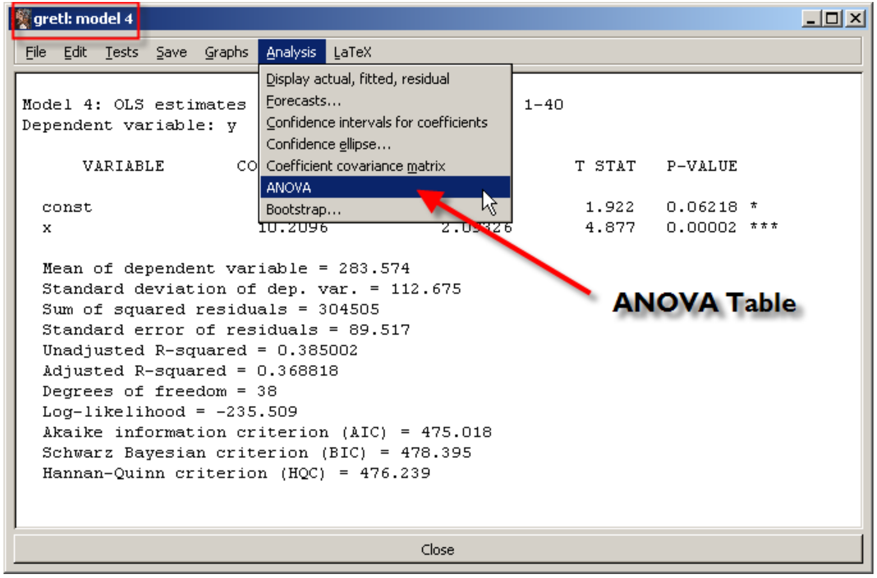
```
ols y const x
genr yhat0=$coeff(const)+20*$coeff(x)
genr sig2 = $ess/$df
genr f = sig2 + sig2/$nobs + ((20-mean(x))^2)*($stderr(x)^2)
genr lb = yhat0-critical(t,$df,0.025)*sqrt(f)
genr ub = yhat0+critical(t,$df,0.025)*sqrt(f)
```

4.2 Coefficient of Determination

One use of regression analysis is to "explain" variation in dependent variable as a function of the independent variable. A summary statistic that is used for this purpose is the coefficient of determination, also known as R^2.

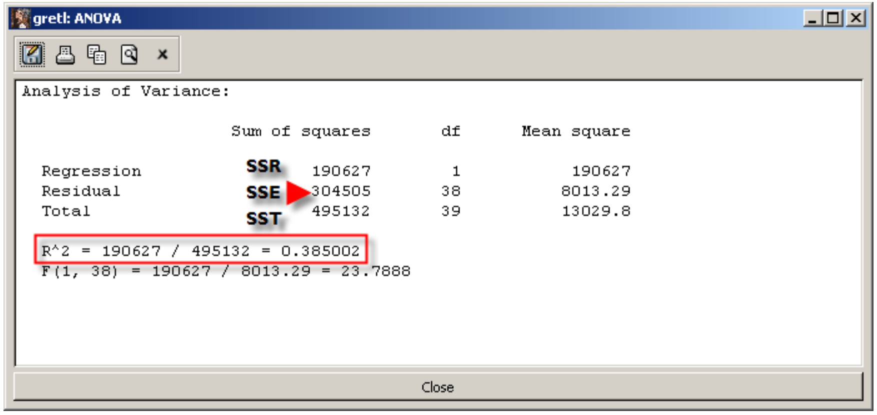
There are a number of different ways of obtaining R^2 in **gretl**. The simplest way to get R^2 is to read it directly off of **gretl**'s regression output. This is shown in Figure 4.3. Another way, and probably most difficult, is to compute it manually using the **analysis of variance** (ANOVA) table. The ANOVA table can be produced after a regression by choosing `Analysis>ANOVA` from the **model** window's pull-down menu as shown in Figure 4.1. The result appears in Figure 4.2. In Figure 4.2 the SSR, SSE, and SST are shown. **Gretl** also does the R^2 computation for you as shown at the bottom of the output. If you want to verify **gretl**'s computation,

Figure 4.1: After estimating the regression, select Analysis>ANOVA from the model window's pull-down menu.



```
Model 4: OLS estimates                                   1-40
Dependent variable: y

     VARIABLE        CO                                 T STAT    P-VALUE

  const                                                  1.922    0.06218 *
  x                  10.2096            2.09326          4.877    0.00002 ***

  Mean of dependent variable = 283.574
  Standard deviation of dep. var. = 112.675
  Sum of squared residuals = 304505
  Standard error of residuals = 89.517
  Unadjusted R-squared = 0.385002
  Adjusted R-squared = 0.368818
  Degrees of freedom = 38
  Log-likelihood = -235.509
  Akaike information criterion (AIC) = 475.018
  Schwarz Bayesian criterion (BIC) = 478.395
  Hannan-Quinn criterion (HQC) = 476.239
```

Figure 4.2: The ANOVA table



```
Analysis of Variance:

                     Sum of squares      df       Mean square

  Regression              190627          1           190627
  Residual                304505         38          8013.29
  Total                   495132         39          13029.8

  R^2 = 190627 / 495132 = 0.385002
  F(1, 38) = 190627 / 8013.29 = 23.7888
```

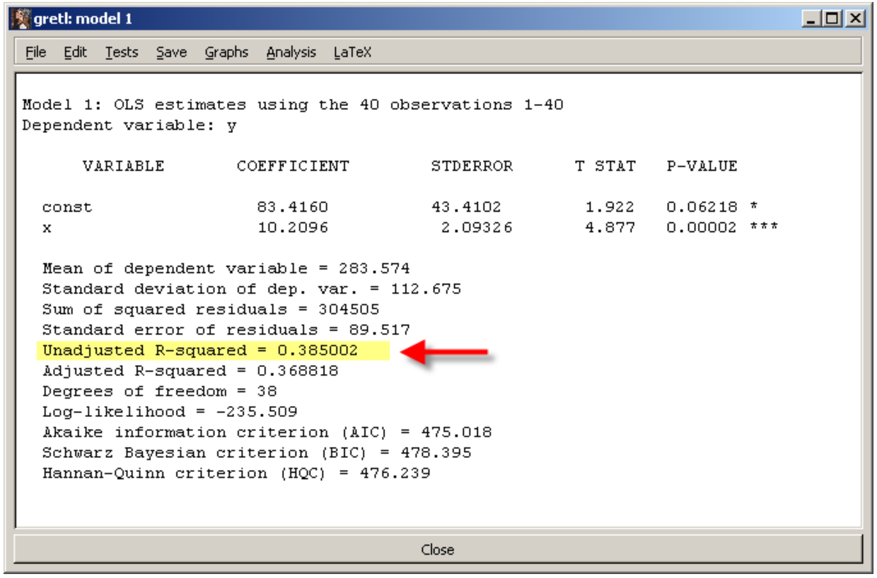
then

$$SST = SSR + SSE = 190627 + 304505 = 495132 \tag{4.4}$$

and

$$\frac{SSR}{SST} = 1 - \frac{SSE}{SST} = \frac{190627}{495132} = .385 \tag{4.5}$$

Figure 4.3: In addition to some other summary statistics, **Gretl** computes the unadjusted R^2 from the linear regression.

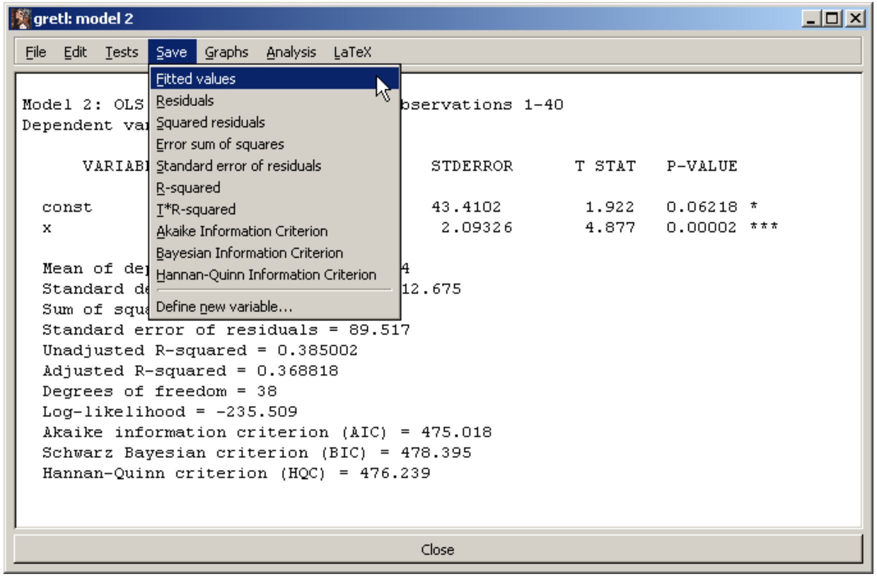


```
Model 1: OLS estimates using the 40 observations 1-40
Dependent variable: y

      VARIABLE       COEFFICIENT        STDERROR       T STAT   P-VALUE

  const                83.4160          43.4102         1.922   0.06218 *
  x                    10.2096           2.09326        4.877   0.00002 ***

  Mean of dependent variable = 283.574
  Standard deviation of dep. var. = 112.675
  Sum of squared residuals = 304505
  Standard error of residuals = 89.517
  Unadjusted R-squared = 0.385002
  Adjusted R-squared = 0.368818
  Degrees of freedom = 38
  Log-likelihood = -235.509
  Akaike information criterion (AIC) = 475.018
  Schwarz Bayesian criterion (BIC) = 478.395
  Hannan-Quinn criterion (HQC) = 476.239
```

Finally, you can think of R^2 is as the squared correlation between your observations on your dependent variable, y, and the predicted values of y based on your estimated model, $\hat{y}$. A **gretl** script to compute this version of the statistic is found below in section 4.6.5.

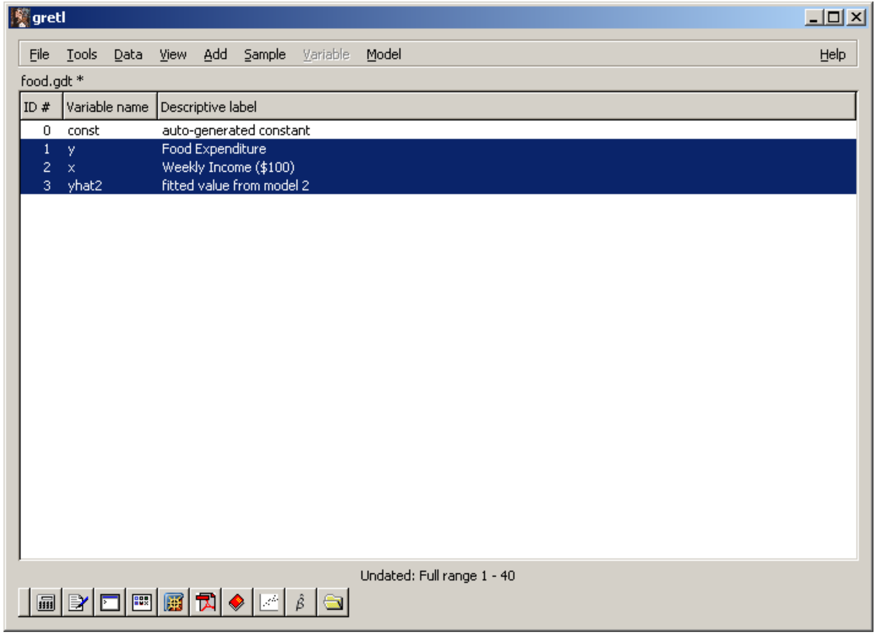
To use the GUI you can follow the steps listed here. Estimate the model using least squares and add the predicted values from the estimated model, $\hat{y}$, to your data set. Then use the **gretl** correlation matrix to obtain the correlation between y and $\hat{y}$. Adding the fitted values to the data set from the pull-down menu in the model window is illustrated in Figure 4.4 below. Highlight the variables y, x, and $yhat2$ by holding the control key down and clicking on each variable in the main **gretl** window as seen in Figure 4.5 below. Then, `View>Correlation Matrix` will produce all the pairwise correlations between each variable you've chosen. These are arranged in a

Figure 4.4: Using the pull-down menu in the Model window to add fitted values to your data set.



```
Model 2: OLS                                        bservations 1-40
Dependent vai

      VARIABI                               STDERROR      T STAT    P-VALUE

  const                                     43.4102        1.922    0.06218 *
  x                                          2.09326       4.877    0.00002 ***

  Mean of dej                              4
  Standard de                              12.675
  Sum of squa
  Standard error of residuals = 89.517
  Unadjusted R-squared = 0.385002
  Adjusted R-squared = 0.368818
  Degrees of freedom = 38
  Log-likelihood = -235.509
  Akaike information criterion (AIC) = 475.018
  Schwarz Bayesian criterion (BIC) = 478.395
  Hannan-Quinn criterion (HQC) = 476.239
```

Figure 4.5: Hold the control key and click on y, x, and $\hat{y} = yhat2$ from the food expenditure regression to select them.

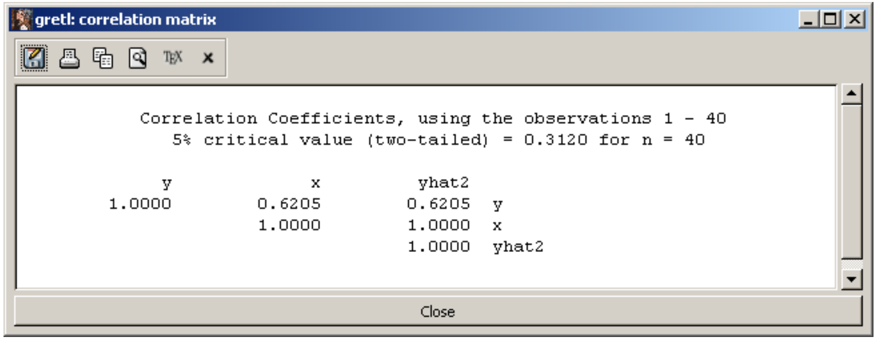


food.gdt *

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	y	Food Expenditure
2	x	Weekly Income ($100)
3	yhat2	fitted value from model 2

Undated: Full range 1 - 40

Figure 4.6: The correlation matrix for y, x, and $\hat{y} = yhat2$ is produced by selecting `View>Correlation matrix` from the pull-down menu.



```
          Correlation Coefficients, using the observations 1 - 40
             5% critical value (two-tailed) = 0.3120 for n = 40

          y             x           yhat2
     1.0000        0.6205          0.6205  y
                   1.0000          1.0000  x
                                   1.0000  yhat2
```

matrix as shown in Figure 4.6 Notice that the correlation between y and x is the same as that between y and $\hat{y}$ (i.e., 0.6205). As shown in your text, this is no coincidence in the simple linear regression model. Also, squaring this number equals R^2 from your regression, $0.6205^2 = .385$.

In Figure 4.4 of *POE* the authors plot y against $\hat{y}$. A positive linear relationship between the two is expected since the correlation their correlation is 0.62. To produce this plot, estimate the regression to open the model window. Add the predicted values of from the regression to the dataset using `Save>Fitted values` from the model window's pull-down menu. Name the fitted value, yhat1 and click OK. Now, return to the main window, use the mouse to highlight the two variables (y and yhat1),[3] then select `View>Graph specified vars>X-Y scatter` from the pull-down menu. This opens the **define graph** dialog box. Choose `yhat1` as the Y-axis variable and `y` as the X-axis variable and click **OK**. A graph appears that looks similar to the one in *POE*. This one actually has a fitted least squares line through the data scatter that, as expected, has a positive slope. In fact, the slope is estimated to be .385, which is the regression's R^2!

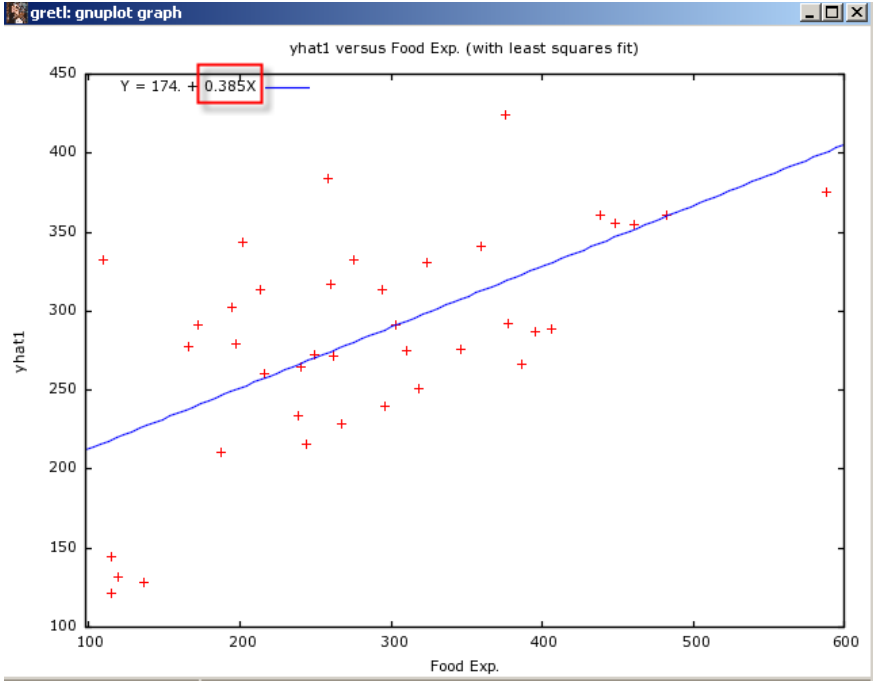
A simpler approach is to open a console window and use the following commands:

```
ols y const x
genr yhat1 = $yhat
gnuplot yhat1 y
```

[3] Remember, press and hold Ctrl, then click on each variable

The first line estimates the regression. The predicted values are saved by **gretl** in `$yhat`. Use the `genr` command to create a new variable, `yhat1`, that uses these. Then, call `gnuplot` with the predicted values, `yhat1`, as the first variable and the actual values of food expenditure, `y`, as the second. The graph is shown below in Figure 4.7. Finally, if you execute these commands using a script, the graph is written to a file on your computer rather than opened in a window. For this reason, I recommend executing these commands from the console rather than from the script file that appears at the end of this chapter.

Figure 4.7: A plot of predicted vs. actual food expenditures produced using **gnuplot** .



4.3 Reporting Results

In case you think **gretl** is just a toy, the program includes a very capable utility that enables it to produce professional looking output. LaTeX, usually pronounced “Lay-tek”, is a typesetting program used by mathematicians and scientists to produce professional looking technical documents. It is widely used by econometricians to prepare manuscripts for wider distribution. In fact, this book is produced using LaTeX.

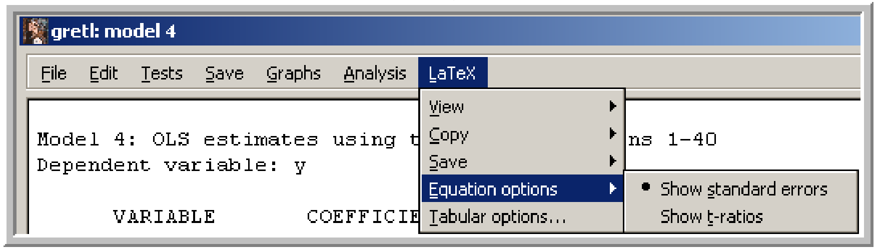
Although LaTeX is free and produces very professional looking documents, it is not widely used by undergraduate and masters students because 1) most degree programs don’t require you to write a lot of technical papers and 2) it’s a computer language and therefore it takes some time to learn its intricacies and to appreciate its nuances. Heck, I’ve been using it for years and still scratch my head when I try to put tables and Figures in the places I’d like them to be!

In any event, **gretl** includes a facility for producing output that can be pasted directly into LaTeX documents. For users of LaTeX, this makes generating regression output in proper format a breeze. If you don’t already use LaTeX, then this will not concern you. On the other hand, if you already use it, **gretl** can be very handy in this respect.

In Figure 4.3 you will notice that on the far right hand side of the menu bar

is a pull-down menu for LaTeX. From here, you click `LaTeX` on the menu bar and a number of options are revealed as shown in Figure 4.8. You can view, copy, or save the regression output in either tabular form or in equation form. You can tell **gretl** whether you want standard errors or t-ratios in parentheses below parameter estimates, and you can define the number of decimal places to be used of output. Nice indeed. Examples of tabular and equation forms of output are found below in Tables 4.1 and 4.2, respectively.

Figure 4.8: Several options for defining the output of LaTeX are available.



```
Model 4: OLS estimates using t                 ns 1-40
Dependent variable: y

       VARIABLE        COEFFICIE
```

Table 4.1: This is an example of LaTeX output in tabular form.

Model 1: OLS estimates using the 40 observations 1–40
Dependent variable: y

Variable	Coefficient	Std. Error	t-statistic	p-value
const	83.4160	43.4102	1.9216	0.0622
x	10.2096	2.09326	4.8774	0.0000

Mean of dependent variable	283.574
S.D. of dependent variable	112.675
Sum of squared residuals	304505.
Standard error of residuals ($\hat{\sigma}$)	89.5170
Unadjusted R^2	0.385002
Adjusted $\bar{R}^2$	0.368818
Degrees of freedom	38
Akaike information criterion	475.018
Schwarz Bayesian criterion	478.395

Table 4.2: Example of LaTeX output in equation form

$$\widehat{\mathrm{y}} = \underset{(1.922)}{83.4160} + \underset{(4.877)}{10.2096}\,\mathrm{x}$$

$$T = 40 \quad \bar{R}^2 = 0.3688 \quad F(1,38) = 23.789 \quad \hat{\sigma} = 89.517$$

(t-statistics in parentheses)

4.4 Functional Forms

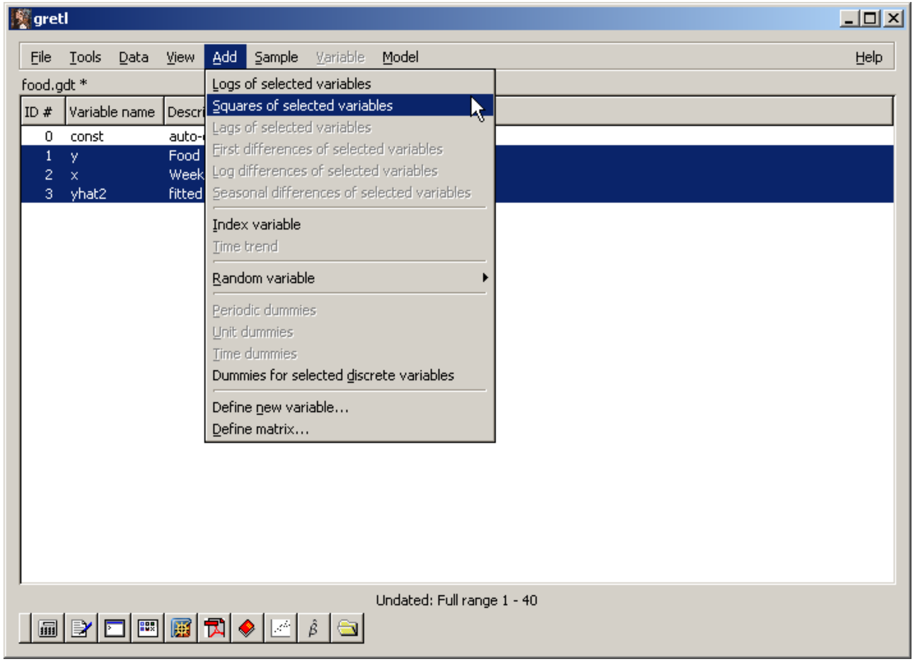
Linear regression is considerably more flexible than its name implies. There is no reason to believe that the relationship between any two variables of interest is necessarily linear. In fact there are many relationships in economics that we know are not linear. The relationship between production inputs and output is governed in the short-run by the law of diminishing returns, suggesting that a convex curve is a more appropriate function to use. Fortunately, a simple transformation of the variables (x, y, or both) can yield a model that is linear in the parameters (but not necessarily in the variables).

Simple transformation of variables can yield regression functions that are quite flexible. The important point to remember is, the functional form that you choose should be consistent with how the data are actually being generated. If you choose an inappropriate form, then your estimated model may at best not be very useful and at worst be downright misleading.

In **gretl** you are given some very useful commands for transforming variables. From the main **gretl** window the `Add` pull-down menu gives you access to a number of transformations; selecting one of these here will automatically add the transformed variable to your data set as well as its description.

Figure 4.9 shows the available selections from this pull-down menu. In the upper part of the panel two options appear in black, the others are greyed out because they are only available is you have defined the **dataset structure** to consist of time series observations. The available options can be used to add the natural logarithm or the squared values of any highlighted variable to your data set. If neither of these options suits you, then the next to last option `Define new variable` can be selected. This dialog uses the `genr` command and the large number of built in functions to transform variables in different ways. Just a few of the possibilities include square roots (sqrt), sine (sin), cosine (cos), absolute value (abs), exponential (exp), minimum (min), maximum (max), and so on. Later in the book, we'll discuss changing the dataset's structure to enable some of the other variable transformation options.

Figure 4.9: The pull-down menu for adding new variables to **gretl**


gretl
File Tools Data View Add Sample Variable Model Help
food.gdt *
ID # Variable name Descri
0 const auto-
1 y Food
2 x Week
3 yhat2 fitted
Logs of selected variables
Squares of selected variables
Lags of selected variables
First differences of selected variables
Log differences of selected variables
Seasonal differences of selected variables
Index variable
Time trend
Random variable
Periodic dummies
Unit dummies
Time dummies
Dummies for selected discrete variables
Define new variable...
Define matrix...
Undated: Full range 1 - 40

4.5 Testing for Normality

Your book, *Principles of Econometrics*, discusses the Jarque-Bera test for normality which is computed using the skewness and kurtosis of the least squares residuals. To compute the Jarque-Bera statistic, you'll first need to estimate your model using least squares and then save the residuals to the data set.

From the **gretl** console

```
ols y const x
genr uhat1 = $uhat
summary uhat1
```

The first line is the regression. The next saves the least squares redsiduals, identified as $uhat, into a variable I have called `uhat1`.[4] You could

[4] You can't use `uhat` instead of `uhat1` because that name is reserved by **gretl**.

also use the point-and-click method to add the residuals to the data set. This is accomplished from the regression's output window. Simply choose `Save>Residuals` from the model pull-down menu to add the estimated residuals to the dataset. The last line of the script produces the summary statistics for the residuals and yields the output in Figure 4.10. One thing to note, **gretl** reports excess kurtosis rather than kurtosis. The excess kurtosis is measured relative to that of the normal distribution which has kurtosis of three. Hence, your computation is

Figure 4.10: The summary statistics for the least squares residuals.

```
? summary uhat1

              Summary Statistics, using the observations 1 - 40
               for the variable 'uhat1' (40 valid observations)

  Mean                        0.00000
  Median                     -6.3245
  Minimum                  -223.03
  Maximum                   212.04
  Standard deviation         88.362
  C.V.                        2.4147E+015
  Skewness                   -0.097319
  Ex. kurtosis               -0.010966
```

$$JB = \frac{T}{6}\left(\text{Skewness}^2 + \frac{(\text{Excess Kurtosis})^2}{4}\right) \tag{4.6}$$

Which is

$$JB = \frac{40}{6}\left(-0.097^2 + \frac{-0.011^2}{4}\right) = .063 \tag{4.7}$$

Gretl also includes a built in test for normality proposed by Doornik and Hansen (1994). Computationally, it is much more complex than the Jarque-Bera test. The Doornik-Hansen test also has a χ^2 distribution if the null hypothesis of normality is true. It can be produced from the **gretl** console after running a regression using the command `testuhat`.[5]

[5] The R software also has a built-in function for performing the Jarque-Bera test. To use it, you have to download and install the *tseries* package from CRAN. Once this is

4.6 Examples

4.6.1 Wheat Yield Example

The results from the example in section 4.3 of your textbook is easily produced in **gretl**. Start by loading the data and estimating the effect of time, *time* on yield *green* using least squares. The following script will load the data file, estimate the model using least square, and generate a graph of the actual and fitted values of yield (`green`) from the model.

```
open c:\userdata\gretl\data\poe\wa-wheat.gdt
ols green const time
gnuplot green time
```

The resulting plot appears below in Figure 4.11. The simple **gnuplot** command works well enough. However, you can take advantage of having declared the dataset structure to be time series to improve the look. In this example we'll reproduce Figure 4.8 of *POE* using two options for **gnuplot**. Figure 4.8 of *POE* plots the residuals, the actual yield, and predicted yield from the regression against time. Estimate the model using least squares and save the predicted values ($yhat) and residuals ($uhat) to new variables using the `genr` command. We'll call these yhat1 and uhat1, respectively. Then use

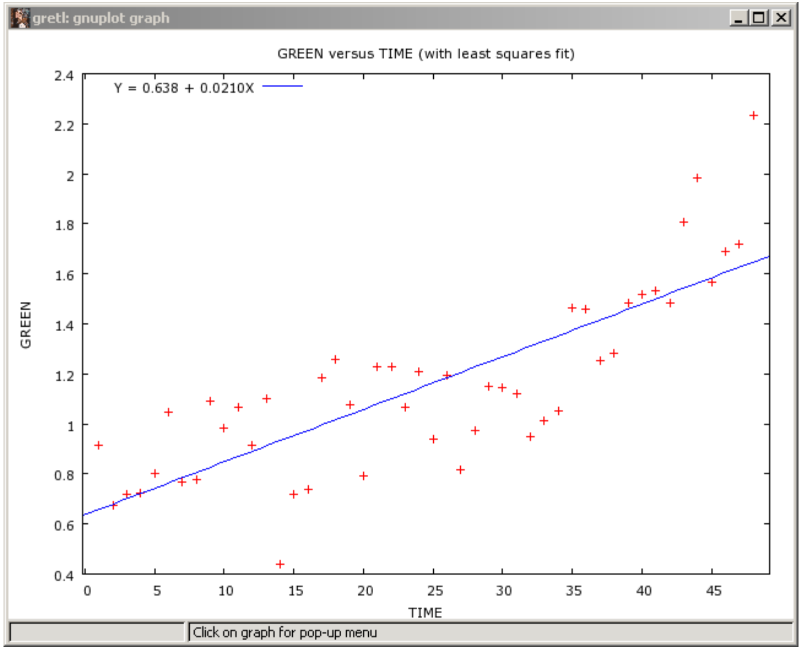
```
gnuplot green yhat1 uhat1 --with-lines --time-series
```

There are two options listed after the plot. The first (`--with-lines`) tells **gnuplot** to connect the points using lines. The second option (`--time-series`) tells *gnuplot* that the graph is of time series. In this case, the dataset's defined time variable will be used to locate each point's position on the X-axis. The graph in Figure 4.10 can be produced similarly. The complete script for Figure 4.8 of *POE* is:

```
open c:\userdata\gretl\data\poe\wa-wheat.gdt
```

done, estimate your model using least squares as discussed in appendix D and execute `jarque.bera.test(fitols$residual)`.

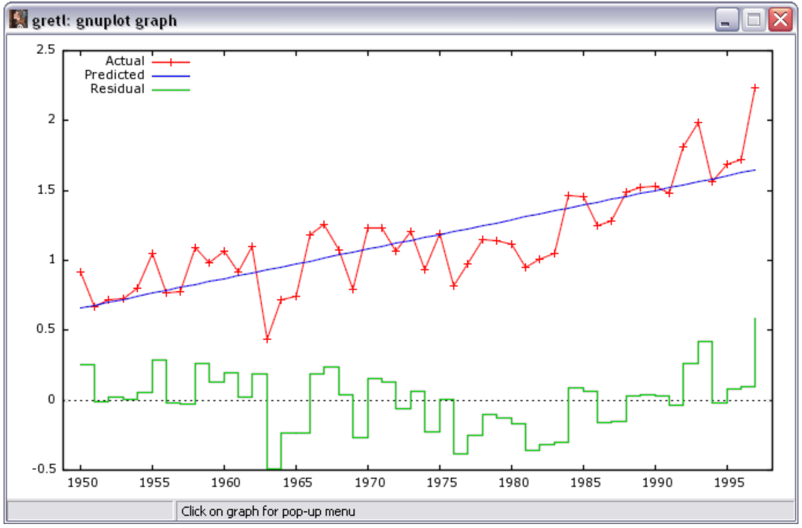
Figure 4.11: The plot of the actual yield and predicted yield from your estimated model



```
ols green const time
genr yhat1 = $yhat
genr uhat1 = $uhat
gnuplot green yhat1 uhat1 --with-lines --time-series
```

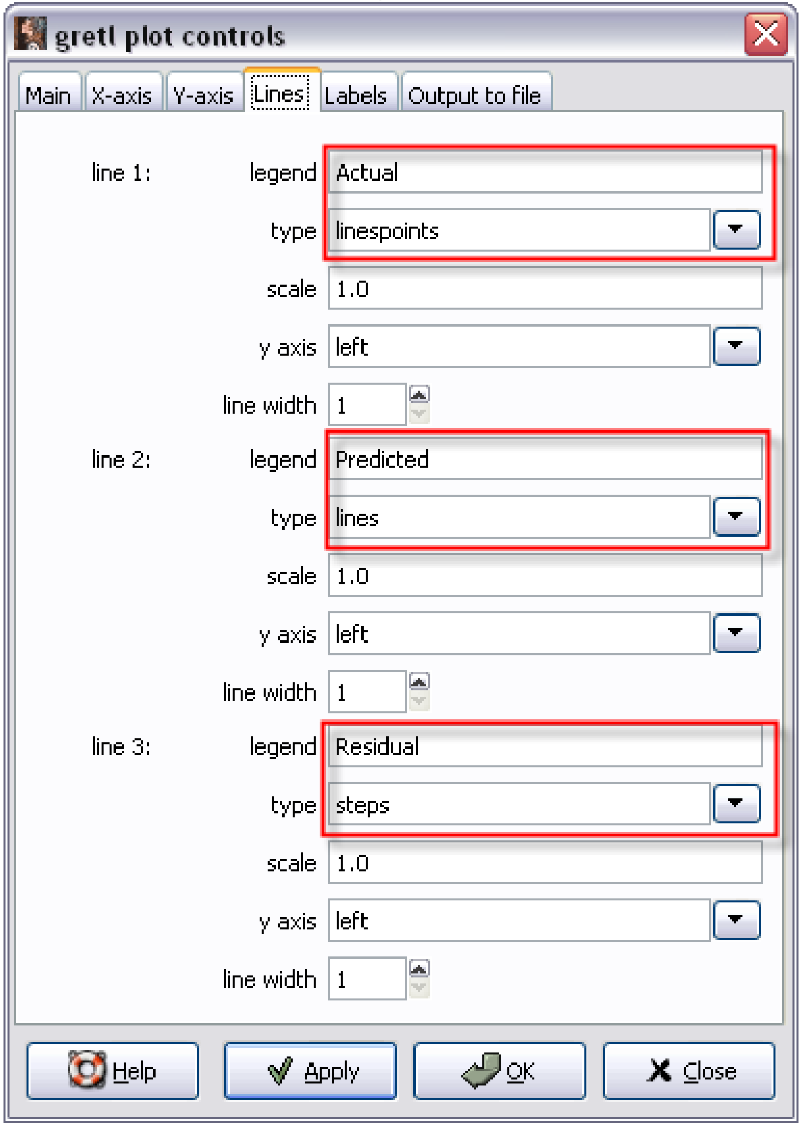
The comparable graph in **gretl** is found in Figure 4.12. Actually, this graph has had a bit of editing done via **gretl**'s graph editing dialog shown in Figure 4.13. From Figure 4.13 we have selected the **lines** tab and changed a few of the defaults. The **legend** for each series is changed from the variable's name to something more descriptive (e.g., *uhat1* is changed to *Residual*). The line styles were also changed. Steps were used for the residuals to mimic the output in Figure 4.9 of *POE* that shows a bar graph of the least squares residuals. From the stepped line, it becomes more obvious that yield is probably not linear in time. The X-axis and Main tabs were also used to change the name of the X-axis from time to Year and to add a title for the graph.

Figure 4.12: The plot of the actual yield and predicted yield from your estimated model using the –time-series option



To explore the behavior of yield further, create a new variable using the `genr` command from $t3 = time^3/1,000,000$ as shown below. The new plot appears in Figure 4.14.

Figure 4.13: The graph dialog box can be used to change characteristics of your graphs. Use the Main tab to give the graph a new name and colors; use the X- and Y-axes tabs to refine the behavior of the axes and to provide better descriptions of the variables graphed.

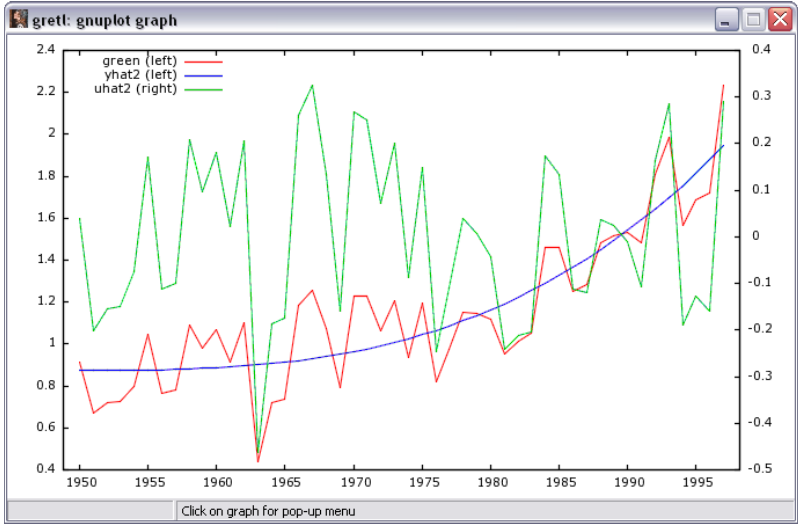


line 1: legend Actual
type linespoints
scale 1.0
y axis left
line width 1

line 2: legend Predicted
type lines
scale 1.0
y axis left
line width 1

line 3: legend Residual
type steps
scale 1.0
y axis left
line width 1

Figure 4.14: The plot of the actual yield and predicted yield from the model estimated with the cubic term



```
genr t3=time^3/1000000
ols green const t3
genr yhat2 = $yhat
genr uhat2 = $uhat
gnuplot green yhat2 uhat2 --with-lines --time-series
```

4.6.2 Growth Model Example

Below you will find a script that reproduces the results from the growth model example in section 4.4.1 of your textbook.

```
open c:\userdata\gretl\data\poe\wa-wheat.gdt
genr lyield = log(green)
ols lyield const time
```

4.6.3 Wage Equation

Below you will find a script that reproduces the results from the wage equation example in section 4.4.2 of your textbook.

```
open c:\userdata\gretl\data\poe\cps1.gdt
genr l_wage = log(wage)
ols l_wage const educ
genr lb = $coeff(educ) - 1.96 * $stderr(educ)
genr ub = $coeff(educ) + 1.96 * $stderr(educ)
print lb ub
```

4.6.4 Predictions in the Log-linear Model

In this example, you use your regression to make predictions about the log wage and the level of the wage for a person having 12 years of schooling.

```
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ
genr lyhat_12 = $coeff(const) + $coeff(educ)*12
genr yhat_12 = exp(lyhat_12)
genr corr_yhat_12 = yhat_12*exp($ess/(2*$df))
```

4.6.5 Generalized R^2

A generalized version of the goodness-of-fit statistic R^2 can be obtained by taking the squared correlation between the actual values of the dependent variable and those predicted by the regression. The following script reproduces the results from section 4.4.4 of your textbook.

```
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ
```

```
genr l_yhat = $yhat
genr y = exp(l_yhat)
genr corr1 = corr(y, wage)
genr Rsquare = corr1^2
```

4.6.6 Prediction Interval

In this script the 95% prediction interval for someone having 12 years of education is estimated.

```
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ
genr lyhat_12 = $coeff(const) + $coeff(educ)*12
genr sig2 = $ess/$df
genr f = sig2 + sig2/$nobs + ((12-mean(educ))^2)*($stderr(educ)^2)
genr sef = sqrt(f)
genr lb = exp(lyhat_12-1.96*sef)
genr ub = exp(lyhat_12+1.96*sef)
print lyhat_12 sig2 f sef lb ub
```

4.7 Script

```
open c:\userdata\gretl\data\poe\food.gdt

ols y const x
genr yhat0 = $coeff(const) + $coeff(x)*20
genr f=8013.2941+(8013.2941/40)+4.3818*(20-19.6047)**2
genr ub=yhat0+2.0244*sqrt(f)
genr lb=yhat0-2.0244*sqrt(f)

#Prediction Intervals
ols y const x
genr yhat0=$coeff(const)+20*$coeff(x)
genr sig2 = $ess/$df
genr f = sig2 + sig2/$nobs + ((20-mean(x))^2)*($stderr(x)^2)
genr lb = yhat0-critical(t,$df,0.025)*sqrt(f)
genr ub = yhat0+critical(t,$df,0.025)*sqrt(f)

#Plot predictions vs actual food exp
#note: the plot will be written to a file.
#To see the plot, open a console window and execute the commands
ols y const x
genr yhatime = $yhat
gnuplot yhat1 y

#Testing normality of errors
ols y const x
genr uhat1 = $uhat
summary uhat1

#Wheat yield example
open c:\userdata\gretl\data\poe\wa-wheat.gdt
ols green const time
gnuplot green time

ols green const time
genr yhat1 = $yhat
genr uhat1 = $uhat
gnuplot green yhat1 uhat1 --with-lines --time-series

genr t3=time^3/1000000
ols green const t3
genr yhat2 = $yhat
genr uhat2 = $uhat
gnuplot green yhat2 uhat2 --with-lines --time-series

#Growth model example
open c:\userdata\gretl\data\poe\wa-wheat.gdt
genr lyield = log(green)
ols lyield const time

#Wage Equation
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
```

```
ols l_wage const educ
genr lb = $coeff(educ) - 1.96 * $stderr(educ)
genr ub = $coeff(educ) + 1.96 * $stderr(educ)
print lb ub

#Predictions in the Log-linear model
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ
genr lyhat_12 = $coeff(const) + $coeff(educ)*12
genr yhat_12 = exp(lyhat_12)
genr corr_yhat_12 = yhat_12*exp($ess/(2*$df))

#Generalized R-Square
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ
genr l_yhat = $yhat
genr y = exp(l_yhat)
genr corr1 = corr(y, wage)
genr Rsquare = corr1^2

#Prediction interval
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ
genr lyhat_12 = $coeff(const) + $coeff(educ)*12
genr sig2 = $ess/$df
genr f = sig2 + sig2/$nobs + ((12-mean(educ))^2)*($stderr(educ)^2)
genr sef = sqrt(f)
genr lb = exp(lyhat_12-1.96*sef)
genr ub = exp(lyhat_12+1.96*sef)
print lyhat_12 sig2 f sef lb ub
```

Chapter 5

Multiple Regression Model

The multiple regression model is an extension of the simple model discussed in Chapter 2. The main difference is that the multiple linear regression model contains more than one explanatory variable. This changes the interpretation of the coefficients slightly and requires another assumption. The general form of the model is shown in equation (5.1) below.

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + e_i \quad i = 1, 2, \ldots, N \tag{5.1}$$

where y_i is your dependent variable, x_{ik} is the i^{th} observation on the k^{th} independent variable, $k = 2, 3, \ldots, K$, e_i is random error, and $\beta_1, \beta_2, \ldots, \beta_K$ are the parameters you want to estimate. Just as in the simple linear regression model, each error, e_i, has an average value of zero for each value of the independent variables; each has the same variance, σ^2, and are uncorrelated with any of the other errors. In order to be able to estimate each of the βs, none of the independent variables can be an exact linear combination of the others. This serves the same purpose as the assumption that each independent variable of the simple linear regression take on at least two different values in your dataset. The error assumptions can be summarized as $e_i|x_{i2}, x_{i3}, \ldots x_{iK}\ iid\ (0, \sigma^2)$. Recall from Chapter 2 that expression *iid* means that the errors are statistically independent from one another (and therefor uncorrelated) and each has the same probability distribution. Taking a random sample from a single population accomplishes this.

The parameters $\beta_2, \beta_3, \ldots, \beta_K$ are referred to as *slopes* and each slope measures the effect of a 1 unit change in x_{ik} on the average value of y_i, *holding all other variables in the equation constant.* The conditional interpretation of the coefficient is important to remember when using multiple linear regression.

The example used in this chapter models the sales for Big Andy's Burger Barn. The model includes two explanatory variables and a constant.

$$S_i = \beta_1 + \beta_2 P_i + \beta_3 A_i + e_i \quad i = 1, 2, \ldots, N \tag{5.2}$$

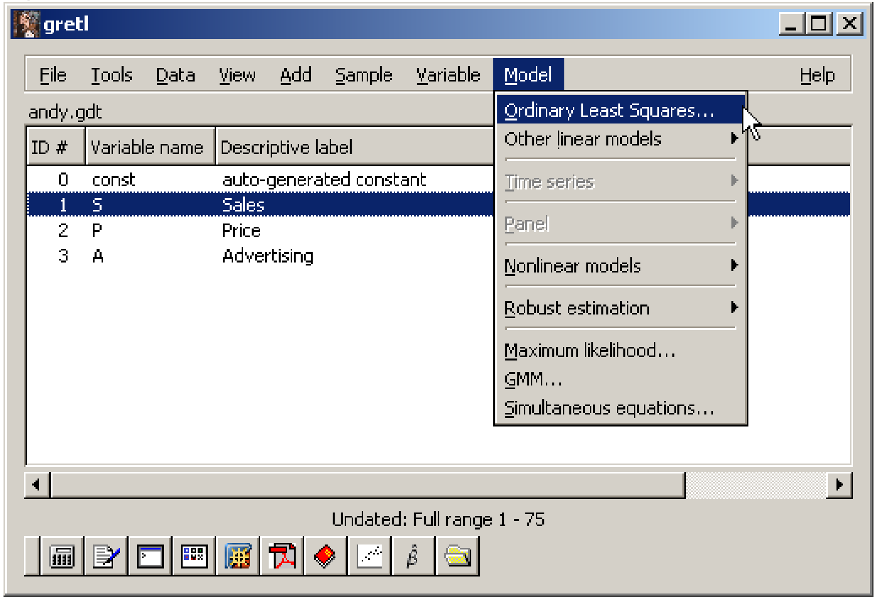
where S_i is monthly sales in a given city and is measured in \$1,000 increments, P_i is price of a hamburger measured in dollars, and A_i is the advertising expenditure also measured in thousands of dollars.

5.1 Linear Regression

The parameters of the model are estimated using least squares which can be done using the pull-down menus and dialog boxes (GUI) or by using **gretl** language itself. Both of these will be demonstrated below. The GUI makes it easy to estimate this model using least squares. There are actually two ways to open the dialog box. The first is to use the pull-down menu. Select `Model>Ordinary Least Squares` from the main **gretl** window as shown below in Figure 5.1. This brings up the dialog box shown in Figure 5.2. As in Chapter 2 you need to put the dependent variable (S) and the independent variables (*const*, P, and A) in the appropriate boxes. Click OK and the model is estimated.

There is a shortcut to get to the specify model dialog box. On the toolbar located at the bottom of the main **gretl** window is a button labeled $\hat{\beta}$. Clicking on this button as shown in Figure 5.3 will open the OLS specify model dialog box in Figure 5.2.

Figure 5.1: Using the pull-down menu to open the ordinary least squares dialog box.

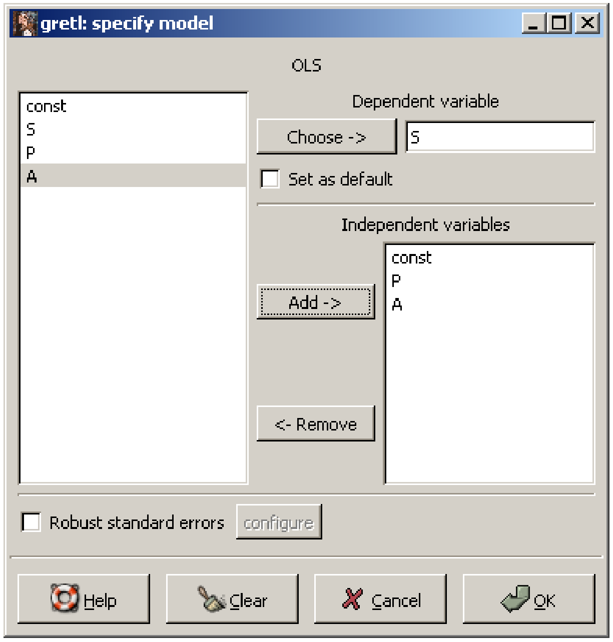

gretl
File Tools Data View Add Sample Variable Model Help
Ordinary Least Squares...
Other linear models
Time series
Panel
Nonlinear models
Robust estimation
Maximum likelihood...
GMM...
Simultaneous equations...

andy.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	S	Sales
2	P	Price
3	A	Advertising

Undated: Full range 1 - 75

Figure 5.2: The `specify model` dialog box for ordinary least squares (OLS)



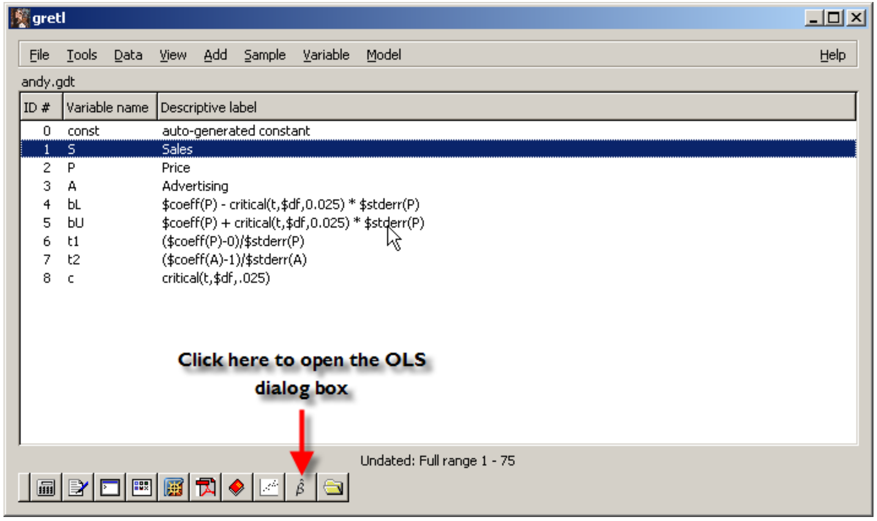
Dependent variable: S

☐ Set as default

Independent variables: const, P, A

☐ Robust standard errors

Figure 5.3: The OLS shortcut



andy.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	S	Sales
2	P	Price
3	A	Advertising
4	bL	$coeff(P) - critical(t,$df,0.025) * $stderr(P)
5	bU	$coeff(P) + critical(t,$df,0.025) * $stderr(P)
6	t1	($coeff(P)-0)/$stderr(P)
7	t2	($coeff(A)-1)/$stderr(A)
8	c	critical(t,$df,.025)

Click here to open the OLS dialog box

Undated: Full range 1 - 75

5.2 Big Andy's Burger Barn

To estimate the model for Big Andy's, we'll use a script file. The following two lines are typed into a script file which is executed by clicking your mouse on the "gear" button of the script window.

```
open c:\userdata\gretl\data\poe\andy.gdt
ols S const P A
```

This assumes that the **gretl** data set `andy.gdt` is installed at `c:\userdata\gretl\data\poe\`. The results, in tablular form, are in Table 5.1 and match those presented in the textbook.

In addition to providing information about how sales change when price or advertising change, the estimated equation can be used for prediction. To predict sales revenue for a price of $5.50 and an advertising expenditure of $1,200 we can use the `genr` to do the computations. From the console,

```
? genr S_hat = $coeff(const) + $coeff(P)*5.5 + $coeff(A)*1.2
Generated scalar S_hat (ID 4) = 77.6555
```

Table 5.1: The regression results from Big Andy's Burger Barn

Model 1: OLS estimates using the 75 observations 1–75
Dependent variable: S

Variable	Coefficient	Std. Error	*t*-statistic	p-value
const	118.914	6.35164	18.7217	0.0000
P	−7.9078	1.09599	−7.2152	0.0000
A	1.86258	0.683195	2.7263	0.0080

Mean of dependent variable	77.3747
S.D. of dependent variable	6.48854
Sum of squared residuals	1718.94
Standard error of residuals ($\hat{\sigma}$)	4.88612
Unadjusted R^2	0.448258
Adjusted $\bar{R}^2$	0.432932
$F(2, 72)$	29.2479
Log-likelihood	−223.87
Akaike information criterion	453.739
Schwarz Bayesian criterion	460.691
Hannan–Quinn criterion	456.515

which matches the result in your text.

5.2.1 SSE, R^2 and Other Statistics

Other important output is included in Table 5.1. For instance, you'll find the sum of squared errors (SSE) which **gretl** refers to as "sum of squared residuals." In this model $SSE = 1718.94$. To obtain the estimated variance, $\hat{\sigma}^2$, divide SSE by the available degrees of freedom to obtain

$$\hat{\sigma}^2 = \frac{SSE}{N-K} = \frac{1718.94}{75-3} = 23.874 \tag{5.3}$$

The square root of this number is referred to by **gretl** as the "Standard error of residuals, $\hat{\sigma}$" and is reported to be 4.88612. **Gretl** also reports R^2 in this table. If you want to compute your own versions of these statistics using the total sum of squares from the model, you'll have to use `Anaylsis>ANOVA` from the model's pull-down menu to generate the ANOVA table. Refer to section 4.2 for details.

To compute your own from the standard **gretl** output recall that

$$\hat{\sigma}_y = \sqrt{\frac{SST}{N-1}} \tag{5.4}$$

The statistic $\hat{\sigma}_y$ is printed by **gretl** and referred to as "S.D. of dependent variable" which is reported to be 6.48854. A little algebra reveals

$$SST = (N-1)\hat{\sigma}_y^2 = 74 * 6.48854 = 3115.485 \tag{5.5}$$

Then,

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{1718.94}{3115.485} = 0.448 \tag{5.6}$$

Otherwise, the goodness-of-fit statistics printed in the **gretl** regression output or the ANOVA table are perfectly acceptable.

Gretl also reports the **adjusted** R^2 in the standard regression output. The adjusted R^2 imposes a small penalty to the usual R^2 when a variable is added to the model. Adding a variable with any correlation to y always reduces SSE and increases the size of the usual R^2. With the adjusted version, the improvement in fit may be outweighed by the penalty and it could become smaller as variables are added. The formula is:

$$\bar{R}^2 = 1 - \frac{SSE/(N-K)}{SST/(N-1)} \tag{5.7}$$

This sometimes referred to as "R-bar squared," (i.e., $\bar{R}^2$) although in **gretl** it is called "adjusted R-squared." For Big Andy's Burger Barn the adjusted R-squared is equal to 0.4329.

5.2.2 Covariance Matrix and Confidence Intervals

Gretl can be used to print the variance-covariance matrix by using the pull-down menu as shown in Figure 2.13. Or, the `--vcv` option can be used with the `ols` command to obtain this result from the console or using a script. The example code is:

```
open c:\userdata\gretl\data\poe\andy.gdt
ols S const P A --vcv
```

Confidence intervals are obtained using the `genr` command in the same way as in Chapter 3. The **gretl** commands

```
genr bL = $coeff(P) - critical(t,$df,0.025) * $stderr(P)
genr bU = $coeff(P) + critical(t,$df,0.025) * $stderr(P)
```

Remember, you can also summon the 95% confidence intervals from the model window using the pull-down menu by choosing `Analysis>Confidence intervals for coefficients`.

5.2.3 t-Tests, Critical Values, and P-values

In Section 3.3 we used the GUI to obtain test statustics, critical values and p-values. However, it is much easier to use the the `genr` command from either the console or as a script to compute these. For t-ratios and one- and two-sided hypothesis tests the appropriate commands are:

```
genr t1 = ($coeff(P)-0)/$stderr(P)
genr t2 = ($coeff(A)-1)/$stderr(A)
```

The critical values for the t_{72} and the p-values for the two statistics can be easily obtained using the command

```
genr c=critical(t,$df,0.025)
pvalue t $df t1
pvalue t $df t2
```

These last three commands produce the output shown below:

```
? genr c=critical(t,$df,.025)
Generated scalar c (ID 8) = 1.99346
? pvalue t $df t1

t(72): area to the right of -7.21524 =~ 1
(to the left: 2.212e-010)
(two-tailed value = 4.424e-010; complement = 1)
? pvalue t $df t2

t(72): area to the right of 1.26257 = 0.105408
(two-tailed value = 0.210817; complement = 0.789183)
```

It is interesting to note that when a negative t-ratio is used in the `pvalue` function, **gretl** returns both the area to its right, the area to its left and the sum of the two areas. So, for the alternative hypothesis that the coefficient on P is less than zero (against the null that it is zero), the p-value is the area to the left of the computed statistic is the desired one.

5.3 Script

```
open c:\userdata\gretl\data\poe\andy.gdt

#Change the descriptive labels and graph labels
setinfo S -d "Monthly Sales revenue ($1000)" -n "Monthly Sales ($1000)"
setinfo P -d "$ Price" -n "Price"
setinfo A -d "Monthy Advertising Expenditure ($1000)" -n \
    "Monthly Advertising ($1000)

#Print the new labels to the screen
labels

#Summary Statistics
summary S P A

#Regression with covariance matrix printed
ols S const P A --vcv

#Prediction
genr S_hat = $coeff(const) + $coeff(P)*5.5 + $coeff(A)*1.2

#Confidence Intervals
  #Price
  genr bL = $coeff(P) - critical(t,$df,0.025) * $stderr(P)
  genr bU = $coeff(P) + critical(t,$df,0.025) * $stderr(P)
  #Advertising
  genr bL = $coeff(A) - critical(t,$df,0.025) * $stderr(A)
  genr bU = $coeff(A) + critical(t,$df,0.025) * $stderr(A)

#t-ratios
  #Two tail tests
  genr t1 = ($coeff(P)-0)/$stderr(P)
  genr t2 = ($coeff(A)-0)/$stderr(A)

  #One tail test
  genr t3 = ($coeff(A)-1)/$stderr(A)

#Ctitical value and p-values
```

```
genr c=critical(t,$df,.025)
pvalue t $df t1 #used for both 1 and 2 tail tests
pvalue t $df t2
pvalue t $df t3
```

Chapter 6

Further Inference in the Multiple Regression Model

In this chapter several extensions of the multiple linear regression model are considered. First, we test joint hypotheses about parameters in a model and then learn how to impose linear restrictions on the parameters. A condition called *collinearity* is also explored.

6.1 F-test

An F-statistic can be used to test multiple hypotheses in a linear regression model. In linear regression there are several different ways to derive and compute this statistic, but each yields the same result. The one used here compares the sum of squared errors (SSE) in a regression model estimated under the null hypothesis (H_0) to the SSE of a model under the alternative (H_1). If the sum of squared errors from the two models are similar, then there is not enough evidence to reject the restrictions. On the other hand, if imposing restrictions implied by H_0 alter SSE substantially, then the restrictions it implies don't fit the data and we reject them.

In the Big Andy's Burger Barn example we estimated the model

$$S_i = \beta_1 + \beta_2 P_i + \beta_3 A_i + e_i \tag{6.1}$$

Suppose we wish to test the hypothesis that price, P_i, has no effect on sales against the alternative that it does. Thus, $H_0 : \beta_2 = 0$ and $H_1 : \beta_2 \neq 0$. Another way to express this is in terms of the models each hypothesis implies.

$$\begin{aligned} H_0 : &\quad \beta_1 + \beta_3 A_i + e_i \\ H_1 : &\quad \beta_1 + \beta_2 P_i + \beta_3 A_i + e_i \end{aligned}$$

The model under H_0 is **restricted** compared to the model under H_1 since in it $\beta_2 = 0$. The F-statistic used to test H_0 versus H_1 estimates each model by least squares and compares their respective sum of squared errors using the statistic:

$$F = \frac{(SSE_r - SSE_u)/J}{SSE_u/(N-K)} \sim F_{J,N-K} \qquad \text{if } H_0 \text{ is true} \tag{6.2}$$

The sum of squared errors from the unrestricted model (H_1) is denoted SSE_u and that of the restricted model (H_0) is SSE_r. The numerator is divided by the number of hypotheses being tested, J. In this case that is 1 since there is only a single restriction implied by H_0. The denominator is divided by the total number of degrees of freedom in the unrestricted regression, $N - K$. N is the sample size and K is the number of parameters in the unrestricted regression. When the errors of your model are (1) independently and identically distributed (*iid*) normals with zero mean and constant variance (e_t *iid* $N(0, \sigma^2)$) and (2) H_0 is true, then this statistic has an F distribution with J numerator and $N-K$ denominator degrees of freedom. Choose a significance level and compute this statistic. Then compare its value to the appropriate critical value from the F table or compare it's p-value to the chosen significance level.

The script to estimate the models under H_0 and H_1 and to compute the test statistic is given below.

```
open C:\userdata\gretl\data\poe\andy.gdt
ols S const P A
genr sseu = $ess
genr unrest_df = $df
```

```
ols S const A
genr sser = $ess

genr Fstat=((sser-sseu)/1)/(sseu/(unrest_df))
pvalue F 1 unrest_df Fstat
```

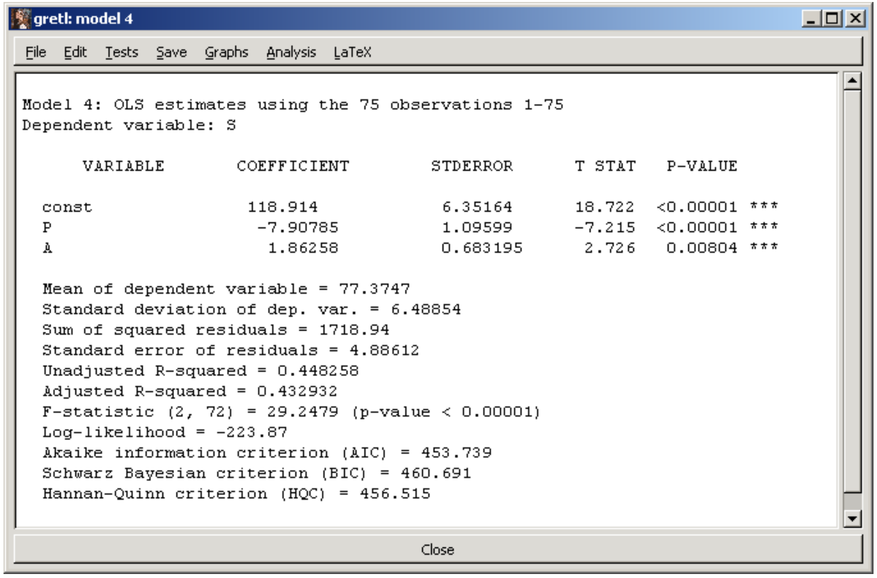
Gretl refers to the sum of squared residuals (SSE) as the "error sum of squares" and it is retrieved from the regression results using the syntax `genr sseu = $ess`. In this case, `$ess` points to the error sum of squares computed in the regression that precedes it. You'll also want to save the degrees of freedom in the unrestricted model so that you can use it in the computation of the p-value for the F-statistic. In this case, the F-statistic has 2 known parameters (J=1 and $N - K$=unrest_df) that are used as arguments in the `pvalue` function.

There are a number of other ways within **gretl** to do this test. These are available through scripts, but it may be useful to demonstrate how to access them through the GUI. First, you'll want to estimate the model using least squares. From the pull-down menu (see Figure 5.1) select `Model>Ordinary Least Squares`, specify the unrestricted model (Figure 5.2), and run the regression. This yields the result shown in Figure 6.1.

You'll notice that along the menu bar at the top of this window there are a number of options that are available to you. Choose `Tests` and the pull-down menu shown in Figure 6.2 will be revealed. The first four options in 6.2 are highlighted and these are the ones that are most pertinent to the discussion here. This menu provides you an easy way to omit variables in the null, add variables to the alternative, test a sum of your coefficients, or to test arbitrary linear restrictions on the parameters of your model.

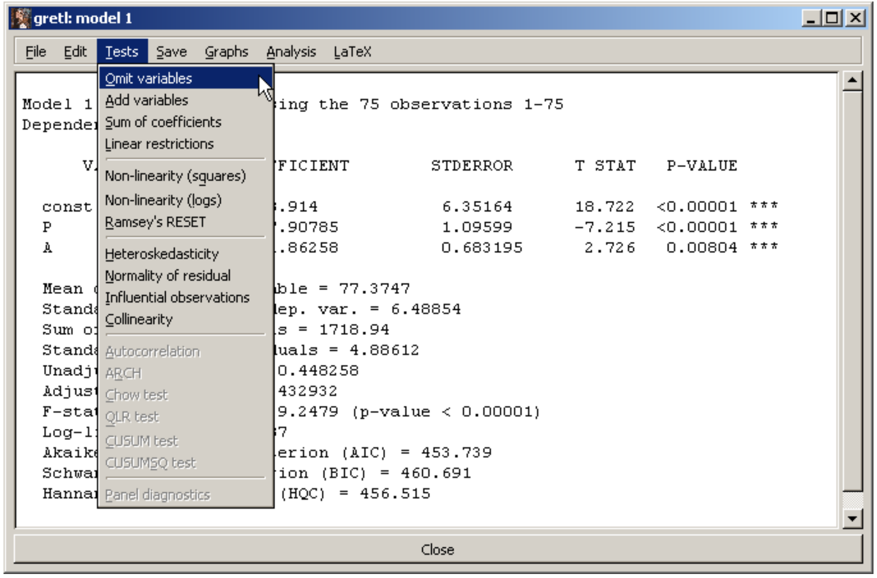
Since this test involves imposing a zero restriction on the coefficient of the variable P, we can use the `omit` option. This brings up the dialog box shown in Figure 6.3. Notice the two radio buttons at the bottom of the window. The first is labeled *Estimate reduced model* and this is the one you want to use to compute equation 6.2. If you select the other, no harm is done. It is computed in a different way, but produces the same answer in a linear model. The only advantage of the Wald test (second option) is that the restricted model does not have to be estimated in order to perform the test. Given **gretl**'s speed, there is not much to be gained here from using

Figure 6.1: The model results from least squares regression using the pull-down menu



```
Model 4: OLS estimates using the 75 observations 1-75
Dependent variable: S

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const               118.914           6.35164      18.722   <0.00001 ***
  P                    -7.90785         1.09599      -7.215   <0.00001 ***
  A                     1.86258         0.683195      2.726    0.00804 ***

  Mean of dependent variable = 77.3747
  Standard deviation of dep. var. = 6.48854
  Sum of squared residuals = 1718.94
  Standard error of residuals = 4.88612
  Unadjusted R-squared = 0.448258
  Adjusted R-squared = 0.432932
  F-statistic (2, 72) = 29.2479 (p-value < 0.00001)
  Log-likelihood = -223.87
  Akaike information criterion (AIC) = 453.739
  Schwarz Bayesian criterion (BIC) = 460.691
  Hannan-Quinn criterion (HQC) = 456.515
```

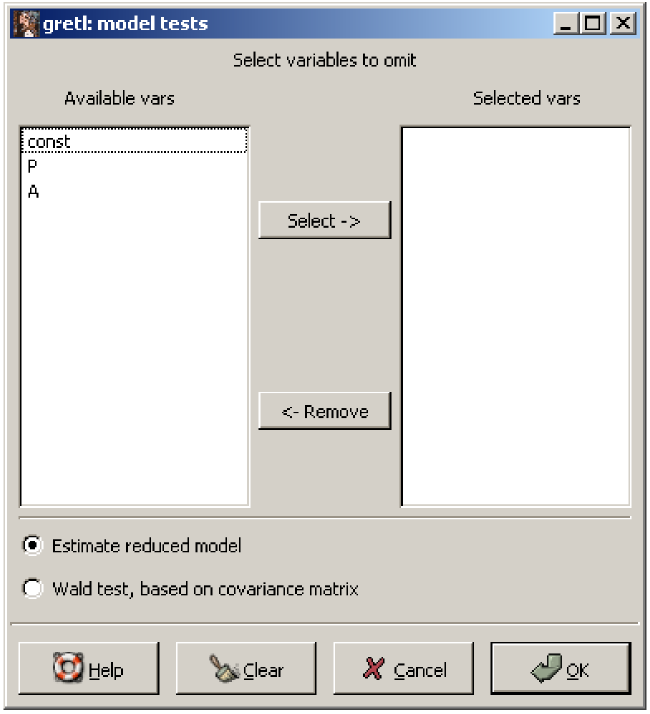
Figure 6.2: Choosing `Tests` from the pull-down menu of the model window reveals several testing options



```
Model 1                    ing the 75 observations 1-75
Depende

      V                    FICIENT        STDERROR      T STAT   P-VALUE

  const                    .914           6.35164      18.722   <0.00001 ***
  P                        .90785         1.09599      -7.215   <0.00001 ***
  A                        .86258         0.683195      2.726    0.00804 ***

  Mean                     ble = 77.3747
  Standa                   ep. var. = 6.48854
  Sum o                    s = 1718.94
  Standa                   uals = 4.88612
  Unadju                   0.448258
  Adjust                   432932
  F-stat                   9.2479 (p-value < 0.00001)
  Log-l                    7
  Akaike                   erion (AIC) = 453.739
  Schwar                   ion (BIC) = 460.691
  Hannan                   (HQC) = 456.515
```

the Wald form of the test, other than it generates less output to view! Select the variable P and click OK to reveal the result shown in Figure 6.4. The interesting thing about this option is that it mimics your manual calculation of the F statistic from the script. It computes the sum of squared errors in the unrestricted and restricted models and computes equation (6.2) based on those regressions. Most pieces of software choose the alternative method (Wald) to compute the test, but you get the same result.

Figure 6.3: The `Omit variables` dialog box available from the `Tests` pull-down menu in the model window.

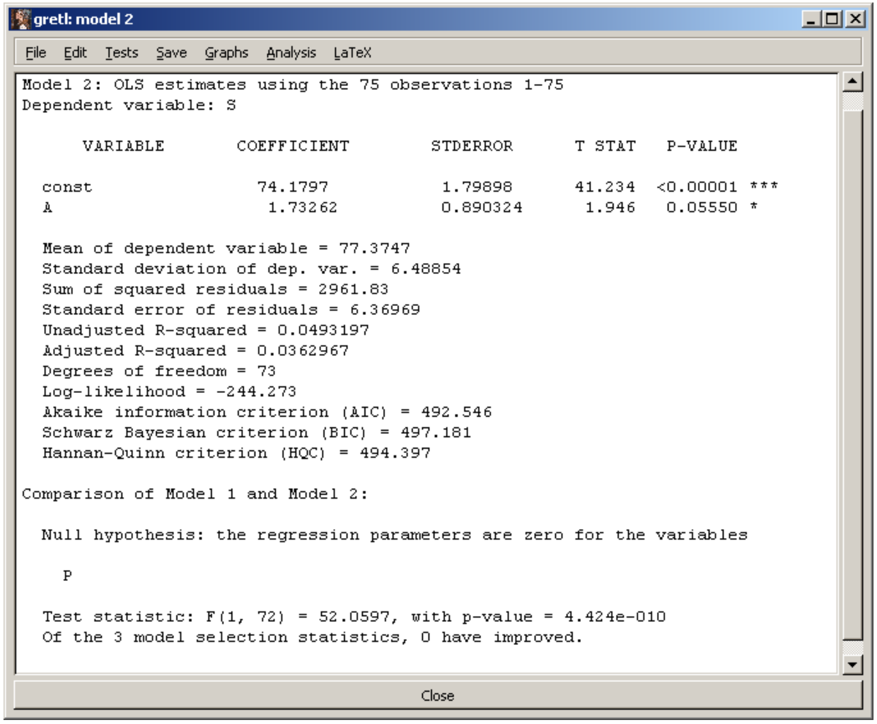


● Estimate reduced model

○ Wald test, based on covariance matrix

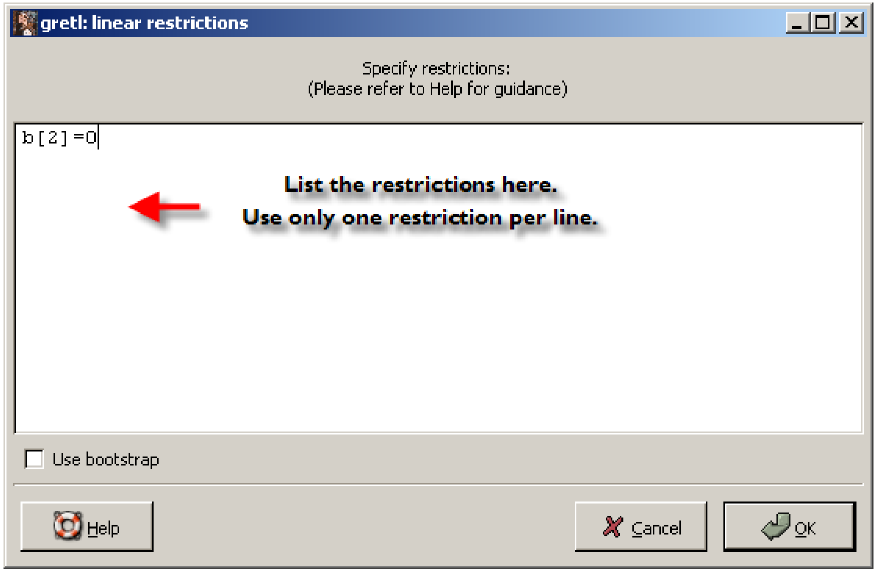
You can also use the `linear restrictions` option from the pull-down menu shown in Figure 6.2. This produces a large dialog box that requires a bit of explanation. The box appears in Figure 6.5. The restrictions you want to impose (or test) are entered here. Each restriction in the set should be expressed as an equation, with a linear combination of parameters on the left and a numeric value to the right of the equals sign. Parameters are referenced in the form `b[variable number]`, where `variable number` represents the position of the regressor in the question, which starts with

Figure 6.4: The results using the `Omit variables` dialog box to test zero restrictions on the parameters of a linear model.



```
Model 2: OLS estimates using the 75 observations 1-75
Dependent variable: S

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                74.1797           1.79898      41.234  <0.00001 ***
  A                     1.73262          0.890324      1.946   0.05550 *

  Mean of dependent variable = 77.3747
  Standard deviation of dep. var. = 6.48854
  Sum of squared residuals = 2961.83
  Standard error of residuals = 6.36969
  Unadjusted R-squared = 0.0493197
  Adjusted R-squared = 0.0362967
  Degrees of freedom = 73
  Log-likelihood = -244.273
  Akaike information criterion (AIC) = 492.546
  Schwarz Bayesian criterion (BIC) = 497.181
  Hannan-Quinn criterion (HQC) = 494.397

Comparison of Model 1 and Model 2:

  Null hypothesis: the regression parameters are zero for the variables

    P

  Test statistic: F(1, 72) = 52.0597, with p-value = 4.424e-010
  Of the 3 model selection statistics, 0 have improved.
```

Figure 6.5: The linear restriction dialog box obtained using the `Linear restrictions` option in the `Tests` pull-down menu.



Specify restrictions:
(Please refer to Help for guidance)

```
b[2]=0
```

List the restrictions here.
Use only one restriction per line.

□ Use bootstrap

1. This means that β_2 is equivalent to `b[2]`. Restricting $\beta_2 = 0$ is done by issuing `b[2]=0` in this dialog. Sometimes you'll want to use a restriction that involves a multiple of a parameter e.g., $3\beta_3 = 2$. The basic principle is to place the multiplier first, then the parameter, using * to multiply. So, in this case the restriction in **gretl** becomes `3*b[3] = 2`.

When you use the console or a script instead of the pull-down menu to impose restrictions, you'll have to tell **gretl** where the restrictions start and end. The restriction(s) starts with a `restrict` statement and ends with `end restrict`. The statement will look like this:

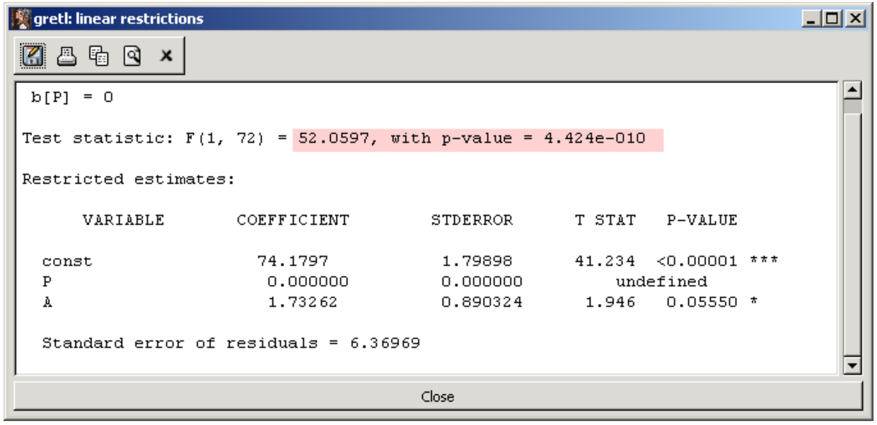
```
open C:\userdata\gretl\data\poe\andy.gdt
ols S const P A

restrict
  b[2] = 0
end restrict
```

When you have more than one restriction to impose or test, put each restriction on its own line. Here is an example of a set of restrictions from a **gretl** script:

```
restrict
    b[1] = 0
    b[2] - b[3] = 0
    b[4] + 2*b[5] = 1
end restrict
```

Of course, if you use the pull-down menu to impose these you can omit the `restrict` and `end restrict` statements. The results you get from using the restrict statements appear in Figure 6.6. The test statistic and its p-value are highlighted in red.

Figure 6.6: The results obtained from using the restrict dialog box.



```
 b[P] = 0

Test statistic: F(1, 72) = 52.0597, with p-value = 4.424e-010

Restricted estimates:

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                 74.1797          1.79898      41.234  <0.00001 ***
  P                      0.000000        0.000000        undefined
  A                      1.73262         0.890324      1.946   0.05550 *

  Standard error of residuals = 6.36969
```

6.2 Regression Significance

To statistically determine whether the regression is actually a model of the average behavior of your dependent variable, you can use the F-statistic. In this case, H_0 is the proposition that y does not depend on your independent variables, and H_1 is that it does.

$$\begin{aligned} H_o: &\quad \beta_1 + e_i \\ H_1: &\quad \beta_1 + \beta_2 x_{i2} + \ldots + \beta_k x_{ik} + e_i \end{aligned}$$

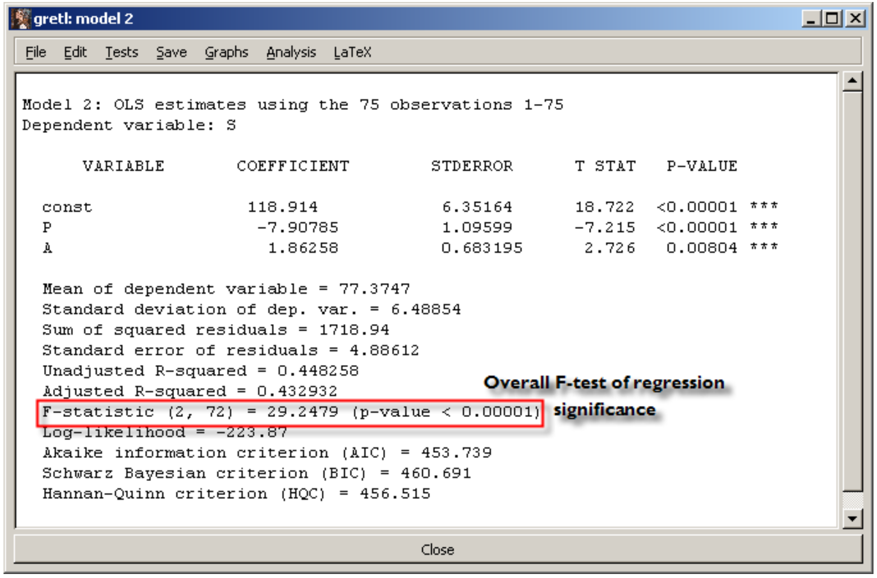
The null hypothesis can alternately be expressed as $\beta_2, \beta_3, \ldots, \beta_K = 0$, a set of $K - 1$ linear restrictions. In Big Andy's Burger Barn the script is

```
open C:\userdata\gretl\data\poe\andy.gdt
ols S const P A
genr sseu = $ess
genr unrest_df = $df

ols S const
genr sser = $ess
genr rest_df = $df

genr J = rest_df - unrest_df
genr Fstat=((sser-sseu)/J)/(sseu/(unrest_df))
pvalue F J unrest_df Fstat
```

The only difference is that you now have two hypotheses to test jointly and the numerator degrees of freedom for the F-statistic is $J = K - 1 = 2$. The saved residual degrees of freedom from the restricted model can be used to obtain the number of restrictions imposed. Each unique restriction in a linear model reduces the number of parameters in the model by one. So, imposing one restriction on a three parameter unrestricted model (e.g., Big Andy's), reduces the number of parameters in the restricted model to two. Let K_r be the number of regressors in the restricted model and K_u the number in the unrestricted model. Subtracting the degrees of freedom in the unrestricted model $(N - K_u)$ from those of the restricted model $(N - K_r)$ will yield the number of restrictions you've imposed, i.e., $(N - K_r) - (N - K_u) = (K_u - K_r) = J$.

The test of regression significance is important enough that it appears on the default output of every linear regression estimated using **gretl**. The statistic and its p-value are highlighted in Figure 6.7. The F-statistic for this test and its p-value are highlighted. Since the p-value is less than = .05, we reject the null hypothesis that the model is insignificant at the five percent level.

Figure 6.7: The overall F-statistic of regression significance is produced by default when you estimate a linear model using least squares.



```
Model 2: OLS estimates using the 75 observations 1-75
Dependent variable: S

     VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const              118.914            6.35164      18.722   <0.00001 ***
  P                   -7.90785          1.09599      -7.215   <0.00001 ***
  A                    1.86258          0.683195      2.726    0.00804 ***

  Mean of dependent variable = 77.3747
  Standard deviation of dep. var. = 6.48854
  Sum of squared residuals = 1718.94
  Standard error of residuals = 4.88612
  Unadjusted R-squared = 0.448258
  Adjusted R-squared = 0.432932
  F-statistic (2, 72) = 29.2479 (p-value < 0.00001)
  Log-likelihood = -223.87
  Akaike information criterion (AIC) = 453.739
  Schwarz Bayesian criterion (BIC) = 460.691
  Hannan-Quinn criterion (HQC) = 456.515
```

Overall F-test of regression significance

6.3 Extended Model

In the extended model, we add the squared level of advertising to the model, A^2, which permits the possibility of diminishing returns to advertising. The model to be estimated is

$$S_i = \beta_1 + \beta_2 P_i + \beta_3 A_i + \beta_4 A_i^2 + e_i \tag{6.3}$$

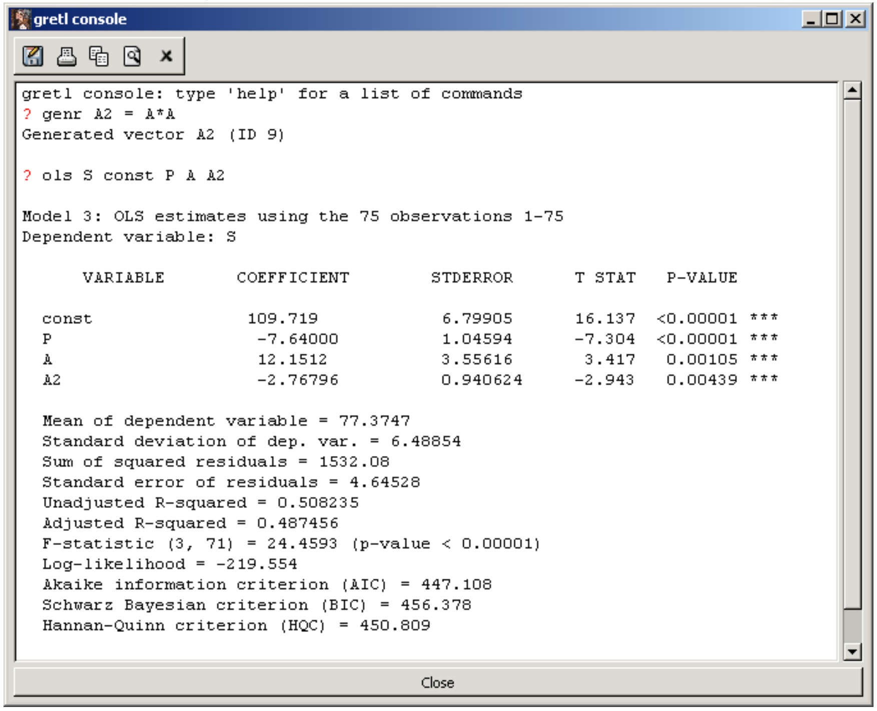
This time, open a console window from the toolbar by clicking on the **open gretl console** button; then, generate a new variable, A^2 using `genr A2 = A*A`. Then estimate (6.3) using the command: `ols S const P A A2`. This yields the output in Figure 6.8:

6.3.1 Is Advertising Significant?

The marginal effect of another unit of advertising on average sales is

$$\frac{\partial E[Sales_i]}{\partial A_i} = \beta_3 + 2\beta_4 A_i \tag{6.4}$$

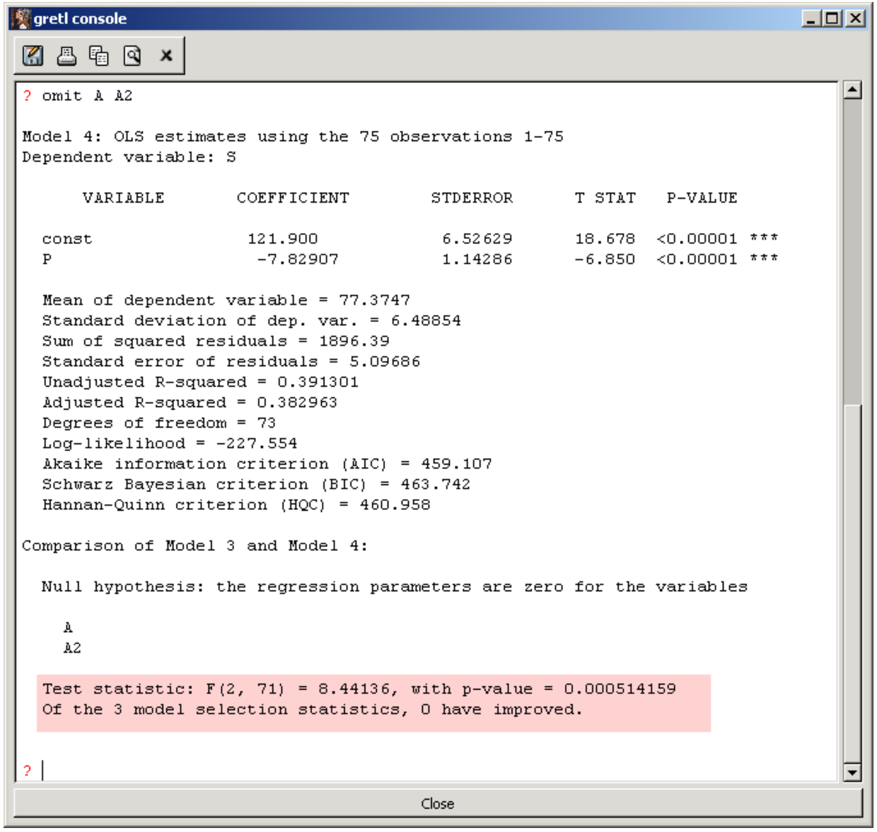
Figure 6.8: The results of the extended model of Big Andy's Burger Barn obtained from the **gretl** console.



```
gretl console: type 'help' for a list of commands
? genr A2 = A*A
Generated vector A2 (ID 9)

? ols S const P A A2

Model 3: OLS estimates using the 75 observations 1-75
Dependent variable: S

      VARIABLE       COEFFICIENT        STDERROR       T STAT   P-VALUE

  const               109.719            6.79905       16.137  <0.00001 ***
  P                    -7.64000          1.04594       -7.304  <0.00001 ***
  A                    12.1512           3.55616        3.417   0.00105 ***
  A2                   -2.76796          0.940624      -2.943   0.00439 ***

  Mean of dependent variable = 77.3747
  Standard deviation of dep. var. = 6.48854
  Sum of squared residuals = 1532.08
  Standard error of residuals = 4.64528
  Unadjusted R-squared = 0.508235
  Adjusted R-squared = 0.487456
  F-statistic (3, 71) = 24.4593 (p-value < 0.00001)
  Log-likelihood = -219.554
  Akaike information criterion (AIC) = 447.108
  Schwarz Bayesian criterion (BIC) = 456.378
  Hannan-Quinn criterion (HQC) = 450.809
```

This means that the effect of another unit of advertising depends on the current level of advertising, A_i. To test for the significance of all levels of advertising requires you to test the joint hypothesis $H_0 : \beta_3 = \beta_4 = 0$ against the alternative $H_a : \beta_3 \neq 0$ or $\beta_4 \neq 0$. From the console, following estimation of the full model, type `omit A A2` and **gretl** will execute the omit variables test discussed in the preceding section. The console window is shown in Figure 6.9 below and the outcome from the omit test is highlighted.

Figure 6.9: Testing the significance of Advertising using the omit statement from the console.



```
? omit A A2

Model 4: OLS estimates using the 75 observations 1-75
Dependent variable: S

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const               121.900            6.52629      18.678  <0.00001 ***
  P                    -7.82907          1.14286      -6.850  <0.00001 ***

  Mean of dependent variable = 77.3747
  Standard deviation of dep. var. = 6.48854
  Sum of squared residuals = 1896.39
  Standard error of residuals = 5.09686
  Unadjusted R-squared = 0.391301
  Adjusted R-squared = 0.382963
  Degrees of freedom = 73
  Log-likelihood = -227.554
  Akaike information criterion (AIC) = 459.107
  Schwarz Bayesian criterion (BIC) = 463.742
  Hannan-Quinn criterion (HQC) = 460.958

Comparison of Model 3 and Model 4:

  Null hypothesis: the regression parameters are zero for the variables

    A
    A2

  Test statistic: F(2, 71) = 8.44136, with p-value = 0.000514159
  Of the 3 model selection statistics, 0 have improved.

? 
```

6.3.2 Optimal Level of Advertising

The optimal level of advertising is that amount where the last dollar spent on advertising results in only 1 dollar of additional sales (we are assuming here that the marginal cost of producing and selling another burger is zero!). Find the level of level of advertising, A_o, that solves:

$$\frac{\partial E[Sales_i]}{\partial A_i} = \beta_3 + 2\beta_4 A_o = \$1 \tag{6.5}$$

Plugging in the least squares estimates from the model and solving for A_o can be done in **gretl**. A little algebra yields

$$A_o = \frac{\$1 - \beta_3}{2\beta_4} \tag{6.6}$$

The script in **gretl** to compute this follows.

```
open C:\userdata\gretl\data\poe\andy.gdt
genr A2 = A*A
ols S const P A A2
genr Ao =(1-$coeff(A))/(2*$coeff(A2))
```

which generates the result:

```
? genr Ao =(1-$coeff(A))/(2*$coeff(A2))
  Generated scalar Ao (ID 7) = 2.01434
```

This implies that the optimal level of advertising is estimated to be approximately \$2014.

To test the hypothesis that \$1900 is optimal (remember, A is measured in \$1000)

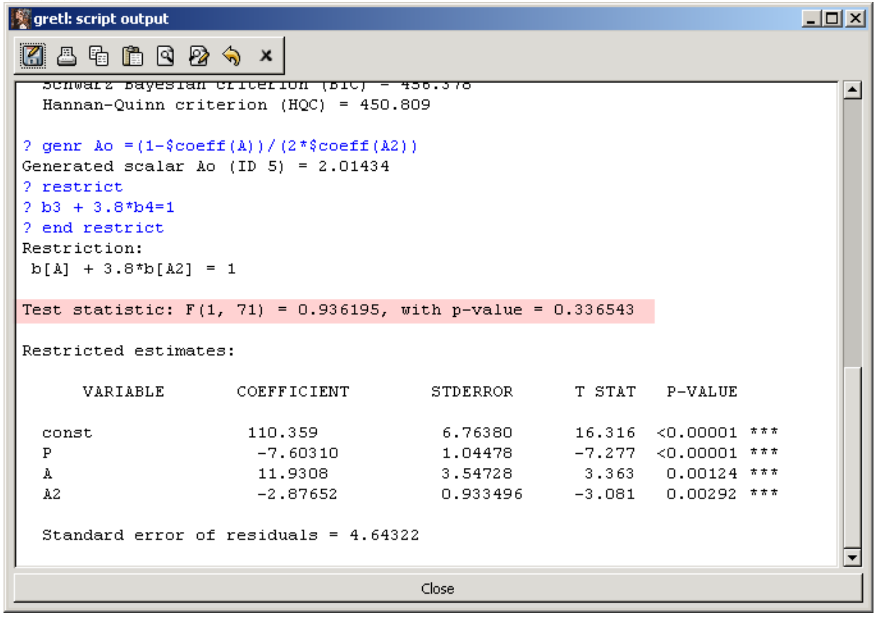
$$\begin{aligned} H_o: &\quad \beta_3 + 2\beta_4 1.9 = 1 \\ H_1: &\quad \beta_3 + 2\beta_4 1.9 \neq 1 \end{aligned}$$

you can use a t-test or an F-test. Following the regression, use

```
restrict
  b[3] + 3.8*b[4]=1
end restrict
```

Remember that `b[3]` refers to the coefficient of the third variable in the regression (`A`) and `b[4]` to the fourth. The output from the script is shown in Figure 6.10.

Figure 6.10: Testing whether $1900 in advertising is optimal using the restrict statement.



```
  Schwarz Bayesian criterion (BIC) = 456.378
  Hannan-Quinn criterion (HQC) = 450.809

? genr Ao =(1-$coeff(A))/(2*$coeff(A2))
Generated scalar Ao (ID 5) = 2.01434
? restrict
? b3 + 3.8*b4=1
? end restrict
Restriction:
 b[A] + 3.8*b[A2] = 1

Test statistic: F(1, 71) = 0.936195, with p-value = 0.336543

Restricted estimates:

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const               110.359           6.76380      16.316  <0.00001 ***
  P                    -7.60310         1.04478      -7.277  <0.00001 ***
  A                    11.9308          3.54728       3.363   0.00124 ***
  A2                   -2.87652         0.933496     -3.081   0.00292 ***

  Standard error of residuals = 4.64322
```

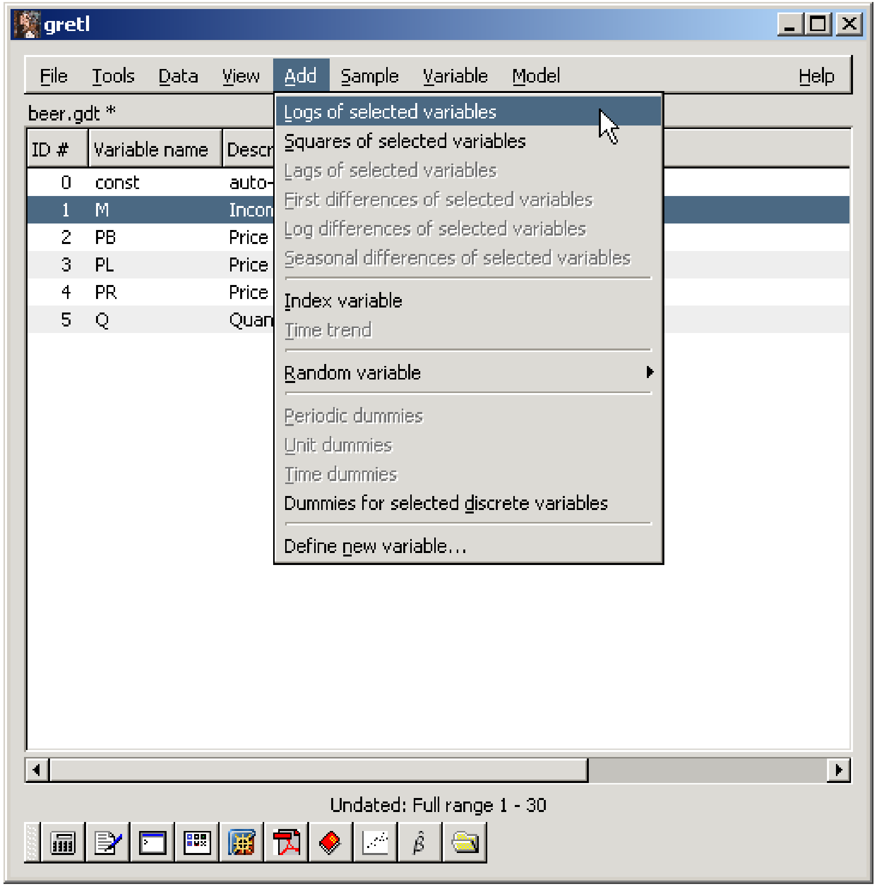
6.4 Nonsample Information

In this section we'll estimate the beer demand model. The data are in `beer.gdt` and are in level form. The model to be estimated is

$$\ln(Q_i) = \beta_1 + \beta_2 ln(PB_i) + \beta_3 ln(PL_i) + \beta_4 ln(PR_i) + \beta_5 ln(M_i) + e_i \quad (6.7)$$

The first thing to do is to convert each of the variables into natural logs. **Gretl** has a built in function for this that is very slick. From the main window, highlight the variables you want to transform with the cursor. Then go to `Add>Logs of selected variables` from the pull-down menu as shown in Figure 6.11. This can also be done is a script or from the console using

Figure 6.11: Use the pull-down menu to add the natural logs of each variable


gretl
File
Tools
Data
View
Add
Sample
Variable
Model
Help
beer.gdt *
Logs of selected variables
Squares of selected variables
Lags of selected variables
First differences of selected variables
Log differences of selected variables
Seasonal differences of selected variables
Index variable
Time trend
Random variable
Periodic dummies
Unit dummies
Time dummies
Dummies for selected discrete variables
Define new variable...

ID #	Variable name	Descr
0	const	auto-
1	M	Incon
2	PB	Price
3	PL	Price
4	PR	Price
5	Q	Quan

Undated: Full range 1 - 30

the command `logs Q PB PL PR M`. The natural log of each of the variables is obtained and the result stored in a new variable with the prefix `l_` ("el" underscore).

No money illusion can be parameterized in this model as $\beta_2 + \beta_3 + \beta_4 + \beta_5 = 0$. This restriction is easily estimated within **gretl** using the restrict dialog or a script as shown below.

```
open C:\userdata\gretl\data\poe\beer.gdt
```

Figure 6.12: **gretl** output for the beer demand

```
? restrict
? b2+b3+b4+b5=0
? end restrict
Restriction:
 b[l_PB] + b[l_PL] + b[l_PR] + b[l_M] = 0

Test statistic: F(1, 25) = 2.49693, with p-value = 0.126639

Restricted estimates:

      VARIABLE  COEFFICIENT     STDERROR      T STAT   P-VALUE

  const           -4.79780       3.71390      -1.292   0.20778
  l_PB            -1.29939       0.165738     -7.840  <0.00001 ***
  l_PL             0.186816      0.284383      0.657   0.51701
  l_PR             0.166742      0.0770752     2.163   0.03989 **
  l_M              0.945829      0.427047      2.215   0.03574 **

  Standard error of residuals = 0.0616756
```

```
logs Q PB PL PR M
ols l_Q const l_PB l_PL l_PR l_M
restrict
  b2+b3+b4+b5=0
end restrict
```

The syntax for the restrictions is new. Instead of b[2]+b[3]+b[4]+b[5]=0 a simpler form is used. This is undocumented in the version I am using (1.6.5) and I am uncertain of whether this will continue to work. It does for now and I've shown it here. Apparently **gretl** is able to correctly parse the variable number from the variable name without relying on the brackets. The output from the **gretl** script output window appears in Figure 6.12.

6.5 Model Specification

There are several issues of model specification explored here. First, it is possible to omit relevant independent variables from your model. A **relevant independent variable** is one that affects the mean of the dependent variable. When you omit a relevant variable that happens to be correlated with any of the other included regressors, least squares suffers from omitted variable bias.

The other possibility is to include **irrelevant variables** in the model. In this case, you include extra regressors that either don't affect y or, if they do, they are not correlated with any of the other regressors. Including irrelevant variables in the model makes least squares less precise than it otherwise would be–this increases standard errors, reduces the power of your hypothesis tests, and increases the size of your confidence intervals.

The example used in the text uses the dataset *edu_inc.gdt*. The first regression

$$faminc_i = \beta_1 + \beta_2 * he_i + \beta_3 we_i + \beta_4 kl6_i + \beta_5 x_{i5} + \beta_6 x_{i6} + e_i \qquad (6.8)$$

where *faminc* is family income, *he* is husband's years of schooling, *we* is woman's years of schooling, and *kl6* are the number of children in the household under age 6. Several variations of this model are estimated. The first includes only *he*, another only *he* and *we*, and one includes the two irrelevant variables, x_5 and x_6. The **gretl** script to estimate these models and test the implied hypothesis restrictions follows. If you type this in yourself, omit the line numbers.

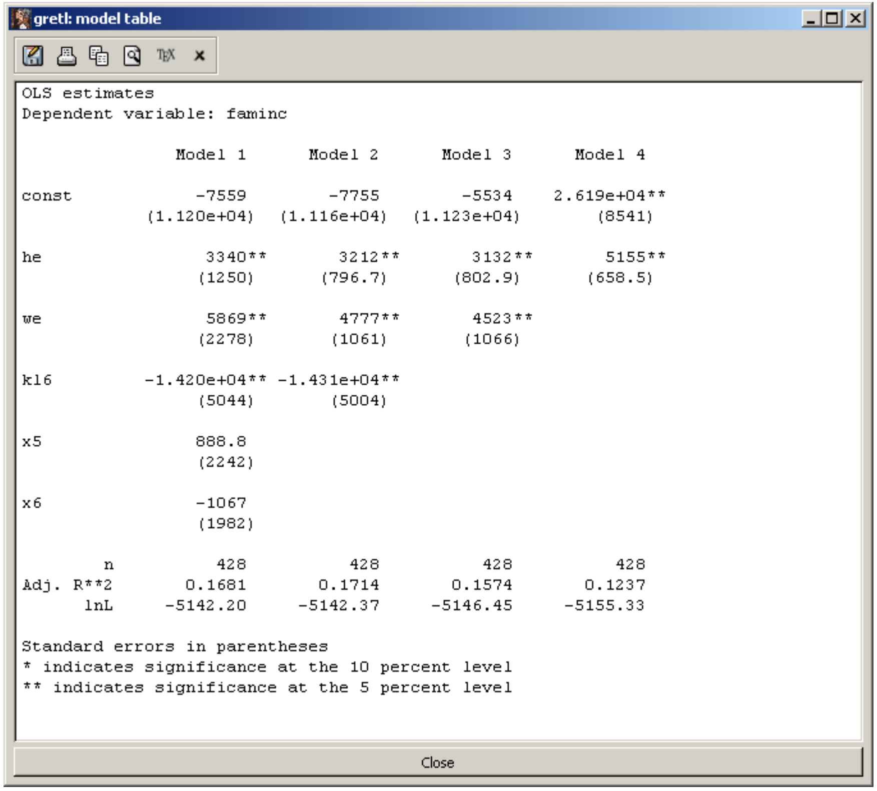
```
#line   code
01      open c:\userdata\gretl\data\poe\edu_inc.gdt
02      ols faminc const he we kl6 x5 x6
03      modeltab add
04      omit x5 x6
05      modeltab add
06      omit kl6
07      modeltab add
08      omit we
09      modeltab add
```

```
10      modeltab show
```

The models can be estimated and saved as icons (`File>Save to session as icon`) within **gretl**. Once they've all been estimated and saved as icons, open a session window (Figure 1.9) and drag each model onto the *model table* icon. Click on the model table icon to reveal the output shown in Figure 6.13.

In the above script, we've used the `modeltab` function after each estimated model to add it to the model table. The final line tells **gretl** to display (`show`) the resulting model table.

Figure 6.13: Save each model as an icon. Open the session window and drag each model to the model table icon. Click on the model table icon to reveal this output.



```
OLS estimates
Dependent variable: faminc

               Model 1      Model 2      Model 3      Model 4

const            -7559        -7755        -5534    2.619e+04**
            (1.120e+04)  (1.116e+04)  (1.123e+04)       (8541)

he                3340**       3212**       3132**       5155**
                 (1250)      (796.7)      (802.9)      (658.5)

we                5869**       4777**       4523**
                 (2278)       (1061)       (1066)

kl6         -1.420e+04** -1.431e+04**
                 (5044)       (5004)

x5               888.8
                 (2242)

x6               -1067
                 (1982)

        n          428          428          428          428
Adj. R**2       0.1681       0.1714       0.1574       0.1237
      lnL     -5142.20     -5142.37     -5146.45     -5155.33

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level
```

One word of caution is in order about the given script and its interpretation. The `omit` statement tests the implied restriction (the coefficient on the omitted variable is zero) versus the estimated model that **immediately precedes it**. Thus, when we test that the coefficient on `kl6` is zero in line 06, the alternative model is the restricted model from line 04, which already excludes $x5$, and $x6$. Thus, only one restriction is being tested. If your intention is to test all of the restrictions (omit $x5$, $x6$ and $kl6$) versus the the completely unrestricted model in line 02 that includes all of the variables, you'll need to modify your code. I'll leave this an an exercise.

6.6 RESET

The **RESET** test is used to assess the adequacy of your functional form. The null hypothesis is that your functional form is adequate. The alternative is that it is not. The test involves running a couple of regressions and computing an F-statistic.

Consider the model

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + e_i \tag{6.9}$$

and the hypothesis

$$\begin{aligned} H_0 : &\quad E[y|x_{i2}, x_{i3}] = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} \\ H_1 : &\quad \text{not } H_0 \end{aligned}$$

Rejection of H_0 implies that the functional form is not supported by the data. To test this, first estimate (6.9) using least squares and save the predicted values, $\hat{y}_i$. Then square and cube $\hat{y}$ and add them back to the model as shown below:

$$\begin{aligned} y_i &= \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \gamma_1 \hat{y}_i^2 + e_i \\ y_i &= \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \gamma_1 \hat{y}_i^2 + \gamma_2 \hat{y}_i^3 + e_i \end{aligned}$$

The null hypotheses to test (against alternative, 'not H_0') are:

$$\begin{aligned} H_0 : &\quad \gamma_1 = 0 \\ H_0 : &\quad \gamma_1 = \gamma_2 = 0 \end{aligned}$$

Estimate the auxiliary models using least squares and test the significance of the parameters of the $\hat{y}^s$. This is accomplished through the following script. Note, the reset command issued after the first regression computes the test associated with $H_0 : \gamma_1 = \gamma_2 = 0$. It is included here so that you can compare the 'canned' result with the one you compute using the two step procedure suggested above. The two results should match.

```
open c:\userdata\gretl\data\poe\cars.gdt
ols mpg const cyl eng wgt
reset

ols mpg const cyl eng wgt
genr y = $yhat
genr y2 = y*y
genr y3 = y2*y

ols mpg const cyl eng wgt y2
omit y2
ols mpg const cyl eng wgt y2 y3
omit y2 y3
```

6.7 Cars Example

The data set cars.gdt is included in package of datasets that are distributed with this manual. The script to reproduce the results from your text is

```
open c:\userdata\gretl\data\poe\cars.gdt

ols mpg const cyl eng wgt
vif
omit cyl eng
```

The test of the individual significance of cyl and eng can be read from the table of regression results. Neither are significant at the 5% level. The joint test of their significance is performed using the omit statement. The F-statistic is 4.298 and has a p-value of 0.0142. The null hypothesis is rejected in favor of their joint significance.

The new statement that requires explanation is `vif`. **vif** stands for *variance inflation factor* and it is used as a collinearity diagnostic by many programs, including **gretl**. The *vif* is closely related to the statistic suggested by Hill et al. (2008) who suggest using the R^2 from auxiliary regressions to determine the extent to which each explanatory variable can be explained as linear functions of the others. They suggest regressing x_j on all of the other independent variables and comparing the R_j^2 from this auxiliary regression to 10. If the R_j^2 exceeds 10, then there is evidence of a collinearity problem.

The vif_j actually reports the same information, but in a less straightforward way. The *vif* associated with the j^{th} regressor is computed

$$vif_j = \frac{1}{1 - R_j^2} \tag{6.10}$$

which is, as you can see, simply a function of the R_j^2 from the j^{th} regressor. Notice that when $R_j^2 > .80$, the $vif_j > 10$. Thus, the rule of thumb for the two rules is actually the same. A vif_j greater than 10 is equivalent to an R^2 greater than .8 from the auxiliary regression.

The output from **gretl** is shown below:

```
Variance Inflation Factors

 Minimum possible value = 1.0
 Values > 10.0 may indicate a collinearity problem

   2)            cyl   10.516
   3)            eng   15.786
   5)            wgt    7.789

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple
correlation coefficient between variable j and the
other independent variables
```

Once again, the **gretl** output is very informative. It gives you the threshold for high collinearity ($vif_j) > 10$) and the relationship between vif_j and R_j^2. Clearly, these data are highly collinear. Two variance inflation factors above the threshold and the one associated with `wgt` is fairly large as well.

The variance inflation factors can be produced from the dialogs as well. Estimate your model then, in the model window, select `Tests>Collinearity` and the results will appear in **gretl**'s output.

6.8 Script

```
open C:\userdata\gretl\data\poe\andy.gdt
ols S const P A
genr sseu = $ess
genr unrest_df = $df

ols S const A
genr sser = $ess

genr Fstat=((sser-sseu)/1)/(sseu/(unrest_df))
pvalue F 1 unrest_df Fstat

ols S const
genr sser = $ess
genr rest_df = $df

genr J = rest_df-unrest_df
genr Fstat=((sser-sseu)/J)/(sseu/(unrest_df))
pvalue F J unrest_df Fstat

genr A2 = A*A
ols S const P A A2
genr Ao =(1-$coeff(A))/(2*$coeff(A2))

restrict
  b3 + 3.8*b4=1
end restrict
```

```
open C:\userdata\gretl\data\poe\beer.gdt
logs Q PB PL PR M
ols l_Q const l_PB l_PL l_PR l_M
restrict
  b2+b3+b4+b5=0
end restrict

open c:\userdata\gretl\data\poe\edu_inc.gdt
ols faminc const he we kl6 x5 x6
modeltab add
omit x5 x6
modeltab add
omit kl6
modeltab add
omit we
modeltab add
modeltab show

open c:\userdata\gretl\data\poe\cars.gdt
ols mpg const cyl eng wgt
reset

ols mpg const cyl eng wgt
genr y = $yhat
genr y2 = y*y
genr y3 = y2*y

ols mpg const cyl eng wgt y2
omit y2
ols mpg const cyl eng wgt y2 y3
omit y2 y3

ols mpg const cyl eng wgt
vif
omit cyl eng
```

Chapter 7

Nonlinear Relationships

In Chapter 7 of *Principles of Econometrics*, the authors consider several methods for modeling nonlinear relationships between economic variables. As they point out, if the slope (the effect of one variable on another) changes for any reason, then the relationship is nonlinear. Specifically, we examine the use of polynomials, dummy variables, and interaction effects to make the basic linear regression model much more flexible.

7.1 Polynomials

The first model considered is a basic wage equation, where the worker's wage depends of his level of education and experience. We suspect that there are diminishing returns to experience and hence that the wage benefit of another year of experience will decline as a work gains experience.

$$wage_t = \beta_1 + \beta_2 educ_t + \beta_3 exper_t + \beta_4 exper_t^2 + e_t \tag{7.1}$$

The marginal effect of another year of experience on the average wage is

$$\frac{\partial E(wage_t)}{\partial exper_t} = \beta_3 + 2\beta_4 exper_t \tag{7.2}$$

Diminishing returns implies that $\beta_3 > 0$ and $\beta_4 < 0$. The maximum is attained when the slope of the function is zero so setting equation (7.2) equal to zero and solving for *exper* defines the level of experience that we expect will maximize wages.

$$\beta_3 + 2\beta_4 exper_t = 0 \quad \text{and} \quad exper_t = -\beta_3/2\beta_4. \tag{7.3}$$

Using **gretl** and the 1000 observations from the 1997 CPS stored in the **gretl** dataset *cps_small.gdt* we use the following script:

```
open c:\userdata\gretl\data\poe\cps_small.gdt
genr exper2 = exper^2
ols wage const educ exper exper2
```

which yields the result

$$\widehat{\text{wage}} = -\underset{(-9.306)}{9.818} + \underset{(17.228)}{1.210}\ \text{educ} + \underset{(6.629)}{0.341}\ \text{exper} - \underset{(-4.252)}{0.0051}\ \text{exper2}$$

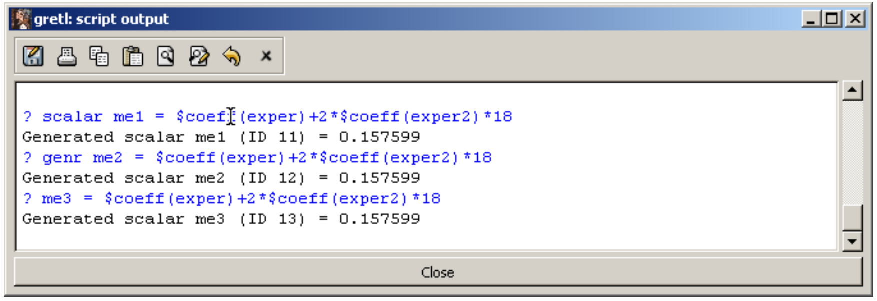
$$T = 1000 \quad \bar{R}^2 = 0.2687 \quad F(3,996) = 123.38 \quad \hat{\sigma} = 5.3417$$

$$(t\text{-statistics in parentheses})$$

The marginal effect for someone with 18 years of experience is obtained by using the statement `scalar me18=$coeff(exper)+2*$coeff(exper2)*18` which yields the desired result. Similarly, the turning point can be computed using the command `scalar turnpt=-$coeff(exper)/(2*$coeff(exper2))`. Notice, the `scalar` command is used instead of `genr` here because the result is a single number rather than a data series. You could use `genr` and obtain exactly the same result. There is little reason to prefer `scalar` over `genr` for the generation of scalars in **gretl**, but I've developed the habit of referring to a single number generated by other software as a scalar and I try to follow this convention in **gretl**. Actually, for scalars in **gretl** you could omit the prefix altogether. In recent versions of **gretl** it is unnecessary as you can see from Figure 7.1. In this figure, the same computation is made using `genr`, `scalar`, and without either!

As noted earlier **gretl** includes a data utility that makes it very easy to add the square of experience to your data file. First, select the variable you want to transform by highlighting it with the cursor. Then from the main **gretl** window use `Add>Squares of selected variables` to add them to the data set.

Figure 7.1: Using genr and scalar



```
? scalar me1 = $coeff(exper)+2*$coeff(exper2)*18
Generated scalar me1 (ID 11) = 0.157599
? genr me2 = $coeff(exper)+2*$coeff(exper2)*18
Generated scalar me2 (ID 12) = 0.157599
? me3 = $coeff(exper)+2*$coeff(exper2)*18
Generated scalar me3 (ID 13) = 0.157599
```

There are a number of other transformations you can make in this way, including add a time trend, logs, lags, differences and dummy variables for units or for panels. The pull-down list is illustrated in Figure 4.9.

As mentioned earlier some of the choices in Figure 4.9 are greyed out, meaning that they can not be selected at this time. This is because they are time series or panel specific functions and can only be used if you have first designated your data as such. To set your data up as time series use the `Data>Dataset structure` pull-down menu which is obtained as shown in Figure 7.2 below. Clicking on `Dataset structure` reveals the dialog box shown in Figure 7.3. If you select time series you will be taken to additional boxes that allow you to define its periodicity (yearly, quarterly, monthly, etc.) and the dates the time series covers. This is a very nice utility and I have used it to convert many of the *POE* datasets to time series for you. We will return to this topic in later chapters.

7.2 Interaction Terms

Another tool for capturing some types of nonlinearity is the creation of interaction terms. An **interaction term** is a variable that is created by multiplying two or more variables together. As discussed in *POE*, interaction terms are useful in allowing the marginal effect of a change in an independent variable on the average value of your dependent variable to be different for different observations in your sample. For instance, you may decide that the average return to another year of schooling is higher the younger a person is, other things being equal. To capture this effect in a model of wages you

Figure 7.2: `Data>Dataset Structure` pull-down menu

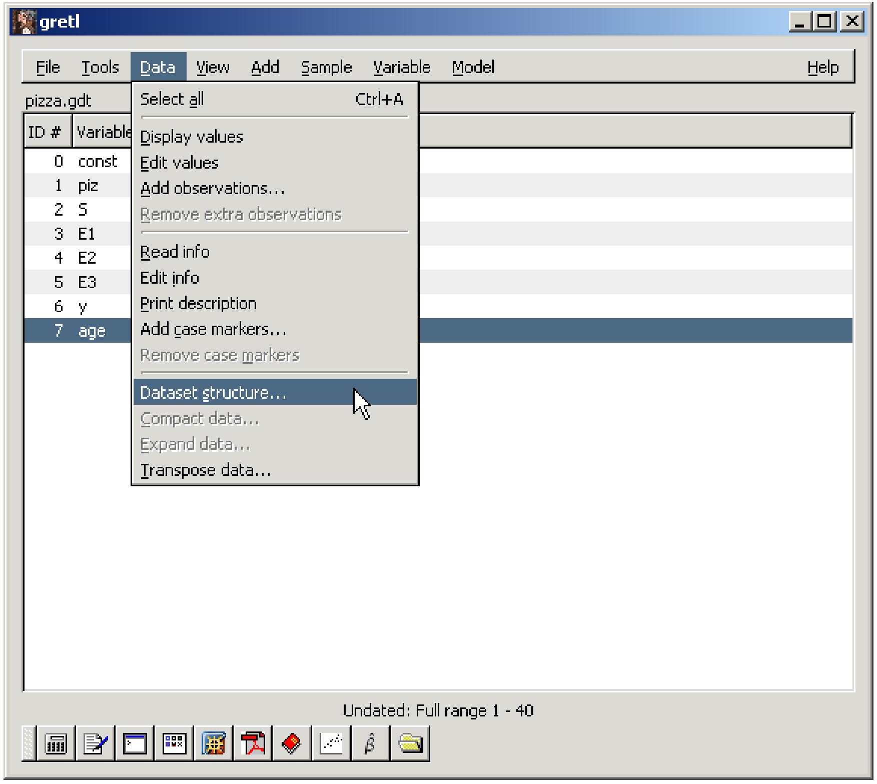
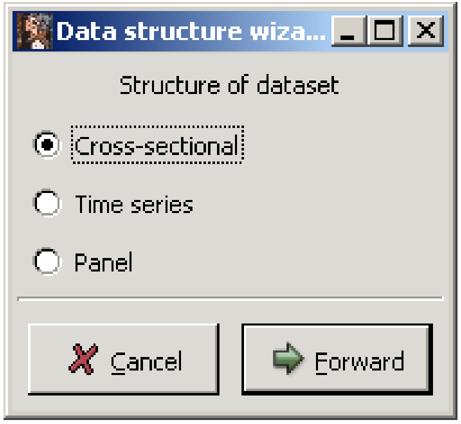


Figure 7.3: Dataset Structure dialog box



● Cross-sectional

○ Time series

○ Panel

could create an interaction between years of schooling (S_i) and a person's age (A_i) by generating a new variable $SA_i = S_i * A_i$ and including it as a regressor in your model. This is the overall gist of the pizza example from your textbook, were a person's age and income are interacted and included in a basic model of pizza demand.

7.3 Examples

7.3.1 Housing Price Example

The model to be estimated is

$$\begin{aligned} price_t = \beta_1 + \delta_1 utown_t + \beta_2 sqft_t + \gamma sqft_t * utown_t \\ + \beta_3 age_t + \delta_2 pool_t + \delta_3 fplace_t + e_t \end{aligned} \tag{7.4}$$

The script to estimate this model is

```
open c:\userdata\gretl\data\poe\utown.gdt
genr p = price/1000
genr sqft_ut = sqft*utown
ols price const utown sqft sqft_ut age pool fplace
```

Notice that the dependent variable, *price*, has been rescaled to be measured in $1,000 increments. This basically reduces the sizes of the estimated coefficients and standard errors by a factor of 1,000. It has no effect on t-ratios or their p-values. The results appear below.

$$\begin{aligned} \widehat{\mathrm{p}} = \underset{(3.957)}{24.5} + \underset{(3.259)}{27.453}\,\mathrm{utown} + \underset{(31.048)}{0.07612}\,\mathrm{sqft} + \underset{(3.913)}{0.01299}\,\mathrm{sqft\_ut} \\ - \underset{(-3.712)}{0.190}\;\mathrm{age} + \underset{(3.658)}{4.377}\,\mathrm{pool} + \underset{(1.697)}{1.649}\,\mathrm{fplace} \end{aligned}$$

$$T = 1000 \quad \bar{R}^2 = 0.8698 \quad F(6, 993) = 1113.2 \quad \hat{\sigma} = 15.225$$

(t-statistics in parentheses)

7.3.2 CPS Example

In this example, the *cps_small.dat* data are used to estimate wage equations. The basic equation is

$$wage_i = \beta_1 + \beta_2 educ_i + \delta_1 black_i + \delta_2 female_i + \gamma black_i * female_i + e_i \quad (7.5)$$

In this specification white-males are the **reference group**. The parameter δ_1 measures the effect of being black, the parameter δ_2 measures the effect of being female, and the parameter γ measures the effect of being black and female, all measured relative to the white-male reference group.

The first part of the script generates the interaction between females and blacks and then uses least squares to estimate the coefficients. The next line uses the `omit` statement to omit the three dummy variables (`black`, `female`, `b_female`) from the model to estimate a restricted version. Further, it performs the F-test of the joint null hypothesis that the three coefficients $(\delta_1, \delta_2, \gamma)$ are zero against the alternative that at least one of them is not.

The next model adds the three regional dummies and tests the null hypothesis that they are jointly zero. Then additional interactions are created between `south` and the other variables. Finally, **gretl**'s `wls` command is used to estimate separate regressions for southerners and non southerners. Here is the script file to compute all of the results for the the CPS examples.

```
open c:\userdata\gretl\data\poe\cps_small.gdt

genr b_female = black*female
ols wage const educ black female b_female
omit black female b_female

ols wage const educ black female b_female south midwest west
omit south midwest west

genr ed_south = educ*south
genr b_south = black*south
genr f_south = female*south
genr b_f_sth = black*female*south
```

```
ols wage const educ black female b_female south \
             ed_south b_south f_south b_f_sth
omit south ed_south b_south f_south b_f_sth
```

The results are collected in model table 7.1 using the modeltab function.

7.3.3 Chow Test

The Chow test is an easy way to test the equivalency of two regressions estimated using different subsets of the sample. In this section I'll show you a trick that you can use for estimating subset regressions and then how to perform the Chow test.

Suppose you wanted to estimate separate wage equations based on the model in equation (7.5): one regression for southerners and another for everyone else. **Gretl** can accomplish this using the weighted least squares, `wls`, estimator. The weighted least squares estimator takes the model

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \ldots + \beta_k x_{ik} + e_i \tag{7.6}$$

and reweighs it using weights, w_i according to

$$w_i * y_i = \beta_1 w_i + \beta_2 w_i * x_{i2} + \beta_3 w_i * x_{i3} + \ldots + \beta_k w_i * x_{ik} + w_i * e_i \tag{7.7}$$

and estimates the coefficients using least squares. This estimator is used later in the book for different purposes, but here it can be used to omit desired observations from your model. Basically, what you want to do is to let $w_i = 1$ for all observations you want to include and $w_i = 0$ for those you want to exclude.

The syntax for the `wls` command is simple.

```
wls w y const x2 x3 x4
```

First call for the weighted least squares estimator with `wls`; next specify the weights to be used (`w`); then, state the regression to be estimated `y const x2 x3 x4`.

Table 7.1: CPS results

OLS estimates
Dependent variable: wage

	Model 1	Model 2	Model 3	Model 4	Model 5
const	-3.230^{**}	-4.912^{**}	-2.456^{**}	-3.230^{**}	-3.578^{**}
	(0.9675)	(0.9668)	(1.051)	(0.9675)	(1.151)
educ	1.117^{**}	1.139^{**}	1.102^{**}	1.117^{**}	1.166^{**}
	(0.06971)	(0.07155)	(0.06999)	(0.06971)	(0.08241)
black	-1.831^{**}		-1.608^{*}	-1.831^{**}	−0.4312
	(0.8957)		(0.9034)	(0.8957)	(1.348)
female	-2.552^{**}		-2.501^{**}	-2.552^{**}	-2.754^{**}
	(0.3597)		(0.3600)	(0.3597)	(0.4257)
b_female	0.5879		0.6465	0.5879	0.06732
	(1.217)		(1.215)	(1.217)	(1.906)
south			-1.244^{**}		1.302
			(0.4794)		(2.115)
midwest			−0.4996		
			(0.5056)		
west			−0.5462		
			(0.5154)		
ed_south					−0.1917
					(0.1542)
b_south					−1.744
					(1.827)
f_south					0.9119
					(0.7960)
b_f_sth					0.5428
					(2.511)
n	1000	1000	1000	1000	1000
$\bar{R}^2$	0.2451	0.2016	0.2482	0.2451	0.2490
ℓ	−3107.86	−3137.43	−3104.33	−3107.86	−3102.81

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level

In the context of equation (7.5) generate a new dummy variable that takes the value 1 for nonsoutherners and zero for southerners; then, use weighted least squares. The following script uses this approach to estimate the two sample subsets. The sum of squared errors are saved for later use.

```
wls nonsouth wage const educ black female b_female
scalar sse_ns = $ess
wls south wage const educ black female b_female
scalar sse_s = $ess
```

If the coefficients for southerners are equal to those for nonsoutherners, then you would **pool** the two subsamples together and estimate the model using the command `ols wage const educ black female b_female`. Otherwise, separate regressions are required. The **Chow test** is used to determine whether the subsamples are really necessary in light of the data you have. To determine whether the regressions were actually equal to one another compute

$$Chow = \frac{SSE_{full} - (SSE_{south} + SSE_{nonsouth})/5}{(SSE_{south} + SSE_{nonsouth})/(n-10)} \sim F_{5,n-10} \tag{7.8}$$

if the two subset regressions are equivalent. You will reject the null hypothesis that the coefficients of the two subsamples are equal if the p-value is less than the desired significance level of the test, α.

The script to compute the Chow test is:

```
ols wage const educ black female b_female
scalar sse_r = $ess
scalar sse_u = sse_ns+sse_s
scalar chowtest = ((sse_r-sse_u)/5)/(sse_u/($nobs-10)
pvalue F 5 $nobs-10 chowtest
```

As you can see, this is just an application of the F-statistic of equation (6.2) discussed in Chapter 6. The unrestricted sum of squares is obtained by adding the sum of squared errors of the two subset regressions. The restricted sum of square errors is from the pooled regression.

Table 7.3: Regression results for the Chow test.

Dependent variable: wage

	Model 7	Model 8	Model 9
	Non-South	South	All
const	-3.578^{**}	-2.275	-3.230^{**}
	(1.211)	(1.555)	(0.9675)
educ	1.166^{**}	0.9741^{**}	1.117^{**}
	(0.08665)	(0.1143)	(0.06971)
black	-0.4312	-2.176^{**}	-1.831^{**}
	(1.418)	(1.080)	(0.8957)
female	-2.754^{**}	-1.842^{**}	-2.552^{**}
	(0.4476)	(0.5896)	(0.3597)
b_female	0.06732	0.6102	0.5879
	(2.004)	(1.433)	(1.217)
n	685	315	1000
$\bar{R}^2$	0.2486	0.2143	0.2451
SSE	22031.3	6981.39	29307.7

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level

The results of the test are:

```
? scalar chowtest = ((sse_r-sse_u)/5)/(sse_u/($nobs-10)
Generated scalar chowtest (ID 20) = 2.01321
? pvalue F 5 $nobs-10 chowtest

F(5, 990): area to the right of 2.01321 = 0.074379
(to the left: 0.925621)
```

7.3.4 Pizza Example

The pizza examples considers the model

$$piz_i = \beta_1 + \beta_2 age_i + \beta_3 y_i + \beta_4 age_i * y_i + e_i \tag{7.9}$$

where $i = 1, 2, \ldots, T$. The marginal effects of age on pizza demand are computed for families having \$25,000 and \$90,000 in income. The **gretl** code to estimate this model using least squares and to obtain the marginal effects is:

```
open c:\userdata\gretl\data\poe\pizza.gdt

ols piz const age y

genr age_inc = age*y
ols piz const age y age_inc

scalar p25 = $coeff(age)+$coeff(age_inc)*25000
scalar p90 = $coeff(age)+$coeff(age_inc)*90000
```

The estimates from the first equation are:

$$\widehat{\text{piz}} = \underset{(4.740)}{342.885} + \underset{(3.947)}{0.00238222}\,\text{y} - \underset{(-3.270)}{7.57556}\,\text{age}$$
$$T = 40 \quad \bar{R}^2 = 0.2930 \quad F(2, 37) = 9.0811 \quad \hat{\sigma} = 131.07$$
$$(t\text{-statistics in parentheses})$$

and those from the second:

$$\widehat{\text{piz}} = \underset{(4.740)}{342.885} + \underset{(3.947)}{0.00238222}\,\text{y} - \underset{(-3.270)}{7.57556}\,\text{age}$$
$$T = 40 \quad \bar{R}^2 = 0.2930 \quad F(2, 37) = 9.0811 \quad \hat{\sigma} = 131.07$$
$$(t\text{-statistics in parentheses})$$

and the computed predictions are:

```
p25 = -6.98270
p90 = -17.3964
```

7.3.5 Log-Linear Wages Example

In the final example a model of log-wages is estimated and the genr command is used to compute the percentage difference between male and female wages and the marginal effect of another year of experience on the log-wage.

```
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ female
scalar pdiff = exp($coeff(female))-1

genr expersq = exper*exper
genr educ_exp = educ*exper
ols l_wage const educ exper educ_exper
scalar me = 100*($coeff(exper)+$coeff(educ_exp)*16)
ols l_wage const educ exper expersq educ_exper
```

The results from the three regressions appear in Table 7.5.

7.4 Script

```
open c:\userdata\gretl\data\poe\cps_small.gdt
genr exper2 = exper^2
ols wage const educ exper exper2

scalar me18 = $coeff(exper)+2*$coeff(exper2)*18
scalar turnpt = -($coeff(exper))/(2*$coeff(exper2))

open c:\userdata\gretl\data\poe\utown.gdt
genr p = price/1000
genr sqft_ut = sqft*utown
ols price const utown sqft sqft_ut age pool fplace

open c:\userdata\gretl\data\poe\cps_small.gdt
genr b_female = black*female
```

Table 7.5: The regression results from the log-linear wages example.

OLS estimates
Dependent variable: ln(wage)

	Model 1	Model 2	Model 3
const	0.9290^{**}	0.1528	-0.2646
	(0.08375)	(0.1722)	(0.1808)
educ	0.1026^{**}	0.1341^{**}	0.1506^{**}
	(0.006075)	(0.01271)	(0.01272)
female	-0.2526^{**}		
	(0.02998)		
exper		0.02492^{**}	0.06706^{**}
		(0.007075)	(0.009533)
educ_exp		-0.0009624^{*}	-0.002019^{**}
		(0.0005404)	(0.0005545)
expersq			-0.0006962^{**}
			(0.0001081)
n	1000	1000	1000
$\bar{R}^2$	0.2654	0.2785	0.3067
ℓ	-670.50	-660.97	-640.54

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level

```
? scalar pdiff = exp($coeff(female))-1
Generated scalar pdiff (ID 11) = -0.223224
? scalar me = 100*($coeff(exper)+$coeff(educ_exp)*16)
Generated scalar me (ID 14) = 0.951838
```

```
ols wage const educ black female b_female
omit black female b_female

ols wage const educ black female b_female south midwest west
omit south midwest west

genr ed_south = educ*south
genr b_south = black*south
genr f_south = female*south
genr b_f_sth = black*female*south

#Use omit statement to test joint hypothesis
ols wage const educ black female b_female south \
          ed_south b_south f_south b_f_sth
omit south ed_south b_south f_south b_f_sth

#Using wls to omit observations
genr nonsouth = 1-south
wls nonsouth wage const educ black female b_female
scalar sse_ns = $ess
wls south wage const educ black female b_female
scalar sse_s = $ess

#Chow test
#Pooled regression (restricted)
ols wage const educ black female b_female
scalar sse_r = $ess
scalar sse_u = sse_ns+sse_s
scalar chowtest = ((sse_r-sse_u)/5)/(sse_u/($nobs-10)
pvalue F 5 $nobs-10 chowtest

#Pizza Example
open c:\userdata\gretl\data\poe\pizza.gdt
ols piz const age y

genr age_inc = age*y
ols piz const age y age_inc

scalar p25 = $coeff(age)+$coeff(age_inc)*25000
scalar p90 = $coeff(age)+$coeff(age_inc)*90000
```

```
#Log wages example
open c:\userdata\gretl\data\poe\cps_small.gdt
genr l_wage = log(wage)
ols l_wage const educ female
scalar pdiff = exp($coeff(female))-1

genr expersq = exper*exper
genr educ_exp = educ*exper
ols l_wage const educ exper educ_exper
scalar me = 100*($coeff(exper)+$coeff(educ_exp)*16)
```

Chapter 8

Heteroskedasticity

The simple linear regression models of Chapter 2 and the multiple regression model in Chapter 5 can be generalized in other ways. For instance, there is no guarantee that the random variables of these models (either the y_i or the e_i) have the same inherent variability. That is to say, some observations may have a larger or smaller variance than others. This describes the condition known as **heteroskedasticity**. The general linear regression model is shown in equation (8.1) below.

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_k x_{iK} + e_i \quad i = 1, 2, \ldots, T \tag{8.1}$$

where y_i is the dependent variable, x_{ik} is the i^{th} observation on the k^{th} independent variable, $k = 2, 3, \ldots, K$, e_i is random error, and $\beta_1, \beta_2, \ldots, \beta_K$ are the parameters you want to estimate. Just as in the simple linear regression model, e_i, have an average value of zero for each value of the independent variables and are uncorrelated with one another. The difference in this model is that the variance of e_i now depends on i, i.e., the observation to which it belongs. Indexing the variance with the i subscript is just a way of indicating that observations may have different amounts of variability associated with them. The error assumptions can be summarized as $e_i|x_{i2}, x_{i3}, \ldots x_{iK}\ iid\ N(0, \sigma_i^2)$.

The intercept and slopes, β_1 and $\beta_2 \ldots \beta_K$, are consistently estimated by least squares even if the data are heteroskedastic. Unfortunately, the usual estimators of the least squares standard errors and tests based on them are inconsistent and invalid. In this chapter, several ways to detect heteroskedasticity are considered. Also, statistically valid ways of estimating the parameters of 8.1 and testing hypotheses about the βs when the data are heteroskedastic are explored.

8.1 Food Expenditure Example

First, a simple model of food expenditures is estimated using least squares. The model is

$$y_i = \beta_1 + \beta_2 x_i + e_i \quad i = 1, 2, \ldots, N. \tag{8.2}$$

where y_i is food expenditure and x_i is income of the i^{th} individual. When the errors of the model are heteroskedastic, then the least squares estimator of the coefficients is consistent. That means that the least squares *point estimates* of the intercept and slope are useful. However, when the errors are heteroskedastic the usual least squares **standard errors** are **inconsistent** and therefor should not be used to form confidence intervals or to test hypotheses.

To use least squares estimates with heteroskedastic data, at a very minimum, you'll need a consistent estimator of their standard errors in order to construct valid tests and intervals. A simple computation proposed by White accomplishes this. Standard errors computed using White's technique are loosely referred to as **robust**, though one has to be careful when using this term; the standard errors are robust to the *presence of heteroskedasticity* in the errors of model (but not necessarily other forms of model misspecification).

Open the *food.gdt* data in **gretl** and estimate the model using least squares.

```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
```

This yields the usual least squares estimates of the parameters, but the wrong standard errors when the data are heteroskedastic. To obtain the robust standard errors, simply add the `--robust` option to the regression as shown in the following **gretl** script. After issuing the `--robust` option, the standard errors stored in `$stderr(x)` are the robust ones.

```
ols y const x --robust

# confidence intervals (Robust)
genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)
print lb ub
```

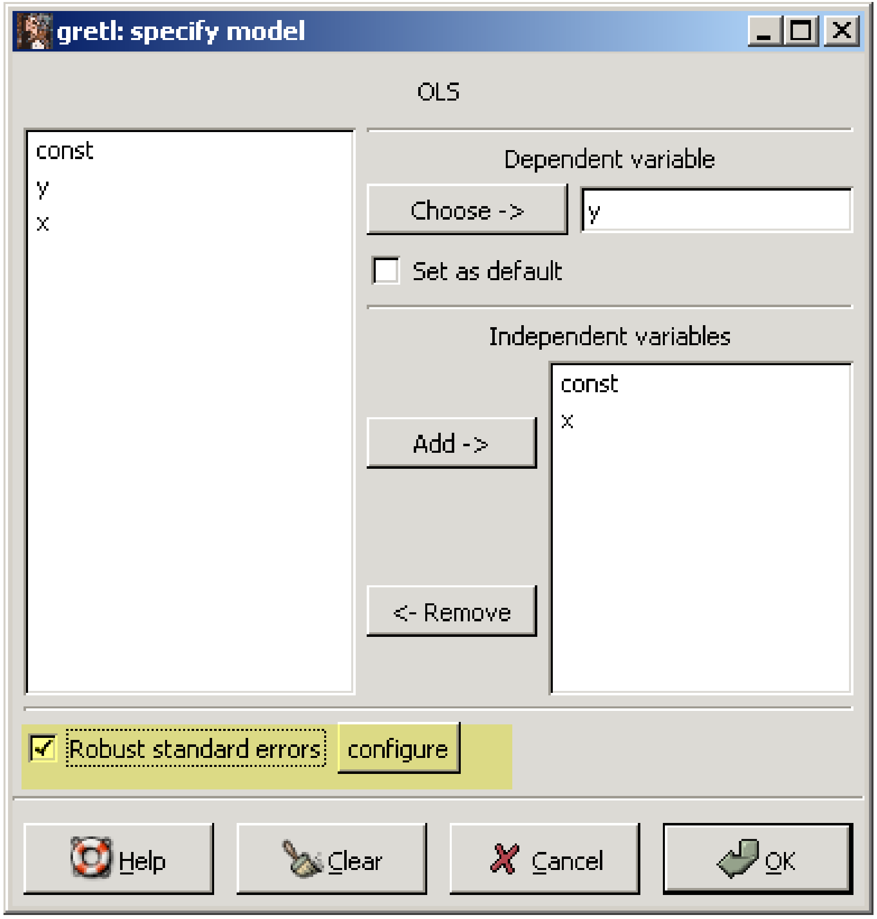
In the script, we've used the `critical(t,$df,0.025)` function to get the desired critical value from the t-distribution. Remember, the degrees of freedom from the preceding regression are stored in `$df`. The first argument in the function indicates the desired distribution, and the last is the desired right-tail probability ($\alpha/2$ in this case).

This can also be done from the pull-down menus. Select `Model>Ordinary Least Squares` (see Figure 2.6) to generate the dialog to specify the model shown in Figure 8.1 below. Note, the check box to generate 'robust standard errors' is highlighted in yellow. You will also notice that there is a button labeled 'configure' just to the right of the check box. Clicking this button reveals a dialog from which several options can be selected. In this case, we can select the particular method that will be used to compute the robust standard errors and even set robust standard errors to be the default computation for least squares. This dialog box is shown in Figure 8.2 below. To reproduce the results in Hill et al. (2008), you'll want to select HC1 from the pull-down list. As you can see, other **gretl** options can be selected here that affect the default behavior of the program.

The model results for the food expenditure example appears in the table below. After estimating the model using the dialog, you can use `Analysis>Confidence intervals for coefficients` to generate 95% confidence intervals. Since you used the `robust` option in the dialog, these will be based on the variant of White's standard errors chosen using the 'configure' button. The result is:

```
t(38, .025) = 2.024
```

Figure 8.1: Check the box for heteroskedasticity robust standard errors.



gretl: specify model

OLS

const
y
x

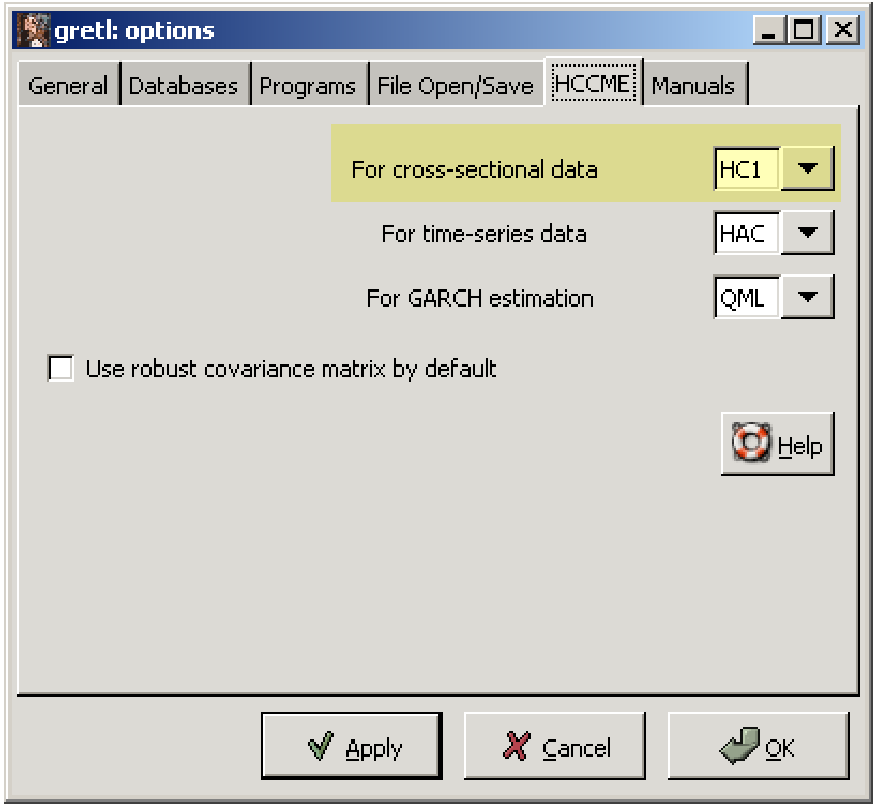
Dependent variable: y

□ Set as default

Independent variables:
const
x

☑ Robust standard errors

Figure 8.2: Set the method for computing robust standard errors.



For cross-sectional data HC1

For time-series data HAC

For GARCH estimation QML

□ Use robust covariance matrix by default

Table 8.1: Least squares estimates with the usual and robust standard errors.

OLS estimates
Dependent variable: y

	Usual Std errors	Robust Std errors
const	83.42*	83.42**
	(43.41)	(27.46)
x	10.21**	10.21**
	(2.093)	(1.809)
n	40	40
R^2	0.3850	0.3850
ℓ	−235.51	−235.51

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level

```
    VARIABLE         COEFFICIENT      95% CONFIDENCE INTERVAL

       const          83.4160         (27.8186, 139.013)
           x          10.2096         (6.54736, 13.8719)
```

8.2 Weighted Least Squares

If you know something about the structure of the heteroskedasticity, you may be able to get more precise estimates using a generalization of least squares. In heteroskedastic models, observations that are observed with high variance don't contain as much information about the location of the regression line as those observations having low variance. The basic idea of generalized least squares in this context is to reweigh the data so that all the observations contain the same level of information (i.e., same variance) about the location of the regression line. So, observations that contain more **noise** are given small weights and those containing more **signal** a higher weight. Reweighing the data in this way is known in some statistical disciplines as *weighted least squares*. This descriptive term is the one used by **gretl** as well.

Suppose that the errors vary proportionally with x_i according to

$$Var(e_i) = \sigma^2 x_i \tag{8.3}$$

The errors are heteroskedastic since each error will have a different variance, the value of which depends on the level of x_i. Weighted least squares reweighs the observations in the model so that each transformed observation has the same variance as all the others. Simple algebra reveals that

$$\frac{1}{\sqrt{x_i}} Var(e_i) = \sigma^2 \tag{8.4}$$

So, multiply equation (8.1) by $1/\sqrt{x_i}$ to complete the transformation. The transformed model is homoskedastic and least squares and the least squares standard errors are statistically valid and efficient.

Gretl makes this easy since it contains a function to reweigh all the observations according to a weight you specify. The command is `wls`, which naturally stands for **w**eighted **l**east **s**quares! The only thing you need to be careful of is how **gretl** handles the weights. **Gretl** takes the square root of the value you provide. That is, to reweigh the variables using $1/\sqrt{x_i}$ you need to use its square $1/x_i$ as the weight. **Gretl** takes the square root of `w` for you. To me, this is a bit confusing, so you may want to verify what **gretl** is doing by manually transforming y, x, and the constant and running the regression. The script file shown below does this.

In the example, you first have to create the weight, then call the function `wls`. The script appears below.

```
open c:\userdata\gretl\data\poe\food.gdt

#GLS using built in function
genr w = 1/x
wls w y const x

genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)
print lb ub

#GLS using OLS on transformed data
genr wi = 1/sqrt(x)
genr ys = wi*y
genr xs = wi*x
genr cs = wi

ols ys cs xs
```

The first argument after `wls` is the name of the weight variable. Then, specify the regression to which it is applied. **Gretl** multiplies each variable (including the constant) by the ***square root*** of the given weight and estimates the regression using least squares.

In the next block of the program, $w_i = 1/\sqrt{x_i}$ is created and used to transform the dependent variable, x and the constant. Least squares regression using this manually weighted data yields the same results as you get with **gretl**'s `wls` command. In either case, you interpret the output of weighted least squares in the usual way.

The weighted least squares estimation yields:

$$\widehat{\mathrm{y}} = \underset{(23.789)}{78.6841} + \underset{(1.3859)}{10.4510}\,\mathrm{x}$$
$$T = 40 \quad \bar{R}^2 = 0.5889 \quad F(1, 38) = 56.867 \quad \hat{\sigma} = 18.75$$
$$\text{(standard errors in parentheses)}$$

and the 95% confidence interval for the slope β_2 is (7.64542, 13.2566).

8.3 Skedasticity Function

A commonly used model for the error variance is the **multipicative heteroskedasticity** model. It appears below in equation 8.5.

$$\sigma_i^2 = \exp\left(\alpha_1 + \alpha_2 z_i\right) \tag{8.5}$$

The variable z_i is an independent explanatory variable that determines how the error variance changes with each observation. You can add additional zs if you believe that the variance is related to them (e.g., $\sigma_i^2 = \exp\left(\alpha_1 + \alpha_2 z_{i2} + \alpha_3 z_{i3}\right)$). It's best to keep the number of zs relatively small. The idea is to estimate the parameters of (8.5) using least squares and then use predictions as weights to transform the data.

In terms of the food expenditure model, let $z_i = ln(x_i)$. Then, taking the natural logarithms of both sides of (8.5) and adding a random error term, v_i, yields

$$\ln\left(\sigma_i^2\right) = \alpha_1 + \alpha_2 z_i + v_i \tag{8.6}$$

To estimate the αs, first estimate the linear regression (8.2) (or more generally, 8.1) using least squares and save the residuals. Square the residuals, then take the natural log; this forms an estimate of $\ln\left(\sigma_i^2\right)$ to use as the dependent variable in a regression. Now, add a constant and the zs to the right-hand side of the model and estimate the αs using least squares.

The regression model to estimate is

$$\ln\left(\hat{e}_i^2\right) = \alpha_1 + \alpha_2 z_i + v_i \tag{8.7}$$

where $\hat{e}_i^2$ are the least squares residuals from the estimation of equation (8.1). The predictions from this regression can then be transformed using the exponential function to provide weights for weighted least squares.

For the food expenditure example, the **gretl** code appears below.

```
ols y const x
genr lnsighat = log($uhat*$uhat)
genr z = log(x)
ols lnsighat const z
genr predsighat = exp($yhat)
genr w = 1/predsighat

wls w y const x
```

The first line estimates the linear regression using least squares. Next, a new variable is generated (`lnsighat`) that is the natural log of the squared residuals from the preceding regression. Then, generate z as the natural log of x. Estimate the skedasticity function using least squares, take the predicted values (`yhat`) and use these in the exponential function (i.e., $\exp(\hat{y}_i)$). The reciprocal of these serve as weights for generalized least squares. Remember, **gretl** automatically takes the square roots of `w` for you in the `wls` function.

This results in:

$$\widehat{\mathrm{y}} = \underset{(9.7135)}{76.0538} + \underset{(0.97151)}{10.6335}\,\mathrm{x}$$

$$T = 40 \quad \bar{R}^2 = 0.7529 \quad F(1,38) = 119.8 \quad \hat{\sigma} = 1.5467$$

(standard errors in parentheses)

8.4 Grouped Heteroskedasticity

Using examples from Hill et al. (2008) a model of grouped heteroskedasticity is estimated and a Goldfeld-Quandt test is performed to determine whether the two sample subsets have the same error variance.

8.4.1 Wage Example

Below, I have written a **gretl** program to reproduce the wage example from Hill et al. (2008) that appears in Chapter 8. The example is relatively straightforward and I'll not explain the script in much detail. It is annotated to help you decipher what each section of the program does.

The example consists of estimating wages as a function of education and experience. In addition, a dummy variable is included that is equal to one if a person lives in a metropolitan area. This is an "intercept" dummy which means that folks living in the metro areas are expected to respond similarly to changes in education and experience (same slopes), but that they earn a premium relative to those in rural areas (different intercept).

Each subset (metro and rural) is estimated separately using least squares and the standard error of the regression is saved for each (`$sigma`). To create weights for weighted least squares, the full sample is restored and the metro and rural dummy variables are each multiplied times their respective regression's standard error. The two variables are added together to form a single variable that is equal to the metro regression standard error for each observation located in a metro area and equal to the rural regression standard error for each observation located in a rural area. The weight is created by taking the reciprocal and squaring. Recall, **gretl** needs the variance rather than the standard error of each observation to perform weighted least squares.

```
open c:\userdata\gretl\data\poe\cps2.gdt
ols wage const educ exper metro

# Use only metro observations
smpl metro --dummy
ols wage const educ exper
scalar stdm = $sigma

#Restore the full sample
smpl full

#Create a dummy variable for rural
genr rural = 1-metro

#Restrict sample to rural observations
smpl rural --dummy
ols wage const educ exper
```

```
scalar stdr = $sigma

#Restore the full sample
smpl full

#Generate standard deviations for each metro and rural obs
genr wm = metro*stdm
genr wr = rural*stdr

#Make the weights (reciprocal)
#Remember, Gretl's wls needs these to be variances
#so you'll need to square them
genr w = 1/(wm + wr)^2

#Weighted least squares
wls w wage const educ exper metro
```

Weighted least squares estimation yields:

$$\widehat{\text{wage}} = \underset{(1.0197)}{-9.39836} + \underset{(0.068508)}{1.19572}\ \text{educ} + \underset{(0.014549)}{0.132209}\ \text{exper} + \underset{(0.34629)}{1.53880}\ \text{metro}$$

$$T = 1000 \quad \bar{R}^2 = 0.2693 \quad F(3,996) = 123.75 \quad \hat{\sigma} = 1.0012$$

(standard errors in parentheses)

The Goldfeld-Quandt statistic is the formed as the ratio of the two variances:

$$F = \frac{\hat{\sigma}_M^2}{\hat{\sigma}_R^2} \sim F_{N_M-K_M,N_R-K_R} \tag{8.8}$$

if the null hypothesis of homoskedasticity is true. Rejection of the Ho means that the subsets have different variances.

```
#Goldfeld Quandt statistic
?scalar fstatistic = stdm^2/stdr^2
Generated scalar fstatistic (ID 17) = 2.08776
```

You could simplify the script a bit by using the regression trick explored in Chapter 7. Create a dummy variable that takes the value of 1 for the desired observations and 0 for the ones you want to drop. Then, use weighted least squares on the entire sample using the dummy variable as your weight. This effectively drops all observations in the sample for which the dummy variable is zero. This trick is useful since it keeps you from having to keep explicit track of which sample is actually in memory at any point in time. Thus,

```
smpl metro --dummy
ols wage const educ exper
```

could be replaced by

```
wls metro wage const educ exper
```

Then there is no need to restore the full sample in the next block of code!

8.4.2 Food Expenditure Example

In this example, the Goldfeld-Quandt test is applied in a more traditional way. Here, you suspect that the variance depends on a specific variable. You sort the data based on this variable and then compare the subset variances to one another using the Goldfeld-Quandt test statistic.

For the food expenditure example, the script follows. Essentially, you want to run two regressions using subsets of the sample. One subset contains observations with low variance, the other observations with high variance. In most cases this means that you'll have to sort your data based on its variability. In our example, one would sort the data based on x_i. **Gretl** gives us another option and that is to create the subsamples based on some criterion. In this case, we want observations separated based on high and low values of x so we can use the `median` function. To pick all observations for which x is above the median, use `smpl x > median(x) --restrict`. Recall that the `smpl` command allows us to manipulate the sample in memory. In this case we use the logical statement that we want observations where x is greater than the median of x, followed by the `--restrict` option. This should give us half the observations.

```
open c:\userdata\gretl\data\poe\food.gdt

#Take subsample where x > median(x)
smpl x > median(x) --restrict

ols y const x
scalar stdL = $sigma
scalar df_L = $df

#Restore the full sample
smpl full

#Take subsample where x < median(x)
smpl x < median(x) --restrict

ols y const x
scalar stdS = $sigma
scalar df_S = $df

#Goldfeld Quandt statistic
scalar fstatistic = stdL^2/stdS^2
pvalue F df_L df_S fstatistic
```

The full sample is restored and the variance for the lower half is saved. Then the test statistic is computed and can be compared to the appropriate critical value. The last statement computes the p-value from the F-distribution. Recall that the degrees of freedom were saved from each subset and they can be used here as the arguments for the numerator and denominator degrees of freedom for F.

The test statistic and p-value are:

```
? scalar fstatistic = stdL^2/stdS^2
Generated scalar fstatistic (ID 7) = 3.61476
? pvalue F df_L df_S fstatistic

F(18, 18): area to the right of 3.61476 = 0.00459643
(to the left: 0.995404)
```

8.5 Other Tests for Heteroskedasticity

The Goldfeld-Quandt test of the null hypothesis of homoskedasticity is only useful when the data can be neatly partitioned into subsamples having different variances. In many circumstances this will not be the case and other tests of the homoskedasticity null hypothesis are more useful. Each of these tests share the same null hypothesis as the Goldfeld-Quandt test: homoskedasticity. They differ in the specification of the *alternative hypothesis.*

The first test considered is based on the estimated multiplicative heteroskedasticity function of section 8.3. The null and alternative hypotheses are

$$H_o: \quad \sigma_i^2 = \quad \sigma^2 \tag{8.9}$$

$$H_1: \quad \sigma_i^2 = \quad \exp\left(\alpha_1 + \alpha_2 z_i\right) \tag{8.10}$$

The homoskedastic null hypothesis is tested against a specific functional relationship. In this case, we know the function (exponential) as well as the variable(s) that causes the variance to vary (z_i). Basically, we want to test whether $\alpha_2 = 0$. If it is, then the errors of the regression model are homoskedastic.

The test of this hypothesis is based on your regression in equation (8.7). The t-ratio on α_2 is approximately normally distributed under Ho so you could use the t-test to test this proposition. If you have multiple zs, use the F-test.

Other equivalent ways of testing this hypothesis are available. As Hill et al. (2008) point out, it is common to test the same hypothesis based on a linear regression

$$\hat{e}_i^2 = \alpha_1 + \alpha_2 z_i + v_i \tag{8.11}$$

Breusch and Pagan have proposed a couple of tests of the homoskedasticity hypothesis (8.9) against the alternative

$$H_1: \sigma_i^2 = h(\alpha_1 + \alpha_2 z_i) \tag{8.12}$$

where $h()$ is some arbitrary function (e.g., linear or exponential). These tests are carried out based on equation (8.11). The alternative hypothesis in (8.12) is more general than that in (8.10) and includes it as a special case. There are two versions of the Breusch-Pagan test. One is used when the errors of the regression are normally distributed and the other when they are not. I suggest using the latter since it is seldom if ever known what the error distribution is. I'll tell you how to do the preferred version in **gretl**.

Basically, estimate (8.11) using least squares and take NR^2 from this regression, where N is your sample size. Under the null hypothesis it has a χ^2_{S-1} distribution, where S is the total number of parameters (the αs) in the estimated equation.

The alternative hypothesis of White's test is even more general than the Breusch-Pagan. The alternative hypothesis is

$$H_1 : \sigma_i^2 \neq \sigma^2 \tag{8.13}$$

Thus the alternative is completely general. The test is similar to the Breusch-Pagan test in that you'll run a regression with $\hat{e}^2$ as a dependent variable and zs as the independent variables. In White's test you will include each z, its square, and their (unique) cross products as regressors. In the food expenditure example that amounts to

$$y_i = \alpha_1 + \alpha_2 z_i + \alpha_3 z_i^2 + v_i \tag{8.14}$$

You can do this in one of two ways. You can run the original regression, save the residuals and square them. Then square z_i to use as an independent variable. Run the regression. For the food expenditure example:

```
open c:\userdata\gretl\data\poe\food.gdt

ols y const x

#Save the residuals
genr ehat = $uhat

#Square the residuals
genr ehat2 = ehat*ehat

#White's test
#Generate squares, cross products (if needed)
```

```
genr x2 = x*x

#Test regression
ols ehat2 const x x2
scalar teststat = $trsq
pvalue X 2 teststat
```

Gretl computes NR^2 in every regression and saves it in `$trsq`. The statistic NR^2 is distributed as a χ^2_{S-1} under the null hypothesis of homoskedasticity and we can use the `pvalue` function to obtain the p-value for the computed statistic. The syntax is `pvalue X df statistic`, with `X` indicating the χ^2, `df` the degrees of freedom, and `statistic` the computed value of NR^2. The script yields a computed test statistic of 7.555, and the p-value of 0.0228789. Homoskedasticity is rejected.

Or, you can use the **gretl** function `lmtest`! In this case, run the original regression and follow it with `lmtest --white` as shown in the script.

```
open c:\userdata\gretl\data\poe\food.gdt

#White's test --built-in
ols y const x
lmtest --white
```

This yields the output:

```
White's test for heteroskedasticity
OLS estimates using the 40 observations 1-40
Dependent variable: uhat^2

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const             -2908.78          8100.11         -0.359   0.72156
  x                   291.746          915.846         0.319   0.75186
  sq_x                 11.1653          25.3095        0.441   0.66167

  Unadjusted R-squared = 0.188877
```

```
Test statistic: TR^2 = 7.555079,
with p-value = P(Chi-square(2) > 7.555079) = 0.022879
```

As you can see, the results from `lmtest --white` and your (labor intensive) script are the same!

8.6 Script

```
open c:\userdata\gretl\data\poe\food.gdt
ols y const x
ols y const x --robust

# confidence intervals (Robust)
genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)
print lb ub

#GLS using built in function
genr w = 1/x
wls w y const x

genr lb = $coeff(x) - critical(t,$df,0.025) * $stderr(x)
genr ub = $coeff(x) + critical(t,$df,0.025) * $stderr(x)
print lb ub

#GLS using OLS on transformed data
genr wi = 1/sqrt(x)
genr ys = wi*y
genr xs = wi*x
genr cs = wi

ols ys cs xs

#Estimating the skedasticity function and GLS
ols y const x
genr lnsighat = log($uhat*$uhat)
genr z = log(x)
ols lnsighat const z
genr predsighat = exp($yhat)
genr w = 1/predsighat

wls w y const x

#---------------------------------------
#Wage Example
open c:\userdata\gretl\data\poe\cps2.gdt
ols wage const educ exper metro

# Use only metro observations
smpl metro --dummy
ols wage const educ exper
scalar stdm = $sigma

#Restore the full sample
smpl full

#Create a dummy variable for rural
genr rural = 1-metro

#Restrict sample to rural observations
smpl rural --dummy
ols wage const educ exper
scalar stdr = $sigma

#Restore the full sample
smpl full

#Generate standard deviations for each metro and rural obs
genr wm = metro*stdm
genr wr = rural*stdr

#Make the weights (reciprocal)
#Remember, Gretl's wls needs these to be variances
#so you'll need to square them
genr w = 1/(wm + wr)^2
```

```
#Weighted least squares
wls w wage const educ exper metro

#Goldfeld Quandt statistic
scalar fstatistic = stdm^2/stdr^2

#----------------------------------------------
#Food Expenditure Example
open c:\userdata\gretl\data\poe\food.gdt

#Take subsample where x > median(x)
smpl x > median(x) --restrict

ols y const x
scalar stdL = $sigma
scalar df_L = $df

#Restore the full sample
smpl full

#Take subsample where x < median(x)
smpl x < median(x) --restrict

ols y const x
scalar stdS = $sigma
scalar df_S = $df

#Goldfeld Quandt statistic
scalar fstatistic = stdL^2/stdS^2
pvalue F df_L df_S fstatistic

#------------------------------------
#LM Test
open c:\userdata\gretl\data\poe\food.gdt

ols y const x

#Save the residuals
genr ehat = $uhat
```

```
#Square the residuals
genr ehat2 = ehat*ehat

#White's test
#Generate squares, cross products (if needed)
genr x2 = x*x

#Test regression
ols ehat2 const x x2
scalar teststat = $trsq
pvalue X 2 teststat

#---------------------------------------------
#White's test
open c:\userdata\gretl\data\poe\food.gdt

#White's test --built-in
ols y const x
lmtest --white
```

Chapter 9

Dynamic Models and Autocorrelation

The multiple linear regression model of equation (5.1) assumes that the observations are not correlated with one another. While this is certainly believable if one has drawn a random sample, it's less likely if one has drawn observations sequentially in time. Time series observations, which are drawn at regular intervals, usually embody a structure where time is an important component. If you are unable to completely model this structure in the regression function itself, then the remainder spills over into the unobserved component of the statistical model (its error) and this causes the errors be correlated with one another.

One way to think about it is that the errors will be **serially correlated** when omitted effects last more than one time period. This means that when the effects of an economic 'shock' last more than a single time period, the unmodelled components (errors) will be correlated with one another. A natural consequence of this is that the more frequently a process is sampled (other things being equal), the more likely it is to be autocorrelated. From a practical standpoint, monthly observations are more likely to be autocorrelated than quarterly observations, and quarterly more likely than yearly ones. Once again, ignoring this correlation makes least squares inefficient at best and the usual measures of precision (standard errors) inconsistent.

In this chapter, several ways to detect autocorrelation in the model's errors are considered. Also, statistically valid ways of estimating the parameters of 8.1 and testing hypotheses about the βs in autocorrelated models are explored.

9.1 Area Response Model for Sugar Cane

Hill et al. (2008) considers a simple model of the area devoted to sugar cane production in Bangladesh. The equation to be estimated is

$$\ln(A_t) = \beta_1 + \beta_2 \ln(P_t) + e_t \qquad t = 1, 2, \ldots, N \tag{9.1}$$

The data consist of 34 annual observations on area (A) and price (P). The error term contains all of the economic factors other than price that affect the area of production. If changes in any of these other factors (shocks) affect area for more than one year, then the errors of the model will not be mutually independent of one another. The errors are said to be *serially correlated* or **autocorrelated**. Least square estimates of the βs are consistent, but the usual computation for the standard errors is not.

If the shock persists for two periods, and the shock is stable in the sense that its influence on the future diminishes as time passes, then we could use a model such as

$$e_t = \rho e_{t-1} + v_t \tag{9.2}$$

where ρ is a parameter and v_t is random error. This says that today's shock is in part due to the shock that happened in the previous period. Thus, there is some *persistence* in the area under tillage that is unrelated to price. The model referred to in equation 9.2 is called **first order autocorrelation** and is abbreviated **AR(1)**.

Stability means that the parameter ρ must lie in the (-1,1) interval (not including the endpoints). If $|\rho|$ is one or greater then the errors are not stable and a shock will send your model spiraling out of control!

As is the case with heteroskedastic errors, there is a way to salvage least squares when your data are autocorrelated. In this case you can use an estimator of standard errors that is robust to both heteroskedasticity and autocorrelation proposed by Newey and West. This estimator is sometimes called **HAC**, which stands for **heteroskedasticity autocorrelated consistent**. This and some issues that surround its use are discussed in the next few sections.

9.1.1 Bandwidth and Kernel

HAC is not quite as automatic as the heteroskedasticity consistent (HC) estimator in Chapter 8. To be robust with respect to autocorrelation you have to specify how far away in time the autocorrelation is likely to be significant. Essentially, the autocorrelated errors over the chosen time window are averaged in the computation of the HAC standard errors; you have to specify how many periods over which to average and how much weight to assign each residual in that average. The language of time series analysis can often be opaque. This is the case here. The weighted average is called a **kernel** and the number of errors to average in this respect is called **bandwidth**. Just think of the kernel as another name for weighted average and bandwidth as the term for number of terms to average.

Now, what this has to do with **gretl** is fairly simple. You get to pick a method of averaging (Bartlett kernel or Parzen kernel) and a bandwidth (`nw1`, `nw2` or some integer). **Gretl** defaults to the Bartlett kernel and the bandwidth $nw1 = 0.75xN^{1/3}$. As you can see, the bandwidth `nw1` is computed based on the sample size, N. The `nw2` bandwidth is $nw2 = 4 \times (N/100)^{2/9}$. This one appears to be the default in other programs like EViews.

Implicity there is a trade-off to consider. Larger bandwidths reduce bias (good) as well as precision (bad). Smaller bandwidths exclude more relevant autocorrelations (and hence have more bias), but use more observations to increase precision (smaller variance). The general principle is to choose a bandwidth that is large enough to contain the largest autocorrelations. The choice will ultimately depend on the frequency of observation and the length of time it takes for your system to adjust to shocks.

The bandwidth or kernel can be changed using the `set` command from the console or in a script. The `set` command is used to change various defaults in **gretl** and the relevant switches for our use are `hac_lag` and `hac_kernel`. The use of these is demonstrated below. The following script could be used to change the kernel to `bartlett` and the bandwidth to `nw2`:

```
open c:\userdata\gretl\data\poe\bangla.gdt
set hac_kernel bartlett
set hac_lag nw2
```

9.1.2 Dataset Structure

The other key to using HAC is that your data must be structured as a time series. This can be done through the dialogs or very simply using the console. First let's look at the **Dataset wizard** provided in the system of menus.

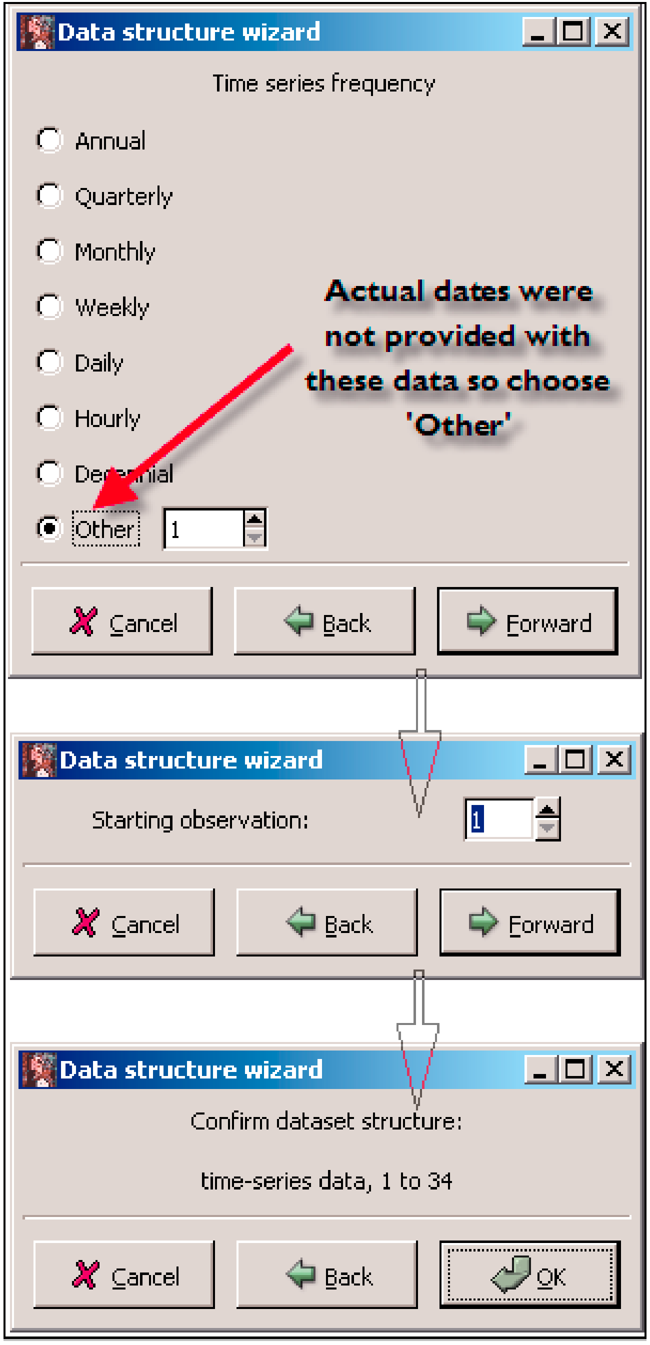
Open the `bangla.gdt` data and choose `Data>Data set structure` from the main pull-down menu (refer to Figure 7.2). This brings up the **Data structure wizard** dialog box (Figure 7.3). Choose **Time series** and click **Forward**. Although the data were collected annually, no actual dates (years) are provided. So in the next box (top of Figure 9.1) choose **other** and click **Forward**. This leads to the second box in the figure. Start the observations at 1, click **Forward** again, and a window confirming your choices (shown at the bottom) will open. If satisfied, click **OK** to close the wizard or **Back** to make changes. If you had chosen annual, quarterly, monthly or other actual time frame in the first of the wizard's boxes, then you would be given the opportunity to select actual dates in the second box. Again, your choices are confirmed in the final box generated by the wizard.

Gretl includes the `setobs` command that will do the same thing. For *bangla.gdt* dataset the command is

```
setobs 1 1 --time-series
```

The first number identifies the periodicity (1=year, 4=quarter, 12=month, and so on). The second number sets the starting date. Since there is no date for this data we start the time counter at 1. Finally, the `--time-series` option is used to declare the data to be time-series. Here are a few other examples:

Figure 9.1: Choose `Data>Dataset structure` from the main window. This starts the Dataset wizard, a series of dialogs that allow you to specify the periodicity and dates associated with your data.



Time series frequency

○ Annual

○ Quarterly

○ Monthly

○ Weekly

○ Daily

○ Hourly

○ Decennial

● Other 1

Actual dates were not provided with these data so choose 'Other'

Starting observation: 1

Confirm dataset structure:

time-series data, 1 to 34

```
setobs 4 1978:3 --time-series
setobs 12 1950:01 --time-series
setobs 1 1949 --time-series
```

The first statement starts a quarterly series in the third quarter of 1978, the second a monthly series beginning in January 1950, and the last a yearly series beginning in 1949. See the help on `setobs` to declare daily or hourly series, or to setup your data as a cross-section or panel.

9.1.3 HAC Standard Errors

Once **gretl** recognizes that your data are time series, then the robust command will automatically apply the HAC estimator of standard errors with the default values of the kernel and bandwidth (or the ones you have set with the `set` command). Thus, to obtain the HAC standard errors simply requires

```
open c:\userdata\gretl\data\poe\bangla.gdt
logs p a
ols l_p const l_a --robust
```

The statement `logs p a` creates the natural logarithms of the variables p and a and puts them into the dataset as `l_p` and `l_a`. These are used in the regression with the `--robust` option to produce least squares estimates with HAC standard errors.

The results appear below:

OLS estimates
Dependent variable: l_a

	OLS (wrong SE)	OLS with HAC
const	3.893^{**}	3.893^{**}
	(0.06058)	(0.06135)
l_p	0.7761^{**}	0.7761^{**}
	(0.3669)	(0.2775)
n	34	34
R^2	0.1965	0.1965
ℓ	−7.15	−7.15

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level

Notice that the standard errors computed using HAC are a little different from those in Hill et al. (2008). No worries, though. They are statistically valid and suggest that EViews and **gretl** are doing the computations a bit differently.

9.2 Nonlinear Least Squares

Perhaps the best way to estimate a linear model that is autocorrelated is using nonlinear least squares. As it turns out, the nonlinear least squares estimator only requires that the errors be stable (not necessarily stationary). The other methods commonly used make stronger demands on the data, namely that the errors be covariance stationary. Furthermore, the nonlinear least squares estimator gives you an unconditional estimate of the autocorrelation parameter, ρ, and yields a simple t-test of the hypothesis of no serial correlation. Monte Carlo studies show that it performs well in small samples as well. So with all this going for it, why not use it?

The biggest reason is that nonlinear least squares requires more computational power than linear estimation, though this is not much of a constraint these days. Also, in **gretl** it requires an extra step on your part. You have to type in an equation for **gretl** to estimate. This is the way one works in EViews and other software by default, so the burden here is relatively low.

Nonlinear least squares (and other nonlinear estimators) use numerical methods rather than analytical ones to find the minimum of your sum of squared errors objective function. The routines that do this are iterative. You give the program a good first guess as to the value of the parameters and it evaluates the sum of squares function at this guess. The program looks at the slope of your sum of squares function at the guess and computes a *step* in the parameter space that takes you closer to a minimum (further down the hill). If an improvement in the sum of squared errors function is found, the new parameter values are used as the basis for another step. Iterations continue until no further significant reduction in the sum of squared errors function can be found.

In the context of the area response equation the AR(1) model is

$$\ln(A_t) = \beta_1(1-\rho) + \beta_2(\ln(P_t) - \rho\ln(P_{t-1})) + \rho\ln(A_{t-1}) + v_t \tag{9.3}$$

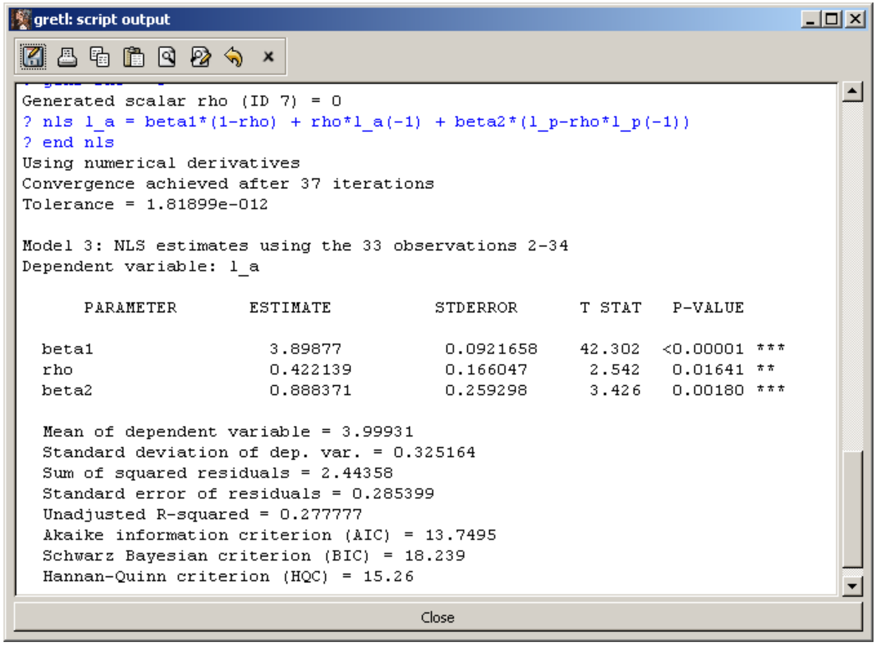
The errors, v_t, are random and the goal is to find β_1, β_2, and ρ that minimize $\sum v_t^2$. Ordinary least squares is a good place to start in this case. The OLS estimates are consistent so we'll start our numerical routine there, setting ρ equal to zero. The **gretl** script to do this follows:

```
open c:\userdata\gretl\data\poe\bangla.gdt
logs p a
ols l_a const l_p --robust

genr beta1 = $coeff(const)
genr beta2 = $coeff(l_p)
genr rho = 0

nls l_a = beta1*(1-rho) + rho*l_a(-1) + beta2*(l_p-rho*l_p(-1))
end nls
```

Magically, this yields the same result from your text!

The `nls` command is initiated with `nls` followed by the equation representing the systematic portion of your model. The command is closed by the statement `end nls`. In the script, I used **gretl**'s built in functions to take lags. Hence, `l_a(-1)` is the variable `l_a` lagged by one period (-1). In this way you can create lags or leads of various lengths in your **gretl** programs without explicitly having to create new variables via the generate command. The results of nonlinear least squares appear below in Figure 9.2.

Figure 9.2: Nonlinear least squares results for the AR(1) regression model.



```
Generated scalar rho (ID 7) = 0
? nls l_a = beta1*(1-rho) + rho*l_a(-1) + beta2*(l_p-rho*l_p(-1))
? end nls
Using numerical derivatives
Convergence achieved after 37 iterations
Tolerance = 1.81899e-012

Model 3: NLS estimates using the 33 observations 2-34
Dependent variable: l_a

      PARAMETER       ESTIMATE          STDERROR      T STAT   P-VALUE

  beta1                 3.89877          0.0921658    42.302  <0.00001 ***
  rho                   0.422139         0.166047      2.542   0.01641 **
  beta2                 0.888371         0.259298      3.426   0.00180 ***

  Mean of dependent variable = 3.99931
  Standard deviation of dep. var. = 0.325164
  Sum of squared residuals = 2.44358
  Standard error of residuals = 0.285399
  Unadjusted R-squared = 0.277777
  Akaike information criterion (AIC) = 13.7495
  Schwarz Bayesian criterion (BIC) = 18.239
  Hannan-Quinn criterion (HQC) = 15.26
```

Equation 9.3 can be expanded and rewritten in the following way:

$$\ln(A_t) = \beta_1(1-\rho) + \beta_2\ln(P_t) - \beta_2\rho\ln(P_{t-1}) + \rho\ln(A_{t-1}) + v_t \tag{9.4}$$

$$\ln(A_t) = \delta + \delta_0\ln(P_t) - \delta_1\ln(P_{t-1})) + \theta_1\ln(A_{t-1}) + v_t \tag{9.5}$$

Both equations contain the same variables, but Equation (9.3) contains only 3 parameters while (9.5) has 4. This means that (9.3) is *nested* within (9.5) and a formal hypothesis test can be performed to determine whether the implied restriction holds. The restriction is $\delta_1 = -\theta_1\delta_0$.[1] To test this hypothesis using **gretl** you can use a variant of the statistic (6.2) discussed in section 6.1. You'll need the restricted and unrestricted sum of squared errors from the models. The statistic is

$$J \times F = \frac{(SSE_r - SSE_u)}{SSE_u/(N-K)} \dot{\sim} \chi^2_J \qquad \text{if } H_0: \ \delta_1 = -\theta_1\delta_0 \text{ is true} \tag{9.6}$$

[1] $\delta = \beta_1(1-\rho), \delta_0 = \beta_2, \delta_1 = -\rho\beta_2, \theta_1 = \rho$

Since $J = 1$ this statistic has an approximate χ^2_1 distribution and it is equivalent to an F test. Note, you will get a slightly different answer that the one listed in your text. However, rest assured that the statistic is asymptotically valid.

For the example, we've generated the output:

```
Replaced scalar fstat (ID 12) = 1.10547
? pvalue X 1 fstat

Chi-square(1): area to the right of 1.10547 = 0.293069
(to the left: 0.706931)
? pvalue F 1 $df fstat

F(1, 29): area to the right of 1.10547 = 0.301752
(to the left: 0.698248)
```

Because the sample is so small (only 29 degrees of freedom) the p-values from the F(1,29) and the χ^2_1 are a bit different. Still, neither is significant at the 5% level.

9.3 Testing for Autocorrelation

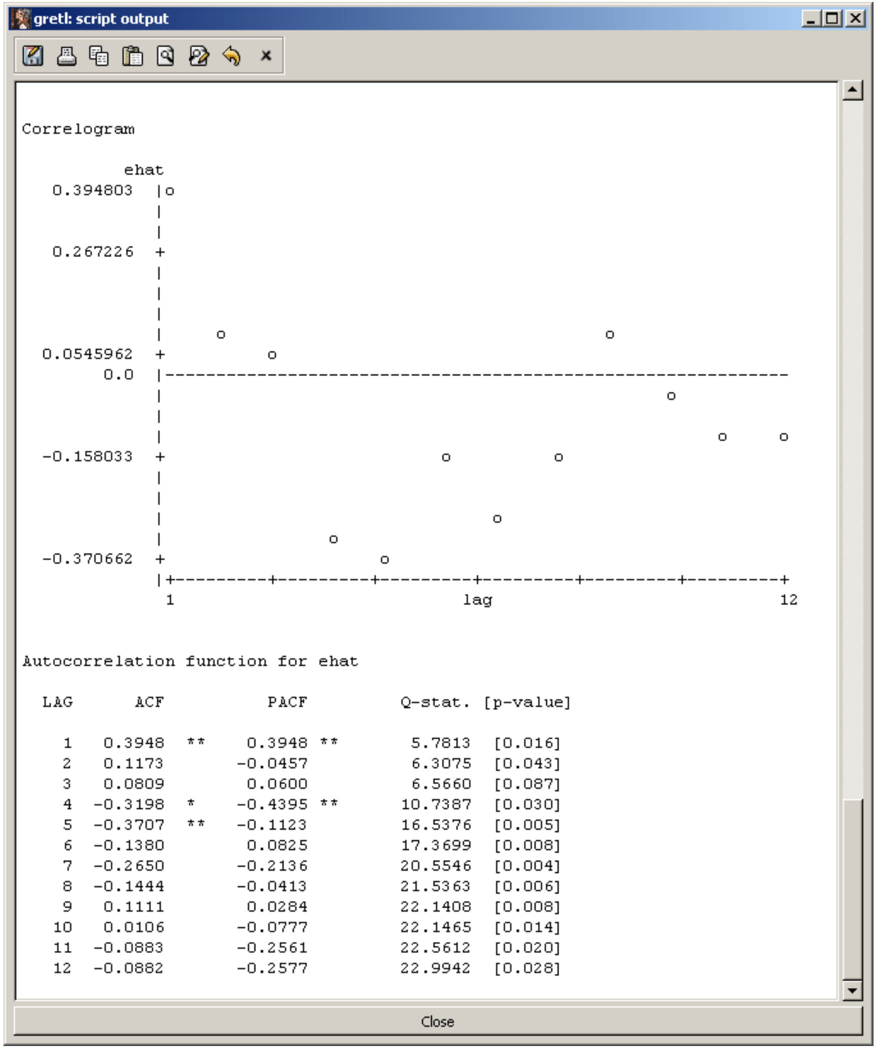
Two methods are used to determine the presence or extent of autocorrelation. The first is to take a look at the **residual correlogram**. A correlogram is a graph that plots series of correlations between $\hat{x}_t$ and $\hat{x}_{t-j}$ against the time interval between the observations, $j = 1, 2, \ldots, m$. A residual correlogram uses residuals from an estimated model as the time series, x_t. So, the first thing to do is to estimate the model using least squares and then save the residuals, $\hat{e}_t$. Once you have the residuals, then use the `corrgm` command to get the correlogram. The syntax follows:

```
open c:\userdata\gretl\data\poe\bangla.gdt
logs p a
ols l_a const l_p --robust
```

```
genr ehat = $uhat
corrgm ehat 12
```

The output is found in Figure 9.3 below. Essentially, the 12 autocorrelations plotted are simple correlations between $\hat{e}_t$ and $\hat{e}_{t-m}$ for $m = 1, 2, \ldots, 12$. Statistical significance at the 5% level is denoted with two asterisks (**) and at the 10% level with one (*). The correlogram is just a way of visualizing

Figure 9.3: Correlogram of the least squares residuals

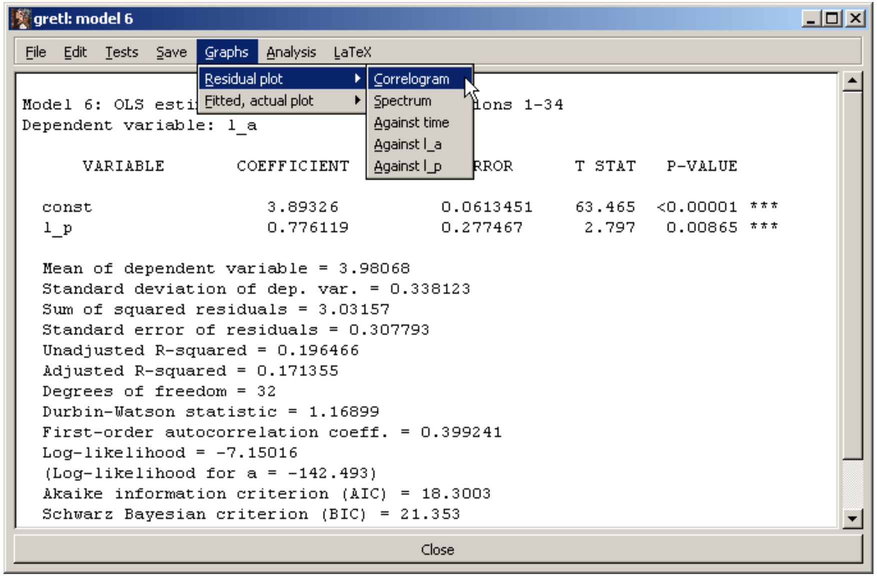


```
Correlogram

          ehat
   0.394803  |o
             |
             |
   0.267226  +
             |
             |
             |
             |     o                                     o
  0.0545962  +          o
        0.0  |-------------------------------------------------------------
             |                                                 o
             |
             |                                                      o     o
  -0.158033  +                           o          o
             |
             |
             |                                o
             |                5
  -0.370662  +                     o
             |+---------+---------+---------+---------+---------+---------+
              1                            lag                            12

Autocorrelation function for ehat

  LAG      ACF          PACF         Q-stat. [p-value]

    1   0.3948  **    0.3948  **      5.7813  [0.016]
    2   0.1173       -0.0457          6.3075  [0.043]
    3   0.0809        0.0600          6.5660  [0.087]
    4  -0.3198  *    -0.4395  **     10.7387  [0.030]
    5  -0.3707  **   -0.1123         16.5376  [0.005]
    6  -0.1380        0.0825         17.3699  [0.008]
    7  -0.2650       -0.2136         20.5546  [0.004]
    8  -0.1444       -0.0413         21.5363  [0.006]
    9   0.1111        0.0284         22.1408  [0.008]
   10   0.0106       -0.0777         22.1465  [0.014]
   11  -0.0883       -0.2561         22.5612  [0.020]
   12  -0.0882       -0.2577         22.9942  [0.028]
```

this, as it plots each of the autocorrelations against its lag number.

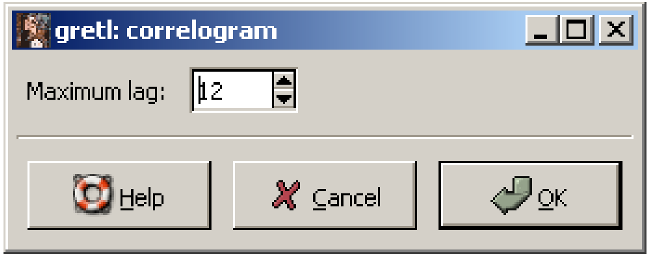
The dialogs yields a much prettier and marginally more informative result. Estimate the model using `Model>Ordinary Least Squares` as shown in Figures 5.1 and 5.2. Click OK to run the regression and the results appear in a model window. Then select `Graphs>Residual plot>Correlogram` from the pull-down menus as shown in Figure 9.4. Select the number of lags to include using the dialog box (Figure 9.5). Click **OK** and **gretl** opens two windows containing results. The first contains the table shown at the bottom half of Figure 9.3, which shows the computed sample autocorrela-

Figure 9.4: From the model window you can obtain the correlogram of the least squares residuals with `Graph>Residual plot>Correlogram`.



```
Model 6: OLS esti                                    ions 1-34
Dependent variable: l_a

      VARIABLE       COEFFICIENT                    RROR      T STAT    P-VALUE

  const                 3.89326          0.0613451     63.465   <0.00001 ***
  l_p                   0.776119         0.277467       2.797    0.00865 ***

  Mean of dependent variable = 3.98068
  Standard deviation of dep. var. = 0.338123
  Sum of squared residuals = 3.03157
  Standard error of residuals = 0.307793
  Unadjusted R-squared = 0.196466
  Adjusted R-squared = 0.171355
  Degrees of freedom = 32
  Durbin-Watson statistic = 1.16899
  First-order autocorrelation coeff. = 0.399241
  Log-likelihood = -7.15016
  (Log-likelihood for a = -142.493)
  Akaike information criterion (AIC) = 18.3003
  Schwarz Bayesian criterion (BIC) = 21.353
```

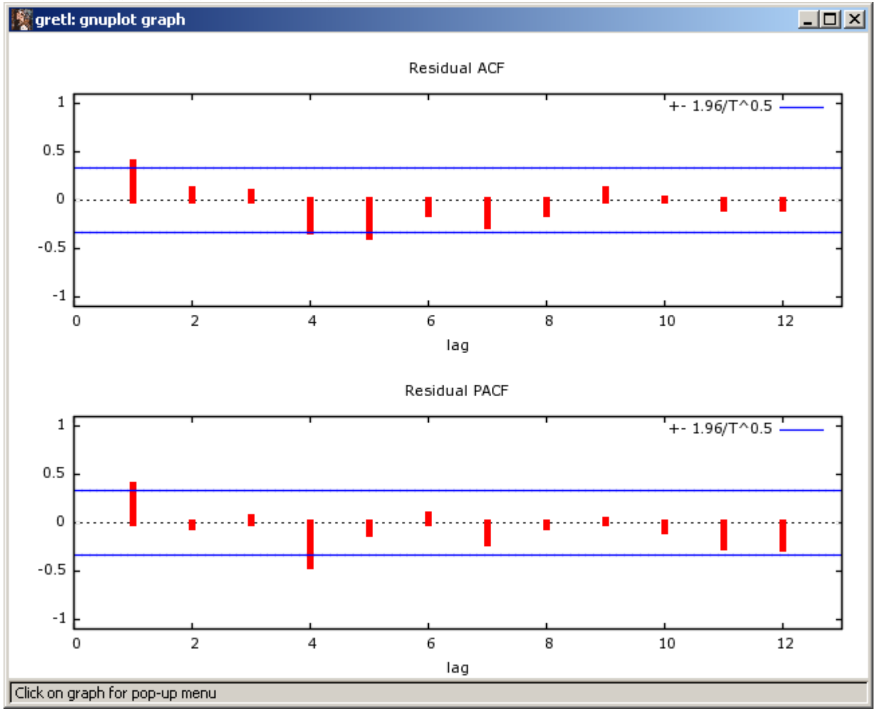
Figure 9.5: Choose the desired number of lags using the dialog box.



Maximum lag: 12

tions (ACF) and partial autocorrelations (PACF). The other is a graph of these along with 95% confidence bands. This graph is depicted in Figure 9.6 below. You can see that the first and fifth autocorrelations lie outside of the confidence band, indicating that they are individually significant at the 5% level.

Figure 9.6: This version of the correlogram is much prettier and includes confidence bands for the autocorrelations.



The other way to determine whether or not your residuals are autocorrelated is to use an LM (Lagrange multiplier) test. For autocorrelation, this test is based on an auxiliary regression where lagged least squares residuals are added to the original regression equation. If the coefficient on the lagged residual is significant then you conclude that the model is autocorrelated. So, for a regression model $y_t = \beta_1 + \beta_2 x_t + e_t$ the first step is to estimate the parameters using least squares and save the residuals, $\hat{e}_t$. An auxiliary regression model is formed using $\hat{e}_t$ as the dependent variable and original regressors and the lagged value $\hat{e}_{t-1}$ as an independent variables.

The resulting auxiliary regression is

$$\hat{e}_t = \beta_1 + \beta_2 x_t + \rho \hat{e}_{t-1} + v_t \tag{9.7}$$

Now, test the hypothesis $\rho = 0$ against the alternative that $\rho \neq 0$ and you are done. The test statistic is NR^2 from this regression which will have a χ^2_1 if Ho: is true. The script to accomplish this is:

```
ols ehat const l_p ehat(-1)
scalar NR2 = $trsq
pvalue X 1 NR2
```

If you prefer to use the dialogs, then estimate the model using least square in the usual way (`Model>Ordinary least squares`) and select `Tests>Autocorrelation` from the resulting model window (i.e., the one in Figure 9.4). Choose the number of lagged values of $\hat{e}_t$ you want to include in (9.7) (in our case only 1) and click OK. This will give you the same result as the script. The result appears in Figure 9.7. Note, the first statistic reported is simply the joint test that all the lagged values of $\hat{e}$ you included in (9.7) are jointly zeros. The second one is the NR^2 version of the test done in the script. **Gretl** also computes a Ljung-Box Q statistic whose null hypothesis is no autocorrelation. It is also insignificant at the 5% level.

9.4 Autoregressive Models and Forecasting

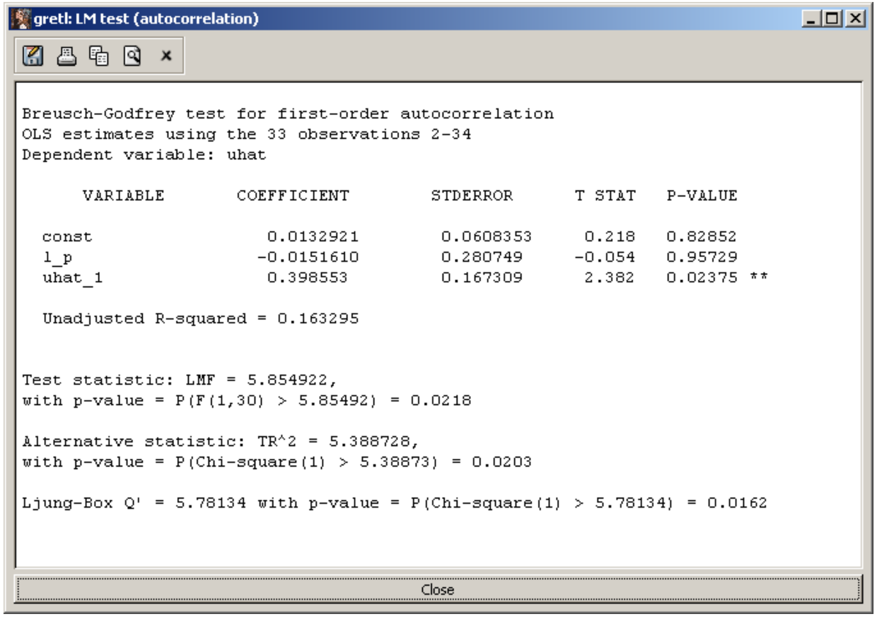
A autoregressive model will include one or more lags of your dependent variable on the right-hand-side of your regression equation. An AR(p) includes p lags of y_t as shown below in equation (9.8).

$$y_t = \delta + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \ldots + \theta_p y_{t-p} + v_t \tag{9.8}$$

In general, p should be large enough so that v_t is white noise.

The dataset *inflation.gdt* includes 270 monthly observations on the CPI from which an inflation variable is computed. To estimate an AR(3) model of inflation, simply use the script

Figure 9.7: Using `Test>Autocorrelation` from the model pull-down menu will generate the following output.



```
Breusch-Godfrey test for first-order autocorrelation
OLS estimates using the 33 observations 2-34
Dependent variable: uhat

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                0.0132921        0.0608353     0.218    0.82852
  l_p                 -0.0151610        0.280749     -0.054    0.95729
  uhat_1               0.398553         0.167309      2.382    0.02375 **

  Unadjusted R-squared = 0.163295


Test statistic: LMF = 5.854922,
with p-value = P(F(1,30) > 5.85492) = 0.0218

Alternative statistic: TR^2 = 5.388728,
with p-value = P(Chi-square(1) > 5.38873) = 0.0203

Ljung-Box Q' = 5.78134 with p-value = P(Chi-square(1) > 5.78134) = 0.0162
```

```
open C:\userdata\gretl\data\poe\inflation.gdt
ols infln const infln(-1 to -3)
```

In this case a bit of shorthand is used to generate the lagged values of inflation to include as regressors. The syntax `infln(-1 to -3)` tells **gretl** to include a range of the variable inflation from lags from 1 to 3. The minus signs indicate lags. This is equivalent to using a list of variables as in

```
ols infln const infln(-1) infln(-2) infln(-3)
```

Obviously, if p were large then using the range version would save a lot of typing.

Using this model to forecast in **gretl** is very simple. The main decision you have to make at this point is how many periods into the future you want to forecast. In **gretl** you have to extend the sample to include future periods under study.

9.4.1 Using the Dialogs

Return to the main **gretl** window and choose `Model>Ordinary least squares`. This will bring up the 'specify model' dialog box. Choose `infln` as the dependent variable as shown.

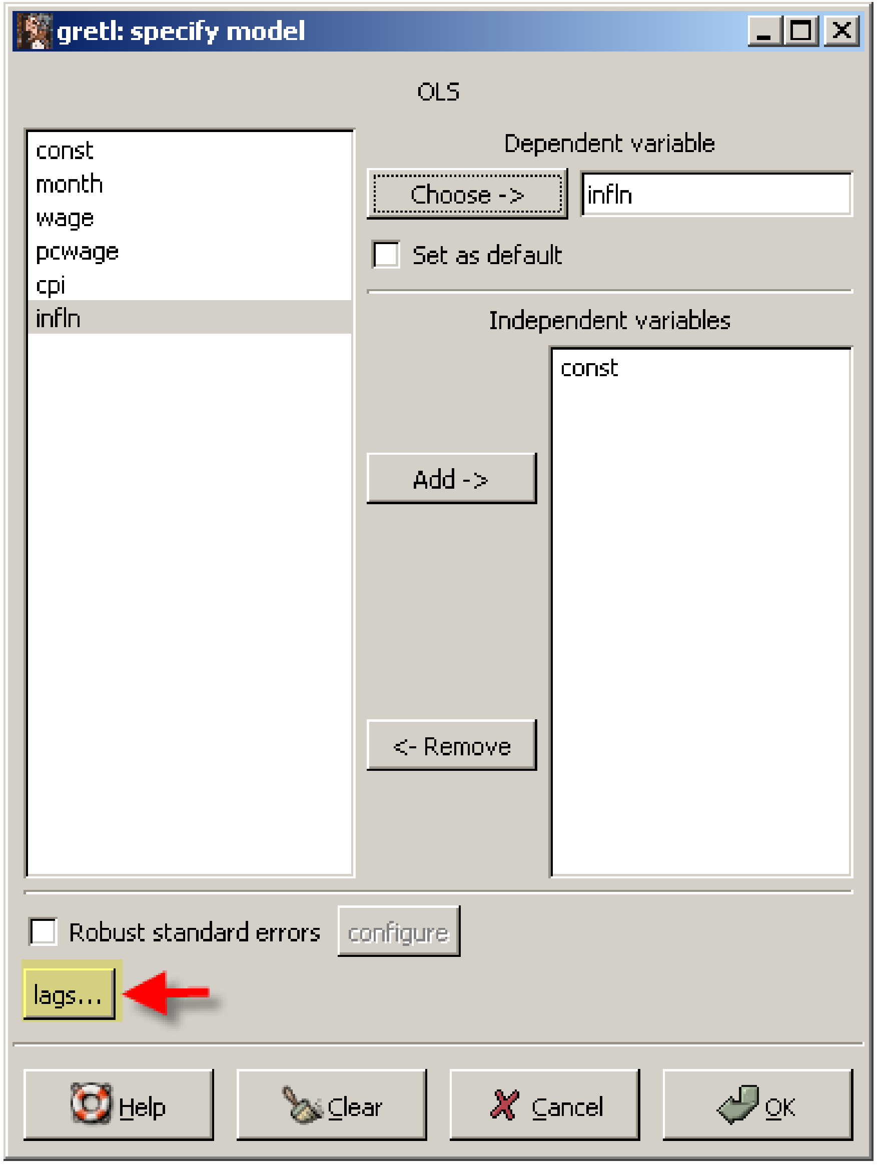
Since your data are defined as time series (recall, you did this through `Data>Dataset structure`) an extra button, labeled 'lags...', appears at the bottom of the dialog as highlighted in Figure 9.8. Click the 'lags...' button in the specify model dialog box and the 'lag order' dialog box shown in Figure 9.9 opens. Click OK and the the 3 lagged values of inflation are added to the model. Now, click OK in the specify model dialog as in Figure 9.8. The model is estimated and the model window shown in Figure 9.10 opens.

Now, we'll use the dialogs to extend the sample and generate the forecasts. From the model window choose `Analysis>Forecasts`. This opens the Add observations dialog box shown in Figure 9.11. To add three observations change the number in the box to 3. Click OK to open the forecast dialog box shown below in Figure 9.12.

By choosing to add 3 observations to the sample, the forecast range is automatically set to 2006:06 to 2006:08. Notice that we've chosen 'automatic forecast (dynamic out of sample).' Click OK and the forecast results appear:

```
For 95% confidence intervals, t(262, .025) = 1.969

  Obs      infln     Forecast     SE        95% C.I.

1998:02   0.000000   0.23350
.
.
.
2006:03   0.350966   0.05914
2006:04   0.598804   0.37476
2006:05   0.446762   0.34046
2006:06              0.26015   0.19724  -0.12823 - 0.64854
2006:07              0.24872   0.21054  -0.16584 - 0.66329
2006:08              0.26972   0.21111  -0.14596 - 0.68541
```

Figure 9.8: From the main window select `Model>Ordinary least squares`. This brings up the specify model dialog box that includes a button for adding lags of the variables to your model



Dependent variable: infln

□ Set as default

Independent variables: const

□ Robust standard errors

Figure 9.9: Check the box labeled 'Lags of dependent variable' and change the second counter to '3' as shown.

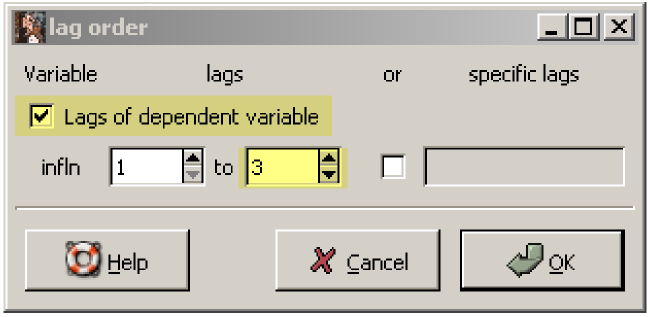
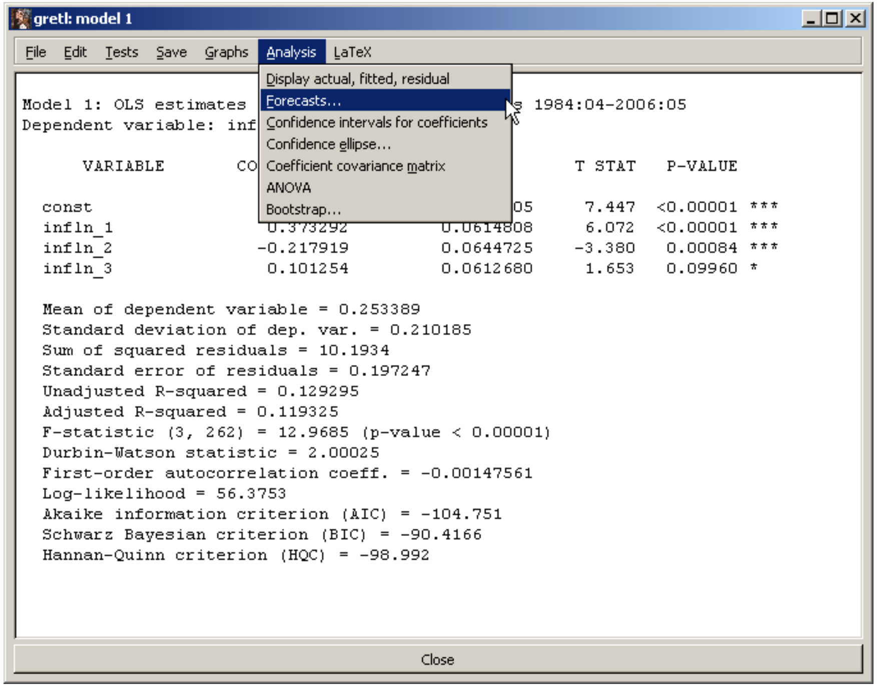
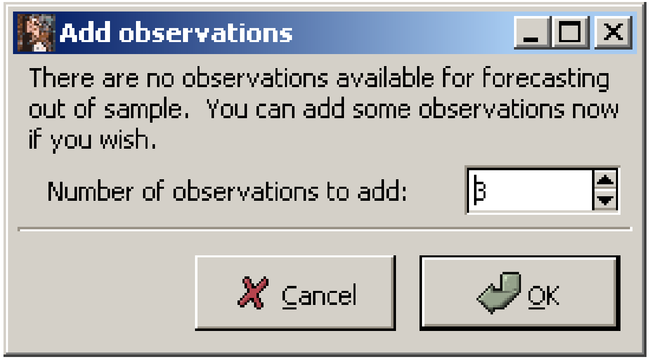


Figure 9.10: Choose `Analysis>Forecasts` from the estimated forecast model to open the forecast dialog box.


gretl: model 1
File Edit Tests Save Graphs Analysis LaTeX
Display actual, fitted, residual
Forecasts...
Confidence intervals for coefficients
Confidence ellipse...
Coefficient covariance matrix
ANOVA
Bootstrap...
Close

```
Model 1: OLS estimates                               s 1984:04-2006:05
Dependent variable: inf

     VARIABLE       CO                                  T STAT    P-VALUE

  const                                          05      7.447   <0.00001 ***
  infln_1               0.373292         0.0614808      6.072   <0.00001 ***
  infln_2              -0.217919         0.0644725     -3.380    0.00084 ***
  infln_3               0.101254         0.0612680      1.653    0.09960 *

  Mean of dependent variable = 0.253389
  Standard deviation of dep. var. = 0.210185
  Sum of squared residuals = 10.1934
  Standard error of residuals = 0.197247
  Unadjusted R-squared = 0.129295
  Adjusted R-squared = 0.119325
  F-statistic (3, 262) = 12.9685 (p-value < 0.00001)
  Durbin-Watson statistic = 2.00025
  First-order autocorrelation coeff. = -0.00147561
  Log-likelihood = 56.3753
  Akaike information criterion (AIC) = -104.751
  Schwarz Bayesian criterion (BIC) = -90.4166
  Hannan-Quinn criterion (HQC) = -98.992
```

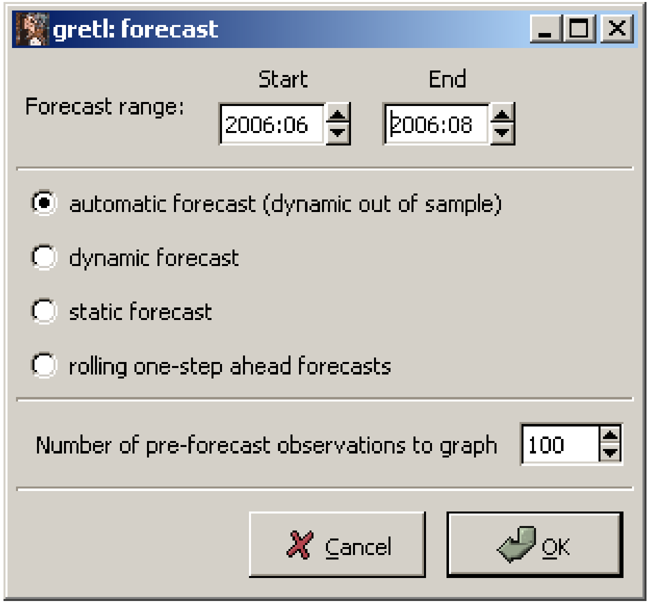
Figure 9.11: Using `Data>Add observations` from the main **gretl** pull-down menu will extend the sample period. This is necessary to generate forecasts.



There are no observations available for forecasting out of sample. You can add some observations now if you wish.

Number of observations to add: 3

Figure 9.12: Forecast dialog box



Forecast range: Start 2006:06 End 2006:08

● automatic forecast (dynamic out of sample)

○ dynamic forecast

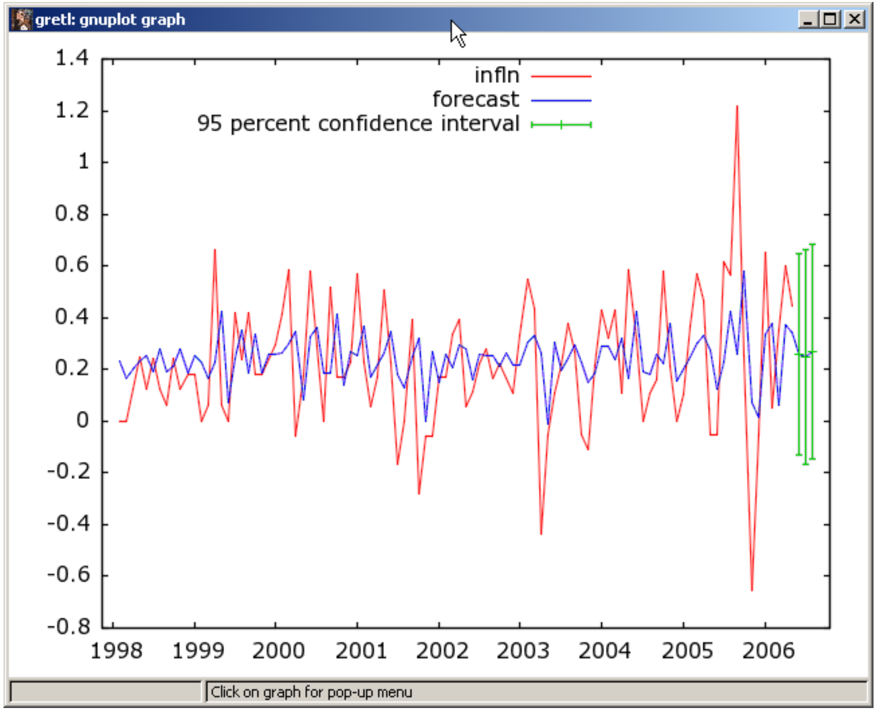
○ static forecast

○ rolling one-step ahead forecasts

Number of pre-forecast observations to graph 100

Miraculously, these match those in *POE*! **Gretl** also uses **gnuplot** to plot the time series and the forecasts (with intervals) as shown in Figure 9.13. The last three observations are forecasts (in blue) and include the 95% confidence intervals shown in green. Actual inflation appears in red.

Figure 9.13: **Gretl** calls **gnuplot** to generate a graph of the time series and the forecast.



9.4.2 Using a Script

Doing all of this using a script is easy as well. Simply estimate the model using `ols infln const infln(-1 to -3)`, use the `addobs 3` command to add 3 observations to the end of the sample, and forecast 3 periods using `fcasterr 2006:06 2006:08`. The `--plot` option ensures that the graph will be produced. The script is:

```
open c:\userdata\gretl\data\poe\inflation.gdt
```

```
ols infln const infln(-1 to -3)
addobs 3
fcasterr 2006:06 2006:08 --plot
```

To estimate the distributed lag model of inflation

$$infl_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + e_t \qquad (9.9)$$

where x_t is the percentage change in wages at time t. The script is:

```
open c:\userdata\gretl\data\poe\inflation.gdt

ols infln const pcwage(0 to -3)
scalar in1 = $coeff(pcwage)+$coeff(pcwage_1)
scalar in2 = in1 + $coeff(pcwage_2)
scalar in3 = in2 + $coeff(pcwage_3)
```

Here, the independent variable is `pcwage`, which is already in the dataset. To add the contemporaneous (lag=0) and 3 lagged values to the list of independent variables, simply add `pcwage(0 to -3)` as shown. The **delay multipliers** are just the coefficients of the corresponding lagged variables. The **interim multiplier** is obtained by cumulatively adding the coefficients together. For instance the interim multiplier at lag 1 is equal to the sum of the delay multipliers (e.g., interim multiplier at lag 1 is ($\beta_0 + \beta_1$). When using the range version (e.g., `pcwage(0 to -3)`) of the language to generate lags, **gretl** appends an underline _ and the corresponding lag number to the variable. So, $pcwage_{t-1}$ is referred to as `pcwage_1`.

9.5 Autoregressive Distributed Lag Model

This model is just a generalization of the ones previously discussed. In this model you include lags of the dependent variable (autoregressive) **and** the contemporaneous and lagged values of independent variables as regressors (distributed lags). The shorthand notation is ARDL(p,q) where p is the maximum distributed lag and q is the maximum autoregressive lag. The model is

$$y_t = \delta + \delta_0 x_t + \delta_1 x_{t-1} + \ldots + \delta_q x_{t-q} + \theta_1 y_{t-1} + \ldots + \theta_p y_{t-p} + v_t \qquad (9.10)$$

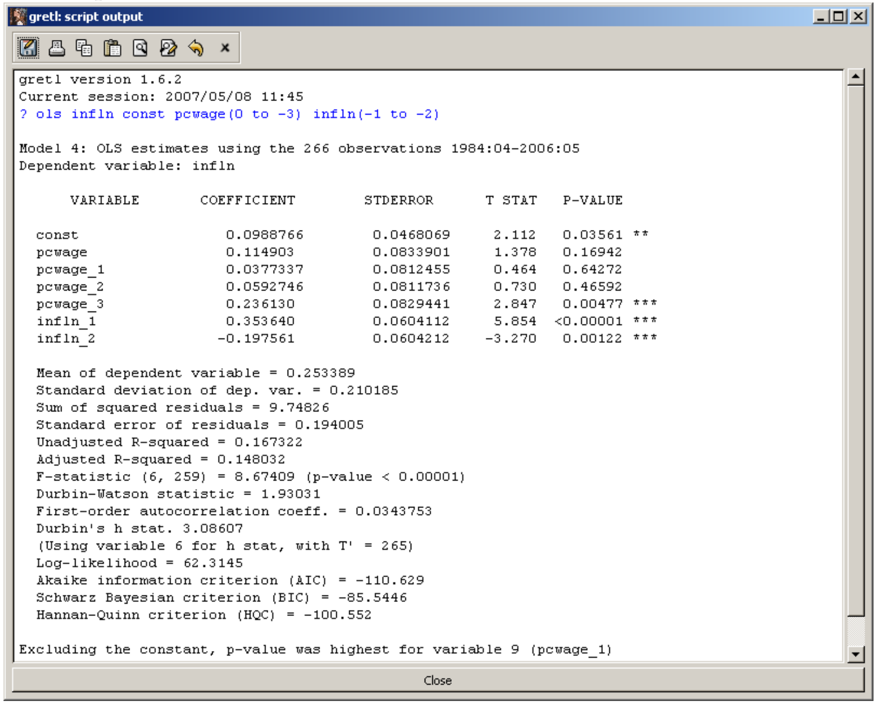
The ARDL(3,2) model of inflation includes the contemporaneous and first 3 lagged values of *pcwage* and the first 2 lags of $infl$ as independent variables.[2]

The script is

```
open c:\userdata\gretl\data\poe\inflation.gdt
ols infln const pcwage(0 to -3) infln(-1 to -2)
```

The result appears in Figure 9.14.

Figure 9.14: Results of the autoregressive distributed lag model produced by the script.



```
gretl version 1.6.2
Current session: 2007/05/08 11:45
? ols infln const pcwage(0 to -3) infln(-1 to -2)

Model 4: OLS estimates using the 266 observations 1984:04-2006:05
Dependent variable: infln

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                 0.0988766        0.0468069     2.112    0.03561 **
  pcwage                0.114903         0.0833901     1.378    0.16942
  pcwage_1              0.0377337        0.0812455     0.464    0.64272
  pcwage_2              0.0592746        0.0811736     0.730    0.46592
  pcwage_3              0.236130         0.0829441     2.847    0.00477 ***
  infln_1               0.353640         0.0604112     5.854   <0.00001 ***
  infln_2              -0.197561         0.0604212    -3.270    0.00122 ***

  Mean of dependent variable = 0.253389
  Standard deviation of dep. var. = 0.210185
  Sum of squared residuals = 9.74826
  Standard error of residuals = 0.194005
  Unadjusted R-squared = 0.167322
  Adjusted R-squared = 0.148032
  F-statistic (6, 259) = 8.67409 (p-value < 0.00001)
  Durbin-Watson statistic = 1.93031
  First-order autocorrelation coeff. = 0.0343753
  Durbin's h stat. 3.08607
  (Using variable 6 for h stat, with T' = 265)
  Log-likelihood = 62.3145
  Akaike information criterion (AIC) = -110.629
  Schwarz Bayesian criterion (BIC) = -85.5446
  Hannan-Quinn criterion (HQC) = -100.552

Excluding the constant, p-value was highest for variable 9 (pcwage_1)
```

[2] Technically, lagged values of inflation are *predetermined* not independent, but we'll leave this discussion for others. Their treatment in a regression is the same, though.

9.6 Script

```
open C:\userdata\gretl\data\poe\bangla.gdt
#declare the data time-series
setobs 1 1 --time-series

#Least squares with wrong std errors
logs p a
ols l_a const l_p

#Least squares with HAC standard errors
#choose lag
set hac_lag nw2
#choose weights
set hac_kernel bartlett
#run regression with robust std errors
ols l_a const l_p --robust

#Nonlinear least squares
#step 1:  set the starting values
genr beta1 = $coeff(const)
genr beta2 = $coeff(l_p)
genr rho = 0

#step 2:  type in the model
nls l_a = beta1*(1-rho) + rho*l_a(-1) + beta2*(l_p-rho*l_p(-1))
end nls

#save restricted sum of squared errors for the hypothesis test
scalar sser=$ess

#get the unrestricted sse
ols l_a const l_p l_p(-1) l_a(-1)
scalar sseu=$ess

scalar fstat = (sser-sseu)/(sseu/$df)
pvalue X 1 fstat
pvalue F 1 $df fstat
```

```
#Correlogram
ols l_a const l_p --robust
genr ehat = $uhat
corrgm ehat 12

#LM test
ols l_a const l_p
genr ehat = $uhat
ols ehat const l_p ehat(-1)
scalar NR2 = $trsq

#Dynamic forecasting in an autoregressive model
open C:\userdata\gretl\data\poe\inflation.gdt
ols infln const infln(-1 to -3)
addobs 3
fcasterr 2006:06 2006:08--plot

#Distributed Lag model and interim multipliers
ols infln const pcwage(0 to -3)
scalar in1 = $coeff(pcwage)+$coeff(pcwage_1)
scalar in2 = in1 + $coeff(pcwage_2)
scalar in3 = in2 + $coeff(pcwage_3)

#ARDL(3,2)
ols infln const pcwage(0 to -3) infln(-1 to -2)
#First 5 lag weights for infinite distributed lag
scalar b0 = $coeff(pcwage)
scalar b1 = $coeff(infln_1)*b0+$coeff(pcwage_1)
scalar b2 = $coeff(infln_1)*b1+$coeff(infln_2)*b0+$coeff(pcwage_2)
scalar b3 = $coeff(infln_1)*b2+$coeff(infln_2)*b1+$coeff(pcwage_3)
scalar b4 = $coeff(infln_1)*b3+$coeff(infln_2)*b2
```

Chapter 10

Random Regressors and Moment Based Estimation

In this chapter you will learn to use instrumental variables to obtain consistent estimates of a model's parameters when its independent variables are correlated with the model's errors.

10.1 Basic Model

Consider the linear regression model

$$y_i = \beta_1 + \beta_2 x_i + e_i \quad i = 1, 2, \ldots, N \tag{10.1}$$

Equation (10.1) suffers from a significant violation of the usual model assumptions when its explanatory variable is contemporaneously correlated with the random error, i.e., $Cov(e_i, x_i) = E(e_i x_i) \neq 0$. In this instance, least squares is known to be both biased and inconsistent.

An **instrument** is a variable, z, that is correlated with x but not with the error, e. In addition, the instrument does not directly affect y and thus does not belong in the actual model. It is common to have more than one instrument for x. All that is required is that these instruments, $z_1, z_2, \ldots, z_s$, be correlated with x, but not with e. Consistent estimation of (10.1) is possible if one uses the **instrumental variables** or **two-stage least squares estimator**, rather than the usual OLS estimator.

10.2 IV Estimation

Gretl handles this estimation problem with ease using what is commonly referred to as *two-stage least squares.* In econometrics, the terms two-stage least squares (TSLS) and instrumental variables (IV) estimation are often used interchangeably. The 'two-stage' terminology is a legacy of the time when the easiest way to estimate the model was to actually use two separate least squares regressions. With better software, the computation is done in a single step to ensure the other model statistics are computed correctly. Since the software you use invariably expects you to specify 'instruments,' it is probably better to think about this estimator in those terms from the beginning. Keep in mind though that **gretl** uses the old-style term *two-stage least squares* (`tsls`) as it asks you to specify instruments in it dialog boxes and scripts.

To perform TSLS or IV estimation you need instruments that are correlated with your independent variables, but not correlated with the errors of your model. First, load the *ch10.gdt* data into **gretl**. Then, to open the basic **gretl** dialog box that computes the IV estimator choose `Model>Other linear models>Two-Stage Least Squares` from the pull-down menu as shown below in Figure 10.1. This opens the dialog box shown in Figure 10.2.

In this example we choose y as the dependent variable, put all of the desired instruments into the *Instruments* box, and put all of the independent variables, including the one(s) measured with error, into the *Independent Variables* box. If some of the right-hand side variables for the model are exogenous, they should be referenced in both lists. That's why the `const` variable (for the constant) appears in both places. Press the OK button and the results are found in Table 10.1. Notice that **gretl** does not report a goodness-of-fit measure, R^2, in the output generated for LaTeX shown here. However, it does appear in the model window, ignoring the sound advice

Table 10.1: TSLS results.

Model 1: TSLS estimates using the 100 observations 1–100
Dependent variable: y
Instruments: z1

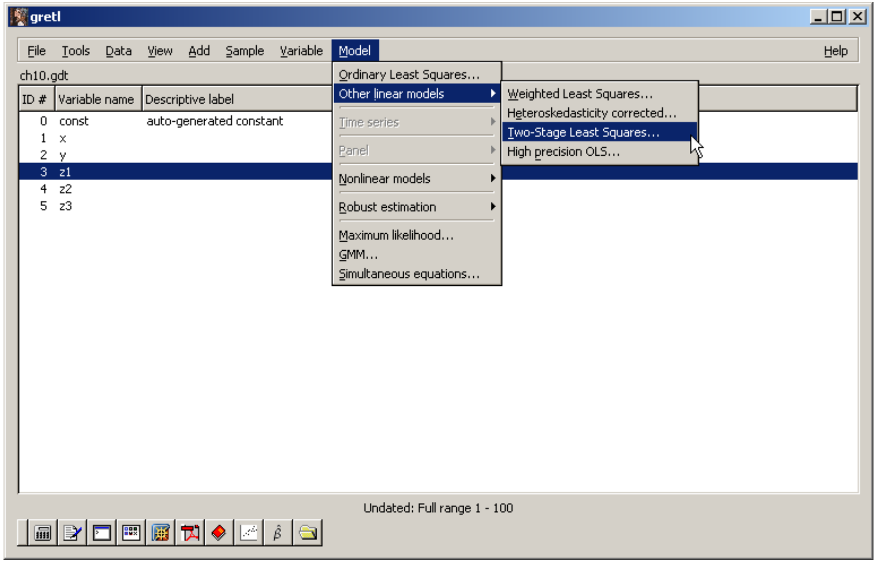
Variable	Coefficient	Std. Error	*t*-statistic	p-value
const	1.10110	0.109128	10.0900	0.0000
x	1.19245	0.194518	6.1302	0.0000

Mean of dependent variable	1.38629
S.D. of dependent variable	1.83882
Sum of squared residuals	95.4985
Standard error of residuals ($\hat{\sigma}$)	0.987155
Degrees of freedom	98
Akaike information criterion	283.182
Schwarz Bayesian criterion	288.392
Hannan–Quinn criterion	285.291

Hausman test –
Null hypothesis: OLS estimates are consistent
Asymptotic test statistic: $\chi^2_1 = 15.0454$
with p-value = 0.000104958

First-stage $F(1, 98) = 38.9197$

Figure 10.1: Two-Stage Least Squares estimator from the pull-down menus



ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	x	
2	y	
3	z1	
4	z2	
5	z3	

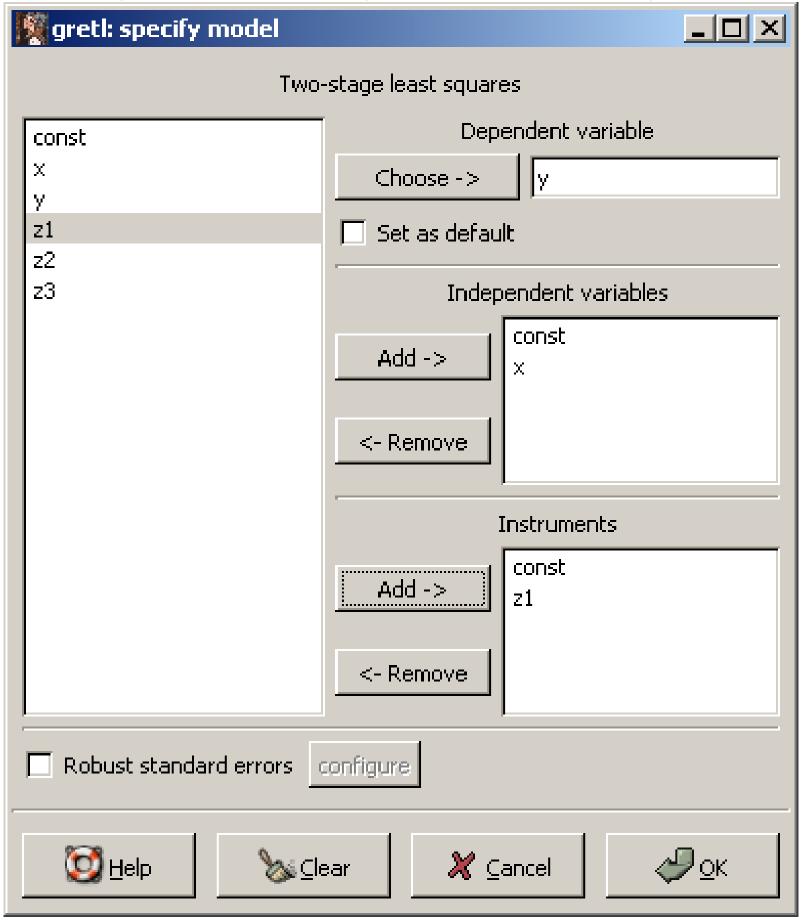
offered by the authors of your textbook to not compute it here. However, **gretl** at least informs you that it is computed as the squared correlation between observed and fitted values of the dependent variable.

If you prefer to use a script, the syntax is very simple. The script for the example above is

```
open c:\userdata\gretl\data\poe\ch10.gdt
tsls y const x;const z1
```

The **gretl** command `tsls` calls for the IV estimator to be used and it is followed by the linear model you wish to estimate. List the dependent variable (`y`) first, followed by the independent variables (`const x`). A semicolon separates the model to be estimated from the list of instruments (`const z1`). Notice that the constant is listed again as an instrument; once again, this is because it is exogenous with respect to the errors of the model and all exogenous variables should be listed in both places.

Figure 10.2: Two-Stage Least Squares dialog box



gretl: specify model

Two-stage least squares

const
x
y
z1
z2
z3

Dependent variable: y

□ Set as default

Independent variables:
const
x

Instruments:
const
z1

□ Robust standard errors

10.3 Specification Tests

There are three specification tests you will find useful with instrumental variables estimation. By default, **Gretl** computes each of these whenever you estimate a model using two-stage least squares. Below I'll walk you through doing it manually and we'll compare the manual results to the automatically generated ones.

10.3.1 Hausman Test

The first test is to determine whether the independent variable(s) in your model is (are) in fact uncorrelated with the model's errors. If so, then least squares is more efficient than the IV estimator. If not, least squares is inconsistent and you should use the less efficient, but consistent, instrumental variable estimator. The null and alternative hypotheses are $H_o:\ Cov(x_i, e_i) = 0$ against $H_a:\ Cov(x_i, e_i) \neq 0$. The first step is to use least squares to estimate

$$x_i = \gamma_1 + \theta_1 z_{i1} + \theta_2 z_{i2} + \nu_i \tag{10.2}$$

and to save the residuals, $\hat{\nu}_i$. Then, add the residuals to the original model

$$y_i = \beta_1 + \beta_2 x_i + \delta\hat{\nu}_i + e_i \tag{10.3}$$

Estimate this equation using least squares and use the t-ratio on the coefficient δ to test the hypothesis. If it is significantly different from zero then the regressor, x_i is not exogenous or predetermined with respect to e_i and you should use the IV estimator (TSLS) to estimate β_1 and β_2. If it is not significant, then use the more efficient estimator, OLS.

The **gretl** script for the Hausman test is:

```
open c:\userdata\gretl\data\poe\ch10.gdt
ols x const z1 z2
genr uhat1 = $uhat
ols y const x uhat1
```

You may have noticed that whenever you use two-stage least squares in **gretl** that the program automatically produces the test statistic for the

Hausman test. There are several different ways of computing this statistic so don't be surprised if it differs from the one you compute manually using the above script.

10.3.2 Testing for Weak Instruments

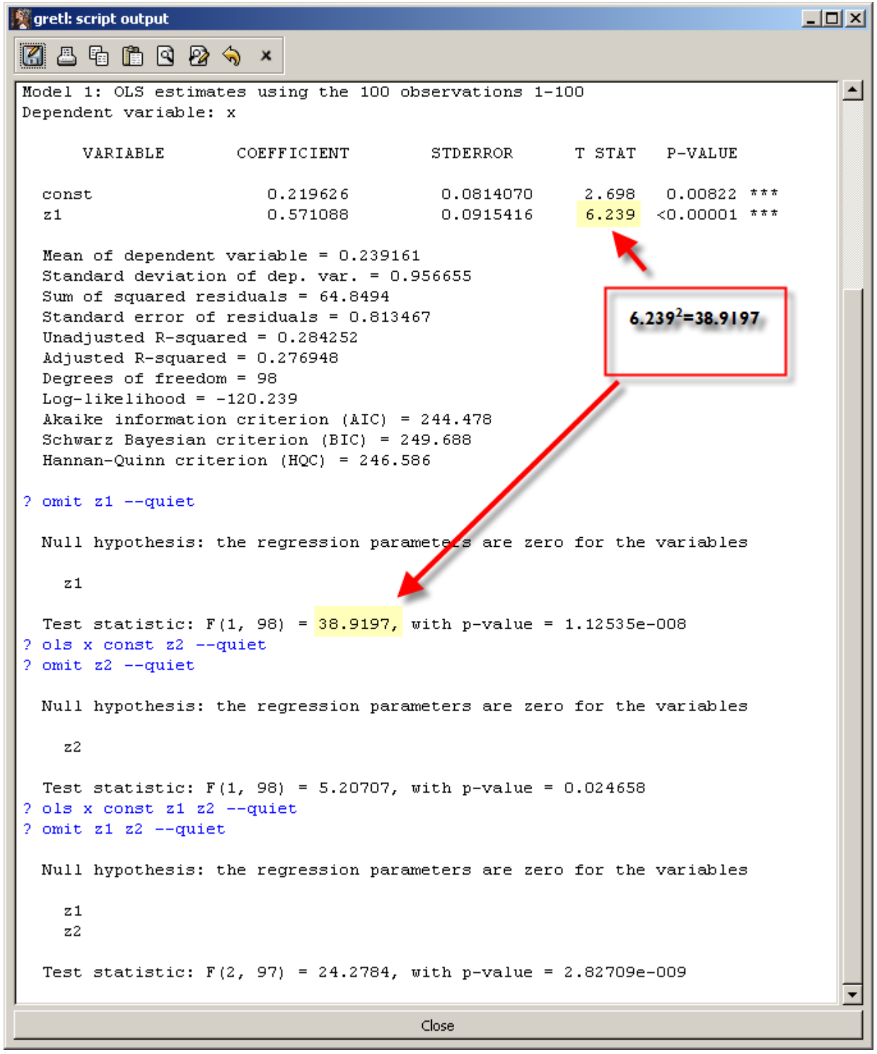
To test for weak instruments, regress each independent variable suspected of being contemporaneously correlated with the error (x) onto all of the instruments ($z's$). If the overall F statistic in this regression[1] is less than 10, then you conclude that the instruments are weak. If it is greater than 10, you conclude that the instruments are strong enough. The following script uses least squares to perform three such tests. The first regression assumes there is only one instrument, $z1$; the second that the single instrument is $z2$; the third assumes both are instruments.

```
open c:\userdata\gretl\data\poe\ch10.gdt
ols x const z1
omit z1 --quiet
ols x const z2 --quiet
omit z2 --quiet
ols x const z1 z2 --quiet
omit z1 z2 --quiet
```

When `omit` follows an OLS regression (e.g., `ols x const z1 z2`), **gretl** estimates a restricted model where the variables listed after it are omitted from the model above. It then performs a joint hypothesis test that the coefficients of the omitted variables are zero against the alternative that one or more are not zero. The `--quiet` option reduces the amount of output you have to wade through by suppressing the regressions; only the test results are printed. The output from **gretl** appears in Figure 10.3 below: Notice that the t-ratio on $z1$ is equal to $0.571088/0.0915416 = 6.23856$ and the F(1,98) statistic associated with the same null hypothesis (i.e., that the coefficient on $z1$ is zero) is 38.9197. In fact there is an exact relationship between these numbers since $t^2_{n-k} = F_{1,n-k}$. This is easily verified here by computing

[1] Recall that the null hypothesis for the overall F statistic is that all slopes are zero.

Figure 10.3: Results from using the `omit` statement after least squares



```
Model 1: OLS estimates using the 100 observations 1-100
Dependent variable: x

      VARIABLE        COEFFICIENT        STDERROR      T STAT    P-VALUE

  const                  0.219626        0.0814070      2.698    0.00822 ***
  z1                     0.571088        0.0915416      6.239   <0.00001 ***

  Mean of dependent variable = 0.239161
  Standard deviation of dep. var. = 0.956655
  Sum of squared residuals = 64.8494
  Standard error of residuals = 0.813467
  Unadjusted R-squared = 0.284252
  Adjusted R-squared = 0.276948
  Degrees of freedom = 98
  Log-likelihood = -120.239
  Akaike information criterion (AIC) = 244.478
  Schwarz Bayesian criterion (BIC) = 249.688
  Hannan-Quinn criterion (HQC) = 246.586

? omit z1 --quiet

  Null hypothesis: the regression parameters are zero for the variables

    z1

  Test statistic: F(1, 98) = 38.9197, with p-value = 1.12535e-008
? ols x const z2 --quiet
? omit z2 --quiet

  Null hypothesis: the regression parameters are zero for the variables

    z2

  Test statistic: F(1, 98) = 5.20707, with p-value = 0.024658
? ols x const z1 z2 --quiet
? omit z1 z2 --quiet

  Null hypothesis: the regression parameters are zero for the variables

    z1
    z2

  Test statistic: F(2, 97) = 24.2784, with p-value = 2.82709e-009
```

6.239^2=38.9197

$6.239^2 \doteq 38.9197$.[2] Since the F value is well beyond 10, we can reject the hypothesis that the instrument $z1$ is weak in favor of the alternative that it is strong enough to be useful.

The second pair of statements in the script assume that $z2$ is the single available instrument and the `omit` statement is again used to elicit the F statistic.

In the last regression, we use both instruments and the `omit` statement in **gretl** to perform the joint test that the instruments are jointly weak.

Gretl proves its worth here. Whenever you estimate a model using two stage least squares, **gretl** will compute the test statistic for the weak instruments test.

10.3.3 Sargan Test

The final test is the Sargan test of the overidentifying restrictions implied by an overidentified model. Recall that to be overidentified just means that you have more instruments than you have endogenous regressors. In our example we have a single endogenous regressor (x) and three instruments ($z1$, $z2$ and $z3$). The first step is to estimate the model using TSLS using all the instruments. Save the residuals and then regress these on the instruments alone. NR^2 from this regression is approximately χ^2 with the number of surplus instruments as your degrees of freedom. **Gretl** does this easily since it saves TR^2 as a part of the usual regression output, where T is the sample size (which we are calling N). The script for the Sargan test follows:

```
open c:\userdata\gretl\data\poe\ch10.gdt
list inst = const z1 z2 z3
tsls y const x; inst
genr uhat2 = $uhat
ols uhat2 inst
genr test = $trsq
pvalue X 3 test
```

[2] The small discrepancy you will find if you try the calculation occurs because of rounding.

This script uses a convenient way to accumulate variables into a set using the list command. The command list inst = const z1 z2 z3 puts the variables contained in const, z1, z2, and z3 into a set called inst. Once defined, the set of variables can be referred to as inst rather than listing them individually as we've done up to this point. In the script above, inst is used in the third line to list the instruments for tsls and again in the fifth line to include these variables in the ols regression.

Rejection of the null hypothesis implies that one or more of the overidentifying restrictions are not valid; you've chosen an inappropriate instrument. If the test statistic is insignificant, then your set of instruments passes muster. Whenever you have extra instruments (the model is overidentified), **gretl** will compute and print out the results from the Sargan test automatically. Unlike the Hausman test, these results should match those you compute manually using the script.

```
Sargan over-identification test -
  Null hypothesis: all instruments are valid
  Test statistic: TR^2 = 13.1107
  with p-value = P(Chi-Square(2) > 13.1107) = 0.00142246
```

10.4 Wages Example

The following script uses the results above to quickly reproduce the results from the wages example in your text. Open the data, restrict the sample to those working ($wage > 0$), and generate logarithm of wages and square experience.

```
open c:\userdata\gretl\data\poe\mroz.gdt

# restrict your sample to include only positive values for wage
smpl wage > 0 --restrict

# generate ln(wage) and experience squared
genr lwage = log(wage)
genr expersq = exper*exper
```

The next thing we'll do is to create lists that contain regressors and instruments. These will simplify the program and help us to avoid having to use the continuation command for long lines of code.

```
# create lists of variables to include in each regression
# Regressors in x
list x = const educ exper expersq
# Instrument sets in z1 and z2
list z1 = const exper expersq mothereduc
list z2 = const exper expersq mothereduc fathereduc
```

The first `list` command puts the regressors `const`, `educ`, `exper`, and `expersq` into a set called `x`. The first set of instruments includes all of the exogenous variables in the list of regressors and adds `mothereduc`; it is called `z1`. The second set, called `z2`, adds `fathereduc` to the list of instruments.

Now, estimate the model using least squares. Notice that the list of regressors has been replaced by the list we created above. If education is endogenous in this regression, then least squares is inconsistent and should not be used.

```
# least squares regression of wage equation
ols lwage x
```

This produces:

$$\widehat{\text{lwage}} = -\underset{(0.19863)}{0.522} + \underset{(0.0141)}{0.107}\,\text{educ} + \underset{(0.0132)}{0.0416}\,\text{exper} - \underset{(0.00039)}{0.000811}\,\text{expersq}$$

$$T = 428 \quad \bar{R}^2 = 0.1509 \quad F(3,424) = 26.286 \quad \hat{\sigma} = 0.66642$$

(standard errors in parentheses)

Estimate the reduced form equation that uses mother's education as the sole instrument along with the other exogenous variables in the model; all of these were collected into `z1`.

```
# least squares regression of the reduced form
ols educ z1
```

This produces:

$$\widehat{\text{educ}} = \underset{(0.424)}{9.775} + \underset{(0.0417)}{0.04886}\,\text{exper} - \underset{(0.00124)}{0.001281}\,\text{expersq} + \underset{(0.031)}{0.268}\,\text{mothereduc}$$
$$T = 428 \quad \bar{R}^2 = 0.1467 \quad F(3, 424) = 25.47 \quad \hat{\sigma} = 2.1111$$
$$\text{(standard errors in parentheses)}$$

Estimate the model using the instrumental variable estimator. The instrumental variable estimators will be consistent if education is endogenous or not. It is not efficient. In the first instance below, only mother's education is used as an instrument and in the second both mother's and father's education are used.

```
# tsls regression using 1 instrument (mother's education)
tsls lwage x ; z1

# tsls using 2 instruments (mother's and father's education)
tsls lwage x ; z2
```

The TSLS results for the regression with one instrument is:

$$\widehat{\text{lwage}} = \underset{(0.473)}{0.1982} + \underset{(0.0374)}{0.04926}\,\text{educ} + \underset{(0.0136)}{0.04486}\,\text{exper} - \underset{(0.000406)}{0.0009221}\,\text{expersq}$$
$$T = 428 \quad \bar{R}^2 = 0.1293 \quad F(3, 424) = 22.137 \quad \hat{\sigma} = 0.6796$$
$$\text{(standard errors in parentheses)}$$

and that for the model with two instruments is:

Model 3: TSLS estimates using the 428 observations 1–428
Dependent variable: lwage
Instruments: mothereduc fathereduc

Variable	Coefficient	Std. Error	*t*-statistic	p-value
const	0.0481	0.4003	0.120	0.904
educ	0.0614	0.0314	1.953	0.051
exper	0.0442	0.0134	3.288	0.001
expersq	−0.0009	0.0004	−2.238	0.025

Mean of dependent variable	1.19017
S.D. of dependent variable	0.723198
Sum of squared residuals	193.020
Standard error of residuals ($\hat{\sigma}$)	0.674712
$F(3, 424)$	24.0965
Akaike information criterion	881.782
Schwarz Bayesian criterion	898.019
Hannan–Quinn criterion	888.195

Hausman test –
  Null hypothesis: OLS estimates are consistent
  Asymptotic test statistic: $\chi^2_1 = 2.8256$
  with p-value = 0.0927721

Sargan over-identification test –
  Null hypothesis: all instruments are valid
  Test statistic: $TR^2 = 0.378071$
  with p-value = $P(\chi^2_1 > 0.378071) = 0.538637$

First-stage $F(2, 423) = 55.4003$

A Hausman test statistic is manually computed to test the validity of the instruments. The least squares residuals from the reduced form equation are regressed on all exogenous and instrumental variables. The residuals are saved and added to the original structural equation. Test the significance of the residuals coefficient using a t-test, or as we've done here, the equivalent F(1,N-K) test using the `omit` statement.

```
# Hausman test (check the t-ratio on ehat for significance)
ols educ z2 --quiet
genr ehat=$uhat
ols lwage x ehat --quiet
omit ehat --quiet
```

This produces:

```
  Test statistic: F(1, 423) = 2.7926, with p-value = 0.09544
```

which is very close the automatic result produced by **gretl** as part of the tsls output.

To test the strength of the instruments we estimate the reduced form equation for education and conduct a joint significance test of the two instruments (mothereduc and fathereduc). Once again, the --quiet option is used to suppress unnecessary output.

```
# test for strength of instruments (coeffs on instruments
# are jointly zero)
ols educ z2 --quiet
omit mothereduc fathereduc --quiet
```

Finally the test of overidentification is done. This requires residuals from the instrumental variable estimator, TSLS. Estimate the model using TSLS and save the residuals. In the second regression, which is estimated using least squares, these residuals are regressed on all exogenous and instrumental variables. NR^2 from this regression is compared to the $\chi^2(2)$ distribution. If the p-value is smaller than the desired α then at least one of the instruments is not appropriate. You'll need to either drop the offending ones or find others to use.

```
# (Sargan's test)
# requires residuals from tsls to use in this test
tsls lwage x; z2
genr vhat=$uhat

ols vhat z2
genr lmstat = $trsq
pvalue X 2 lmstat
```

The script will produce the same results you get from **gretl**'s tsls command.

10.5 Script

```
open c:\userdata\gretl\data\poe\ch10.gdt
```

```
tsls y const x; const z1

#Hausman test
ols x const z1 z2
genr uhat1 = $uhat
ols y const x uhat1

#Testing for weak instruments
open c:\userdata\gretl\data\poe\ch10.gdt
ols x const z1
omit z1 --quiet
ols x const z2 --quiet
omit z2 --quiet
ols x const z1 z2 --quiet
omit z1 z2 --quiet

#Sargan Test
open c:\userdata\gretl\data\poe\ch10.gdt
list inst = const z1 z2 z3
tsls y const x; inst
genr uhat2 = $uhat
ols uhat2 inst
genr test = $trsq
pvalue X 3 test

#Wages Example
open c:\userdata\gretl\data\poe\mroz.gdt

# restrict your sample to include only positive values for wage
smpl wage > 0 --restrict

# generate ln(wage) and experience squared
genr lwage = log(wage)
genr expersq = exper*exper

# create lists of variables to include in each regression
# Regressors in x
list x = const educ exper expersq
# Instrument sets in z1 and z2
list z1 = const exper expersq mothereduc
list z2 = const exper expersq mothereduc fathereduc

# least squares regression of wage equation
ols lwage x

# least squares regression of the reduced form
ols educ z1

# tsls regression using 1 instrument (mother’s education)
tsls lwage x; z1

# tsls using 2 instruments (mother’s and father’s education)
tsls lwage x ; z2
genr vhat = $uhat

# Hausman test (check the t-ratio on ehat for significance)
ols educ z2 --quiet
genr ehat=$uhat
ols lwage x ehat --quiet
omit ehat --quiet

# test for strength of instruments (coeffs on instruments
# are jointly zero)
ols educ z2 --quiet
omit mothereduc fathereduc --quiet

#Repeat using HCCME
ols educ z2 --robust --quiet
omit mothereduc fathereduc --quiet

# test for validity of instruments using residuals from tsls
# (Sargan’s test)
ols vhat z2 --quiet
genr lmstat = $trsq
pvalue X 2 lmstat
```

Chapter 11

Simultaneous Equations Models

In Chapter 11 of *POE* the authors present a model of supply and demand. The econometric model contains two equations and two dependent variables. The distinguishing factor for models of this type is that the values of two (or more) of the variables are jointly determined. This means that a change in one of the variables causes the other to change and vice versa. The model is demonstrated using the truffle example which is explained below.

11.1 Truffle Example

Consider a supply and demand model for truffles:

$$Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_i^d \tag{11.1}$$

$$Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_i^s \tag{11.2}$$

The first equation (11.1) is demand and Q us the quantity of truffles traded in a particular French market, P is the market price of truffles, PS is the market price of a substitute good, and DI is per capita disposable income of the local residents. The supply equation (11.2) contains the variable PF, which is the price of a factor of production. Each observation is indexed by i, $i = 1, 2, \ldots N$. As explained in the text, prices and quantities in a market are jointly determined; hence, in this econometric model P and Q are both endogenous to the system.

11.2 The Reduced Form Equations

The reduced form equations express each endogenous variable as a linear function of every exogenous variable in the entire system. So, for our example

$$Q_i = \pi_{11} + \pi_{21} PS_i + \pi_{31} DI_i + \pi_{41} PF_i + \nu_{i1} \tag{11.3}$$

$$P_i = \pi_{12} + \pi_{22} PS_i + \pi_{32} DI_i + \pi_{42} PF_i + \nu_{i2} \tag{11.4}$$

Since each of the independent variables is exogenous with respect to Q and P, the reduced form equations (11.3) and (11.4) can be estimated using least squares. In **gretl** the script is

```
open c:\userdata\gretl\data\POE\truffles.gdt
ols q const ps di pf
ols p const ps di pf
```

The **gretl** results appear in Table 11.1

11.3 The Structural Equations

The structural equations are estimated using two-stage least squares. The basic **gretl** commands for this estimator are discussed in Chapter 10. The instruments consist of ***all*** exogenous variables, i.e., the same variables you use to estimate the reduced form equations (11.3) and (11.4).

The **gretl** commands to open the truffle data and estimate the structural equations using two-stage least squares are:

```
open c:\userdata\gretl\data\poe\truffles.gdt
tsls q const p ps di; const ps di pf
tsls q const p pf; const ps di pf
```

Table 11.1: The least squares estimates of the reduced form equations.

$$\widehat{\text{q}} = \underset{(2.434)}{7.89510} + \underset{(4.605)}{0.656402}\,\text{ps} + \underset{(3.094)}{2.16716}\,\text{di} - \underset{(-4.181)}{0.506982}\,\text{pf}$$

$$T = 30 \quad \bar{R}^2 = 0.6625 \quad F(3,26) = 19.973 \quad \hat{\sigma} = 2.6801$$

$$(t\text{-statistics in parentheses})$$

$$\widehat{\text{p}} = \underset{(-4.072)}{-32.5124} + \underset{(4.868)}{1.70815}\,\text{ps} + \underset{(4.409)}{7.60249}\,\text{di} + \underset{(4.536)}{1.35391}\,\text{pf}$$

$$T = 30 \quad \bar{R}^2 = 0.8758 \quad F(3,26) = 69.189 \quad \hat{\sigma} = 6.5975$$

$$(t\text{-statistics in parentheses})$$

The second line of the script estimates the demand equation. The **gretl** command `tsls` calls for the two-stage least squares estimator and it is followed by the structural equation you wish to estimate. List the dependent variable (`q`) first, followed by the regressors variables (`const p ps di`). A semicolon separates the model to be estimated from the list of instruments (`const ps di pf`). Don't forget to list the constant again as an instrument. The third line uses the same format to estimate the parameters of the supply equation. Refer to section 10.2, and Figures 10.1 and 10.2 specifically, about using the GUI to estimate the model.

The results from two-stage least squares appear below in Table 11.2

11.4 Fulton Fish Example

The following script estimates the reduced form equations using least squares and the demand equation using two-stage least squares for Graddy's Fulton Fish example.

In the example, $ln(quantity)$ and $ln(price)$ are endogenously determined. There are several potential instruments that are available. The variable *stormy* may be useful in identifying the demand equation. In order for

Table 11.2: Two-stage least square estimates of the demand and supply of truffles.

Demand

$$\widehat{\text{q}} = \underset{(-0.772)}{-4.27947} - \underset{(-2.273)}{0.374459}\,\text{p} + \underset{(3.649)}{1.29603}\,\text{ps} + \underset{(2.196)}{5.01398}\,\text{di}$$

$$T = 30 \quad \bar{R}^2 = 0.1376 \quad F(3,26) = 2.5422 \quad \hat{\sigma} = 4.93$$

(t-statistics in parentheses)

Supply

$$\widehat{\text{q}} = \underset{(16.379)}{20.0328} + \underset{(13.563)}{0.337982}\,\text{p} - \underset{(-12.128)}{1.00091}\,\text{pf}$$

$$T = 30 \quad \bar{R}^2 = 0.8946 \quad F(2,27) = 124.08 \quad \hat{\sigma} = 1.4976$$

(t-statistics in parentheses)

the demand equation to be identified, there must be at least one variable available that effectively influences the supply of fish without affecting its demand. Presumably, stormy weather affects the fishermen's catch without affecting people's appetite for fish! Logically, stormy may be a good instrument.

The model of demand includes a set of dummy variables for day of the week. Friday is omitted to avoid the dummy variable trap. These day of week variables are not expected to affect supply; fishermen catch the same amount on average on any working day. They may affect demand though, since people in some cultures buy more fish on some days than others.

In both demand and supply equations, *ln*(*price*) is the right-hand side endogenous variable. Identification of the demand equation requires *stormy* to be significantly correlated with *lprice*. This can be determined by looking at the t-ratio in the *lprice* reduced form equation.

For supply to be identified, at least one of the day of the week dummy variables (*mon tue wed thu*), which are excluded from the supply equation, has to be significantly correlated with *lprice* in the reduced form. If not, the supply equation cannot be estimated; it is not identified.

Proceeding with the analysis, open the data and estimate the reduced form equations for *lquan* and *lprice*. Go ahead and conduct the joint test of the day of the week variables using the `--quiet` option. The results of this test can help determine whether the supply equation is identified.

```
open C:\userdata\gretl\data\poe\fultonfish.gdt

#Estimate the reduced form equations
ols lquan const stormy mon tue wed thu
ols lprice const stormy mon tue wed thu
omit mon tue wed thu --quiet
```

The reduced form results for *lquan* appear below:

Model 1: OLS estimates using the 111 observations 1–111
Dependent variable: lquan

Variable	Coefficient	Std. Error	t-statistic	p-value
const	8.810	0.147	59.922	0.000
stormy	−0.388	0.144	−2.698	0.008
mon	0.101	0.207	0.489	0.626
tue	−0.485	0.201	−2.410	0.018
wed	−0.553	0.206	−2.688	0.008
thu	0.054	0.201	0.267	0.790

Standard error of residuals ($\hat{\sigma}$)	0.681790
Unadjusted R^2	0.193372
$F(5, 105)$	5.03429
p-value for $F()$	0.000356107

and the results for *lprice*

Model 2: OLS estimates using the 111 observations 1–111
Dependent variable: lprice

Variable	Coefficient	Std. Error	t-statistic	p-value
const	−0.272	0.076	−3.557	0.001
stormy	0.346	0.075	4.639	0.000
mon	−0.113	0.107	−1.052	0.295
tue	−0.041	0.105	−0.394	0.695
wed	−0.012	0.107	−0.111	0.912
thu	0.050	0.104	0.475	0.636

Unadjusted R^2	0.178889
$F(5, 105)$	4.57511
p-value for $F()$	0.000815589

In this equation, stormy is highly significant with a t-ratio of 4.639, but the daily dummy variables are not. A joint test of their significance reveals that they are not jointly significant, either; the F-statistic has a p-value of only .65. Supply is not identified and can't be estimated without better instruments.

The two-stage least squares estimates of the demand equation are obtained using:

```
#TSLS estimates of demand
tsls lquan const lprice mon tue wed thu; \
              const stormy mon tue wed thu
```

to produce the result:

Model 3: TSLS estimates using the 111 observations 1–111
Dependent variable: lquan
Instruments: stormy

Variable	Coefficient	Std. Error	t-statistic	p-value
const	8.506	0.166	51.189	0.000
mon	−0.025	0.215	−0.118	0.906
tue	−0.531	0.208	−2.552	0.011
wed	−0.566	0.213	−2.662	0.008
thu	0.109	0.209	0.523	0.601
lprice	−1.119	0.429	−2.612	0.009

Mean of dependent variable	8.52343
S.D. of dependent variable	0.741672
Sum of squared residuals	52.0903
Standard error of residuals ($\hat{\sigma}$)	0.704342
$F(5, 105)$	5.13561
p-value for $F()$	0.000296831

Hausman test –
Null hypothesis: OLS estimates are consistent
Asymptotic test statistic: $\chi^2_1 = 2.4261$
with p-value $= 0.119329$

First-stage $F(1, 105) = 21.5174$

11.5 Script

```
open c:\userdata\gretl\data\PoE\truffles.gdt
# Least Squares
ols q const ps di pf
ols p const ps di pf

#Two Stage Least Squares
open c:\userdata\gretl\data\PoE\truffles.gdt
tsls q const p ps di;const ps di pf
tsls q const p pf;const ps di pf

# Fulton Fish example
open C:\userdata\gretl\data\PoE\fultonfish.gdt

#Estimate the reduced form equations
ols lquan const stormy mon tue wed thu
ols lprice const stormy mon tue wed thu
omit mon tue wed thu --quiet

#TSLS estimates of demand
tsls lquan const lprice mon tue wed \
           thu;const stormy mon tue wed thu
```

Chapter 12

Analyzing Time Series Data and Cointegration

The main purpose this chapter is to use **gretl** to explore the time series properties of your data. One of the basic points we make in econometrics is that the properties of the estimators and their usefulness for point estimation and hypothesis testing depend on how the data behave. For instance, in a linear regression model where errors are correlated with regressors, least squares won't be consistent and consequently it should not be used for either estimation or subsequent testing.

In time series regressions the data need to be **stationary**. Basically this requires that the means, variances and covariances of the data series cannot depend on the time period in which they are observed. For instance, the mean and variance of the probability distribution that generated GDP in the third quarter of 1973 cannot be different from the one that generated the 4th quarter GDP of 2006. Observations on stationary time series can be correlated with one another, but the nature of that correlation can't change over time. U.S. GDP is growing over time (not mean stationary) and may have become less volatile (not variance stationary). Changes in information technology and institutions may have shortened the persistence of shocks in the economy (not covariance stationary). Nonstationary time series have to be used with care in regression analysis. Methods to effectively deal with this problem have provided a rich field of research for econometricians in recent years.

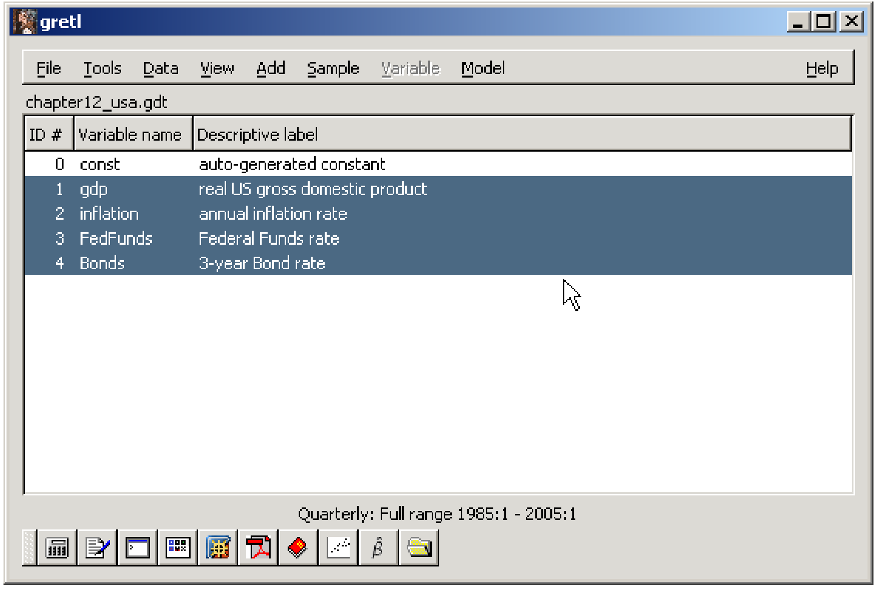
12.1 Series Plots

The first thing to do when working with time series is to take a look at the data graphically. A time series plot will reveal potential problems with your data and suggest ways to proceed statistically. In **gretl** time series plots are simple to generate since there is a built in function that performs this task. Open the data file `usa.gdt`.

```
open C:\userdata\gretl\data\poe\usa.gdt
```

Then, use your mouse to select all of the series as shown in Figure 12.1 below. Then, select `Add>First differences of selected variables` from the pull-down menu as shown in Figure 12.2. The first differences of your time series are added to the data set and each of the differenced series is prefixed

Figure 12.1: Select all of the series.

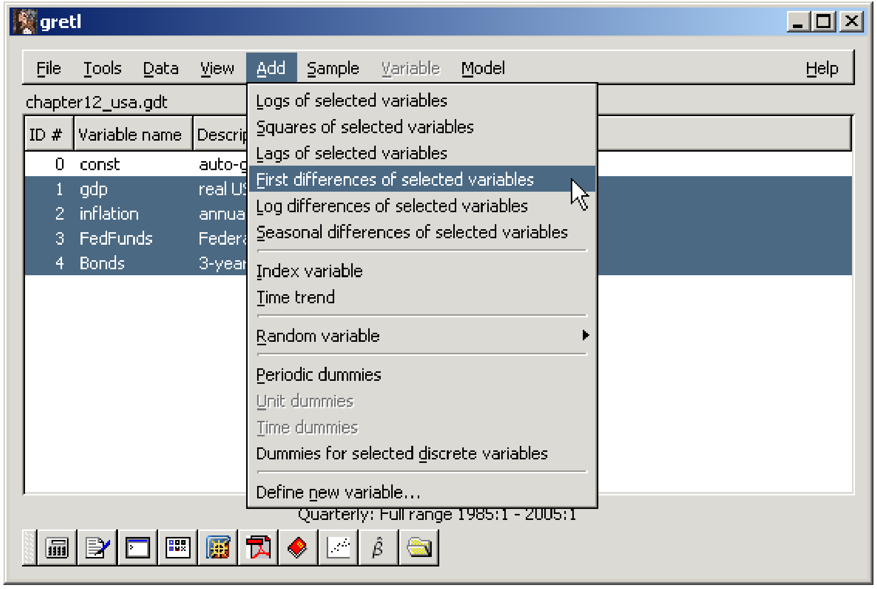


chapter12_usa.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	gdp	real US gross domestic product
2	inflation	annual inflation rate
3	FedFunds	Federal Funds rate
4	Bonds	3-year Bond rate

Quarterly: Full range 1985:1 - 2005:1

Figure 12.2: Add the first differences of the selected series from the pull-down menu.


gretl
File
Tools
Data
View
Add
Sample
Variable
Model
Help
Logs of selected variables
Squares of selected variables
Lags of selected variables
First differences of selected variables
Log differences of selected variables
Seasonal differences of selected variables
Index variable
Time trend
Random variable
Periodic dummies
Unit dummies
Time dummies
Dummies for selected discrete variables
Define new variable...

chapter12_usa.gdt

ID #	Variable name	Descrip
0	const	auto-g
1	gdp	real US
2	inflation	annua
3	FedFunds	Federa
4	Bonds	3-year

Quarterly: Full range 1985:1 - 2005:1

with 'd_', e.g., $gdp_t - gdp_{t-1} = d\_gdp$. Plotting the series can be done in any number of ways. The easiest is to use `view>multiple graphs>Time series` from the pull-down menu. This will allow you to graph the eight series in two batches. Two batches are required since the maximum number of series that can be graphed simultaneously is currently limited to six.

Select `gdp, inflation, d_gdp, and d_inflation` as shown in Figure 12.3. The result appears in Figure 12.4. Repeat this exercise for the remaining series to get the result shown in Figure 12.5.

12.2 Tests for Stationarity

The (augmented) Dickey-Fuller test can be used to test for the stationarity of your data. To perform this test, a few decisions have to be made regarding the time series. The decisions are usually made based on visual inspection of the time series plots. By looking at the plots you can determine whether the time series have a linear or quadratic trend. If the trend

Figure 12.3: Graphing multiple time series using the selection box.

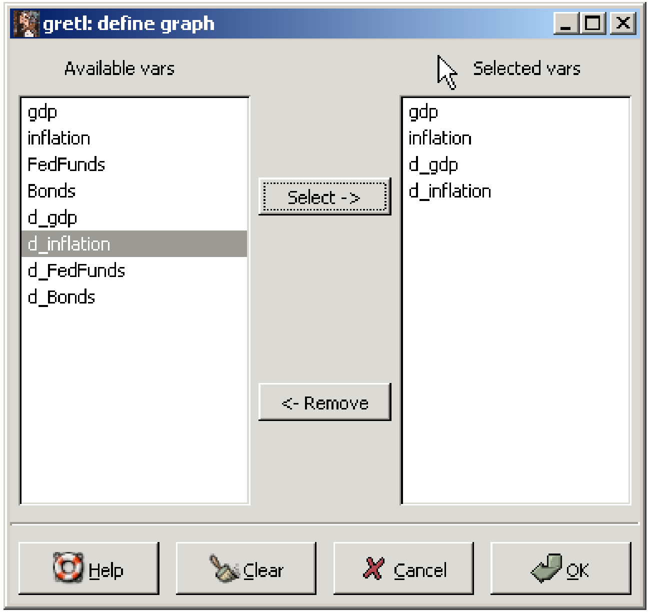


Figure 12.4: Multiple time series graphs.

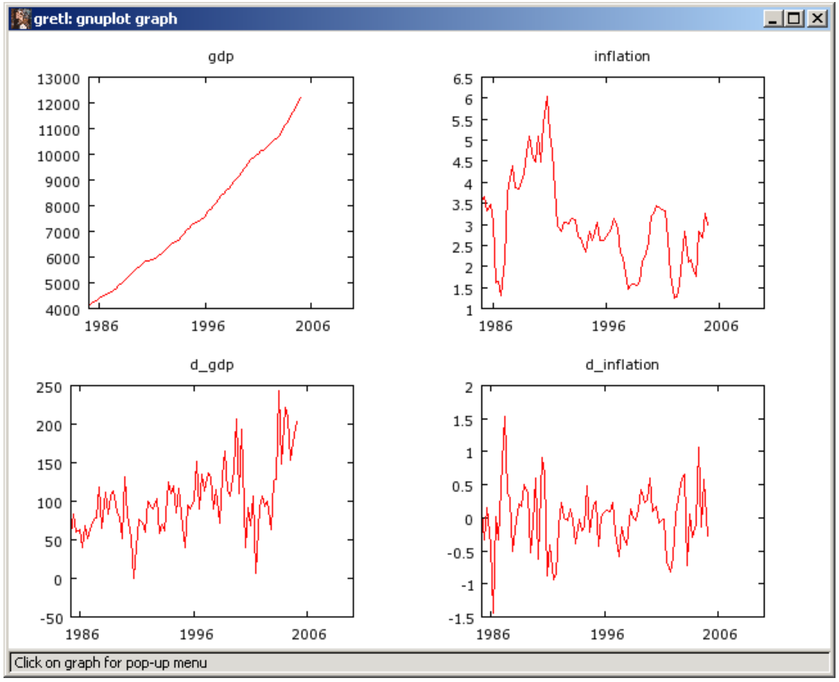
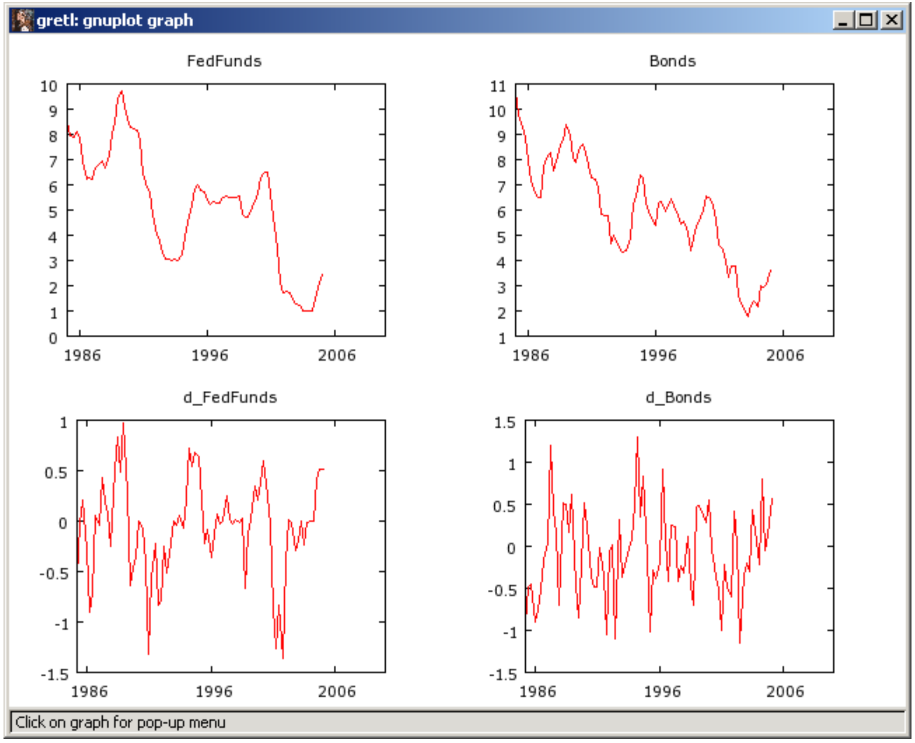


Figure 12.5: Multiple time series graphs for Fed Funds rate and 3 year bonds.



in the series is quadratic then the differenced version of the series will have a linear trend in them. In Figure 12.5 you can see that the Fed Funds rate appears to be trending downward and its difference appears to wander around some constant amount. Ditto for bonds. This suggests that the Augmented Dickey Fuller test regressions for each of the series should contain a constant, but not a time trend.

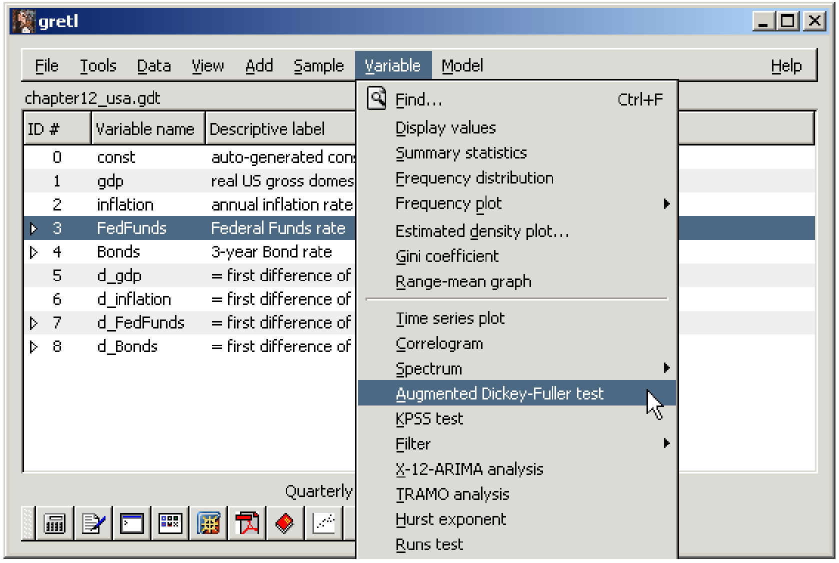
The GDP series in the upper left side of Figure 12.4 appears to be slightly quadratic in time. The differenced version of the series that appears below it has a slight upward drift and hence I would choose an ADF test that included a constant and a time trend. As you may have guessed, analyzing time series in this way is a bit like reading entrails and there is something of an art to it. Our goal is to reduce some of the uncertainty using formal tests whenever we can, but realize that choosing the appropriate test specification requires some judgement by the econometrician.

The next decision is to pick the number of lagged terms to include in the ADF regressions. Again, this is a judgement call, but the residuals from the ADF regression should be void of any autocorrelation. **Gretl** is helpful in this respect since it reports the outcome of an autocorrelation test whenever the built-in ADF routines are used. Below is the example from your text, where the stationarity of the Fed Funds rate and the three year bond series are explored.

To perform the ADF test on the Fed Funds rate, use the cursor to highlight the series and click `Variable>Augmented Dickey Fuller test` from the pull-down menu as shown in Figure 12.6 below. This brings up the dialog box shown in the next Figure, 12.7. Notice that here is where you inform **gretl** whether you want to include a constant, trend, trend squared, seasonal dummies, etc. We have chosen to use only 1 lag, and to include a constant in the ADF regression. Also, we've checked the box to have **gretl** report the results from the regression itself in order to make the results a bit more transparent.

At the bottom of the dialog you must choose whether you want to use the level or the difference of the variable. Choosing the level, as shown in the box, puts the difference on the left-hand side of the regression. This can be a bit confusing, but in reality it should not be. Remember, you are testing to see whether the levels values of the series are stationary. Choosing this box is telling **gretl** that you want to first test levels.

Figure 12.6: Choose the ADF test from the pull-down menu.



chapter12_usa.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated con
1	gdp	real US gross domes
2	inflation	annual inflation rate
3	FedFunds	Federal Funds rate
4	Bonds	3-year Bond rate
5	d_gdp	= first difference of
6	d_inflation	= first difference of
7	d_FedFunds	= first difference of
8	d_Bonds	= first difference of

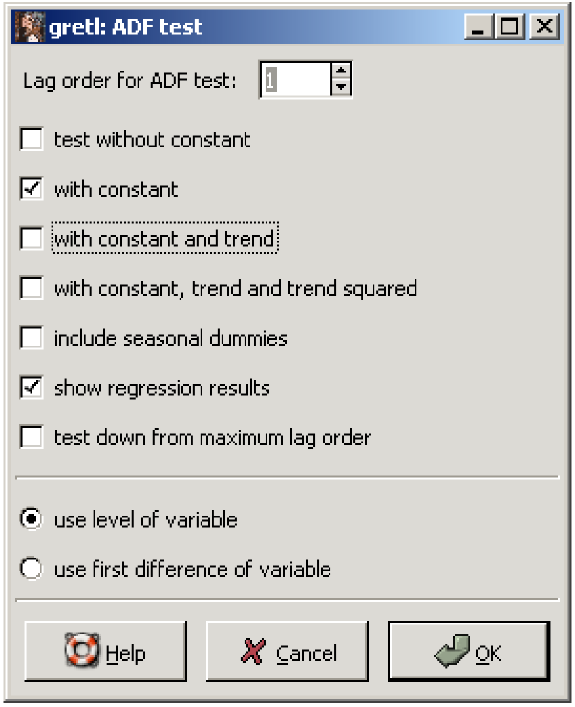
If you want to check to see whether the differences are nonstationary, then click the radio button below the one indicated. Click OK and the results appear as in Figure 12.8.

The test results are quite informative. First it tells you that you are performing a test based on a regression with a constant. It provides you with an estimate of γ, which it refers to as `a-1`, the t-ratio for γ, and the correct p-value for the statistic as computed by Davidson and MacKinnon. It also reports an estimated autocorrelation coefficient for the errors (0.061) which should be small if you have chosen the correct number of lags in the ADF regression.

The null hypothesis of the ADF test is that the time series has a unit root and is not stationary. *If you reject this hypothesis then you conclude that the series is stationary.* To not reject the null means that the level is *not* stationary. Here, the test statistic for the stationarity of the Fed Funds rate is -2.090 which has a p-value of 0.24875. Nonstationarity of the Fed Funds rate can not be rejected in this case at the usual 5 or 10% levels of significance.

One more thing should be said about the ADF test results. **Gretl** ex-

Figure 12.7: The ADF test dialog box.



gretl: ADF test

Lag order for ADF test: 1

☐ test without constant

☑ with constant

☐ with constant and trend

☐ with constant, trend and trend squared

☐ include seasonal dummies

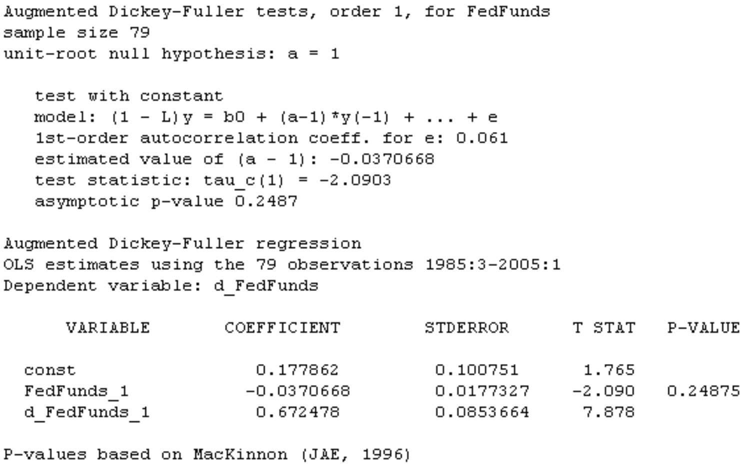
☑ show regression results

☐ test down from maximum lag order

● use level of variable

○ use first difference of variable

Figure 12.8: The ADF test results.



```
Augmented Dickey-Fuller tests, order 1, for FedFunds
sample size 79
unit-root null hypothesis: a = 1

   test with constant
   model: (1 - L)y = b0 + (a-1)*y(-1) + ... + e
   1st-order autocorrelation coeff. for e: 0.061
   estimated value of (a - 1): -0.0370668
   test statistic: tau_c(1) = -2.0903
   asymptotic p-value 0.2487

Augmented Dickey-Fuller regression
OLS estimates using the 79 observations 1985:3-2005:1
Dependent variable: d_FedFunds

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                0.177862          0.100751      1.765
  FedFunds_1          -0.0370668         0.0177327    -2.090    0.24875
  d_FedFunds_1         0.672478          0.0853664     7.878

P-values based on MacKinnon (JAE, 1996)
```

presses the model in a slightly different way than your textbook. The model is

$$(1 - L)y_t = \beta_0 + (\alpha - 1)y_{t-1} + \alpha_1 \Delta y_{t-1} + e_t \qquad (12.1)$$

The coefficient β_0 is included because you believe the series has a trend, $(\alpha - 1) = \gamma$ is the coefficient of interest in the Dickey-Fuller regression, and α_1 is the term that 'augments' the Dickey-Fuller regression. It is included to eliminate autocorrelation in the model's errors, e_t, and more lags can be included if needed to accomplish this. The notation on the left side of the equation $(1 - L)y_t$ makes use of the lag operator, L. The lag operator performs the magic $Ly_t = y_{t-1}$. Thus, $(1-L)y_t = y_t - Ly_t = y_t - y_{t-1} = \Delta y_t$!

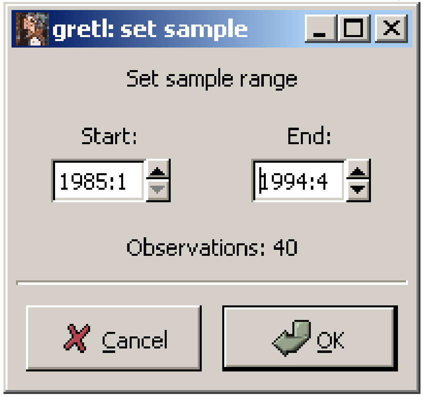
The next thing to do is to create a set of summary statistics. In this case, the textbook has you produce summary statistics for subsamples of the data. The first subsample consists of the 40 observations from 1985:1 to 1994:4. The second also contains 40 observations (a decade!) and continues from 1995:1 to 2004:4. The `summary` command is used to obtain the summary statistics on the desired subsample. In the script, open the data file *usa.gdt* and change the sample to 1985:1-1994:4 using the command `smpl 1985:1 1994:4`. Issue the `summary` command to print the summary statistics of all variables in memory to the screen. Finally, restore the sample to the full range using `smpl full`.

Gretl's `smpl` functions are cumulative. This means that whatever modifications you make to the sample are made based on the sample that is already in memory. So, to get summary statistics on the second subsample (which is not in memory) you have to restore the full sample first using `smpl full`. It is a little clumsy, but it makes sense once you know how it works.

```
open C:\userdata\gretl\data\poe\usa.gdt

smpl 1985:1 1994:4
summary
smpl full

smpl 1995:1 20054:4
summary
smpl full
```

The sample can be manipulated through the dialogs as well. Open the dataset and select `Sample>Set range` from the pull-down menu to reveal the dialog in Figure 12.9. Use the scroll buttons to change the ending date to 1994:4. The observation counter will change and show that the selected sample has 40 observations. Click OK and you are returned to the main **gretl** window. This is shown in the next Figure, 12.10. Now, select all of the variables either using **Crtl+A** from the keyboard or `Data>Select all` from the pull-down menu. Finally, `View>Summary statistics` will reveal the desired information, which is shown below:

Figure 12.9: Choose `Sample>Set range` to reveal the Set sample dialog box. Use the scroll buttons to set the desired sample range.



Set sample range

Start: 1985:1

End: 1994:4

Observations: 40

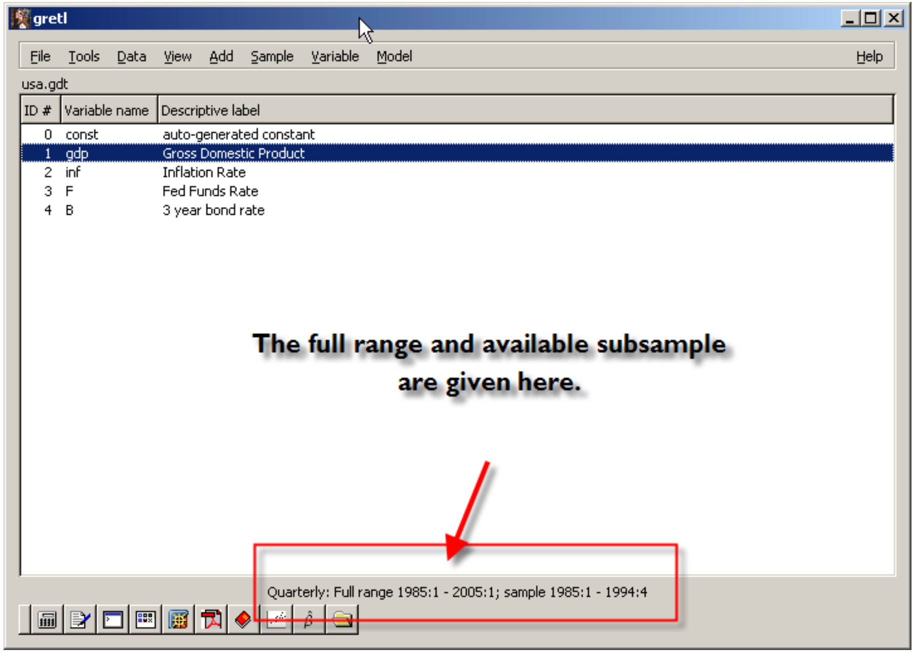
Summary Statistics, using the observations 1985:1–1994:4

Variable	Mean	Median	Minimum	Maximum
gdp	5587.70	5650.35	4119.50	7232.20
inf	3.55601	3.50904	1.30831	6.03757
F	6.28808	6.65667	2.99000	9.72667
B	7.22700	7.49000	4.32000	10.6767

Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
gdp	922.950	0.165175	0.0439792	−1.1688
inf	1.09067	0.306713	0.0926796	−0.455736
F	2.08741	0.331963	−0.301024	−1.1781
B	1.62734	0.225175	−0.224819	−0.713900

Now restore the full sample using `Sample>Restore full range` from the

Figure 12.10: Any changes to the sample should be visible in the main window. Here you can see that the data in memory consist of the 1985:1-1994:4 subsample.



usa.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	gdp	Gross Domestic Product
2	inf	Inflation Rate
3	F	Fed Funds Rate
4	B	3 year bond rate

The full range and available subsample are given here.

Quarterly: Full range 1985:1 - 2005:1; sample 1985:1 - 1994:4

pull-down menu and repeat, changing the sample range to 1995:1 - 2004:4 using the set sample dialog. The results are

Summary Statistics, using the observations 1985:1–2005:1

Variable	Mean	Median	Minimum	Maximum
gdp	7584.25	7298.30	4119.50	12198.8
inf	2.99005	2.90131	1.24379	6.03757
F	5.16955	5.50667	0.996667	9.72667
B	5.94741	6.00000	1.77333	10.6767

Variable	Std. Dev.	C.V.	Skewness	Ex. kurtosis
gdp	2312.62	0.304924	0.266928	−1.1168
inf	1.05382	0.352444	0.571135	0.0136503
F	2.29634	0.444206	−0.199304	−0.760287
B	2.03711	0.342521	−0.109496	−0.553230

12.3 Spurious Regressions

It is possible to estimate a regression and find a statistically significant relationship even if none exists. In time series analysis this is actually a common occurrence when data are not stationary. This example uses two data series, *rw1* and *rw2*, that were generated as independent random walks.

$$rw_1 : \qquad y_t = y_{t-1} + v_{1t} \tag{12.2}$$

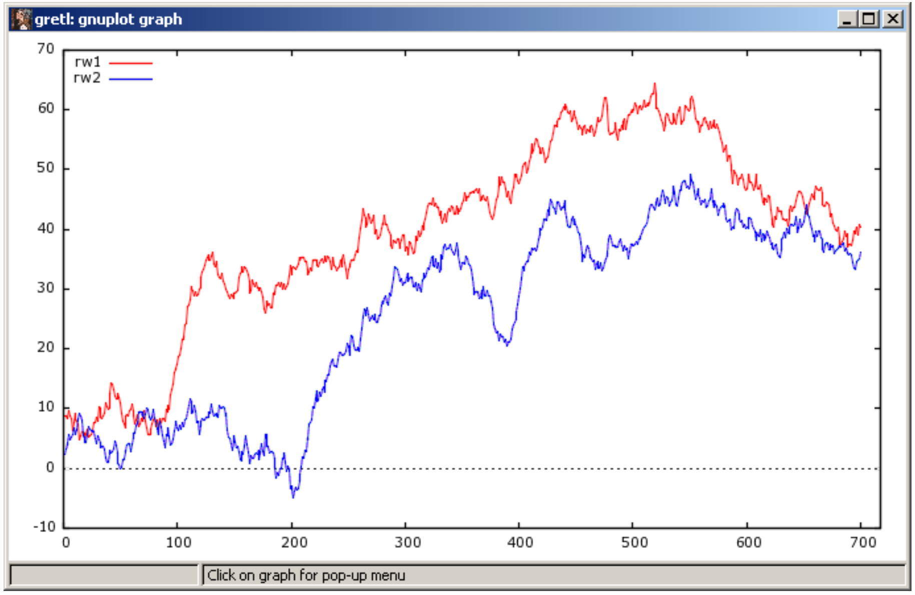
$$rw_2 : \qquad x_t = x_{t-1} + v_{2t} \tag{12.3}$$

The errors are independent standard normal random deviates generated using a pseudo-random number generator. As you can see, x_t and y_t are not related in any way. To explore the empirical relationship between these unrelated series, load the *spurious.dta* data, create a time variable, and declare the data to be time series.

```
open C:\userdata\gretl\data\poe\spurious.gdt
```

The sample information at the bottom of the main **gretl** window indicates that the data have already been declared as time series and that the full range (1-700) is in memory. The first thing to do is to plot the data using a time series plot. To place both series in the same time series graph, select `View>Graph specified vars.>Time series plots` from the pull-down menu. This will reveal the 'define graph' dialog box. Place both series into the 'Selected vars' box and click OK. The result appears in Figure 12.11 below. A scatter plot is revealing as well. Select `View>Graph specified vars.>X-Y scatters` and place rw2 on the X-axis, rw1 on the Y-axis to produce the next graph (Figure 12.12. The linear regression confirms this. Left click on the graph to reveal the pop-up menu shown in Figure 12.13. Select the OLS estimates option to reveal the regression results in Table 12.1.

Figure 12.11: These random walk series appear to be correlated but they are not. It is not uncommon to observe spurious relationships between nonstationary series.



The coefficient on *rw2* is positive (.842) and significant ($t = 40.84 > 1.96$). However, these variables are not related! The observed relationship is purely spurious. The cause of the spurious result is the nonstationarity of the two series. This is why you must check your data for stationarity whenever you use time series in a regression.

Figure 12.12: The scatter plot of the random walk series makes them appear to be related, but they are not. They are nonstationary and the relationship is spurious.

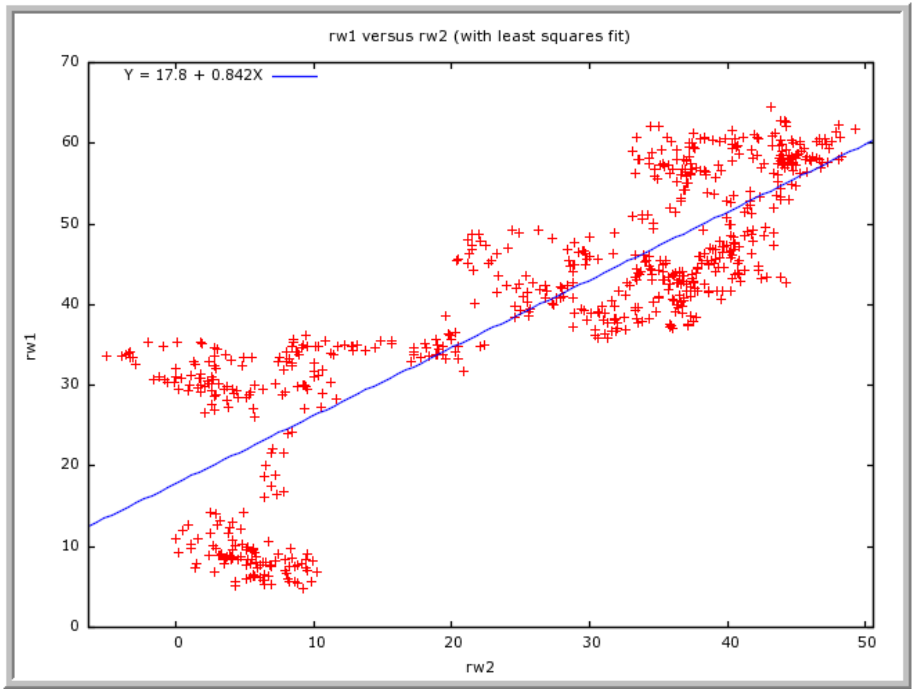


Figure 12.13: Left-click on the graph to reveal this menu. Choose `OLS estimates` to reveal the underlying least squares results that produce the regression line in the graph.

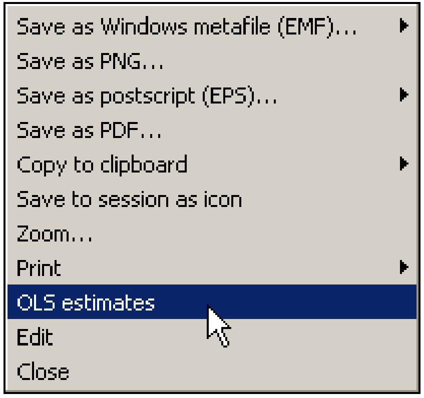


Table 12.1: OLS estimates of a spurious relationship using the 700 observations of the *spurious.gdt* dataset.

Dependent variable: rw1

Variable	Coefficient	Std. Error	t-statistic	p-value
const	17.8180	0.620478	28.7167	0.0000
rw2	0.842041	0.0206196	40.8368	0.0000

Unadjusted R^2	0.704943
Adjusted $\bar{R}^2$	0.704521

The script to produce these graphs is very simple. Use

```
open C:\userdata\gretl\data\poe\spurious.gdt

gnuplot rw1 rw2 --with-lines --time-series
gnuplot rw1 rw2
ols rw1 rw2 const
```

The first plot applies lines and uses the time-series option to use time as the X-axis measurement. The second plot is a simple scatter with the first variable on the Y-axis and the second on the X-. The final statement estimates the regression.

12.4 Cointegration

Two nonstationary series are cointegrated if they tend to move together through time. For instance, we have established that the levels of the Fed Funds rate and the 3-year bond are nonstationary, whereas their differences are stationary. In the opaque terminology used in time series literature, each series is said to be "integrated of order 1" or I(1). If the two nonstationary series move together through time then we say they are "cointegrated." Economic theory would suggest that they should be tied together via arbitrage, but that is no guarantee. In this context, testing for cointegration amounts to a test of the substitutability of these assets.

The basic test is very simple. Regress one I(1) variable on another using least squares. If the series are cointegrated, the residuals from this regression will be stationary. This is verified using augmented Dickey-Fuller test.

The null hypothesis is that the residuals are nonstationary, which implies that the series are not cointegrated. Rejection of this leads to the conclusion that the series are cointegrated. The `coint` function in **gretl** carries out each of the three steps in this test. First, it carries out a Dickey-Fuller test of the null hypothesis that each of the variables listed has a unit root. Then it estimates the cointegrating regression using least squares. Finally, it runs a Dickey Fuller test on the residuals from the cointegrating regression. This procedure, referred to as the Engle-Granger cointegration test and discussed in chapter 12 of Hill et al. (2008), is the one done in **gretl** by default. **Gretl** can also perform cointegration tests based on maximum likelihood estimation of the cointegrating relationships proposed by Johansen and summarized in (Hamilton, 1994, Chapter 20). The Johansen tests use the `coint2` command, which is explained in **gretl**'s documentation.

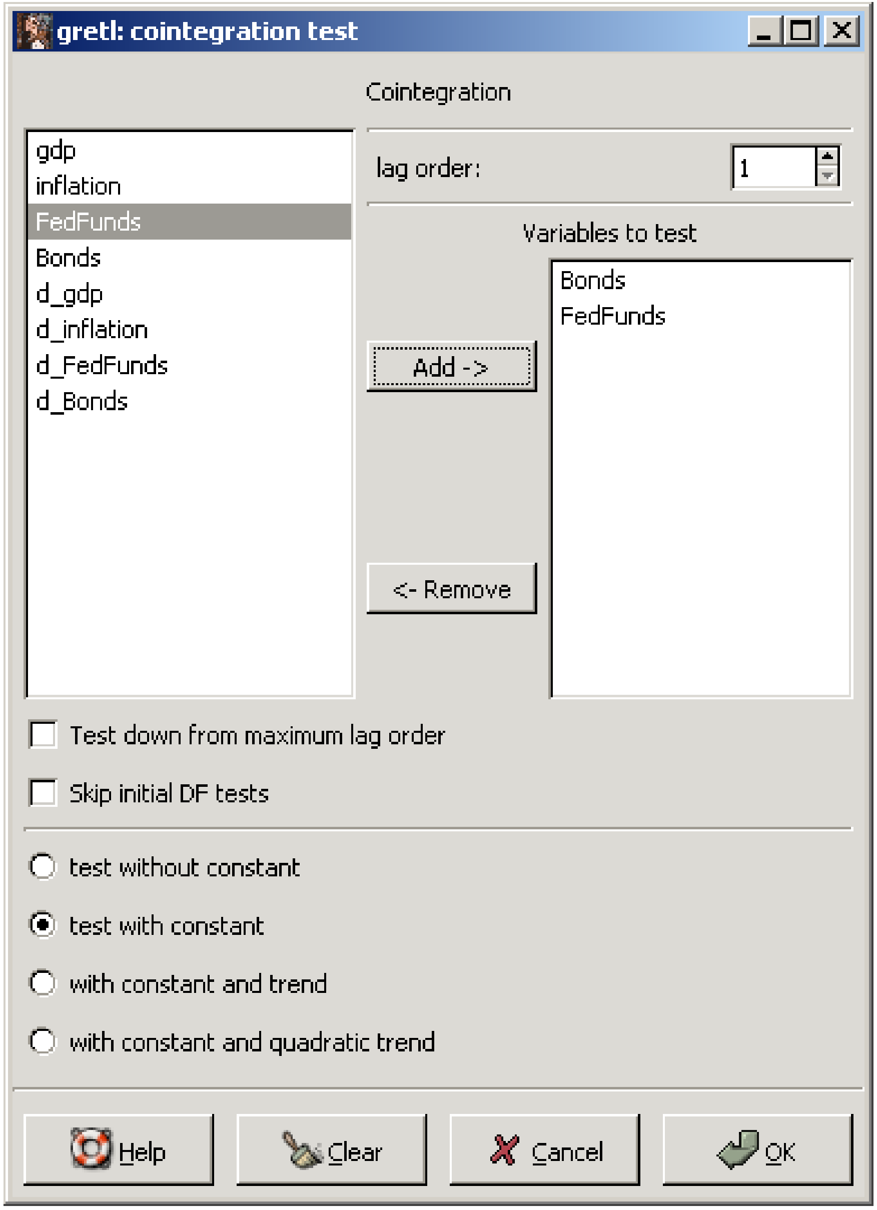
Figure 12.14 shows the dialog box used to test cointegration in this way. To obtain it use `Model>Time series>Cointegration test>Engle-Granger` from the main **gretl** window. In the dialog box you have to indicate how many lags you want in the initial Dickey-Fuller regressions on the the variables, which variables you want to include in the cointegrating relationship, and whether you want a constant, trend, or quadratic trend in the regressions.

12.5 The Analysis Using a Gretl Script

Below, you will find a summary of the **gretl** commands used to produce the results for the *usa.gdt* data from Chapter 12.

```
open C:\userdata\gretl\data\poe\usa.gdt

# Difference each variable
```

Figure 12.14: The dialog box for the cointegration test.



Cointegration

gdp
inflation
FedFunds
Bonds
d_gdp
d_inflation
d_FedFunds
d_Bonds

lag order: 1

Variables to test

Bonds
FedFunds

□ Test down from maximum lag order

□ Skip initial DF tests

○ test without constant

● test with constant

○ with constant and trend

○ with constant and quadratic trend

```
diff gdp inflation FedFunds Bonds

# Augmented Dickey Fuller regressions
# This is the manaul way of doing this regression
ols d_FedFunds const FedFunds(-1) d_FedFunds(-1)
ols d_Bonds const Bonds(-1) d_Bonds(-1)

# Augmented Dickey Fuller regressions using the built-in function
# Note: 1 lag is called for and a constant is included (--c)
adf 1 FedFunds --c --verbose
adf 1 Bonds --c --verbose

# Dickey-Fuller regressions for first differences
# Note: adf 0 indicates no lags for the difference version
adf 0 FedFunds --nc --verbose --difference
adf 0 Bonds --nc --verbose --difference

# Engle-Granger test of cointegration
# Note: one lag is used in the adf portion of the test
coint 1 Bonds FedFunds
```

The `diff` function takes the first difference of each series. The `adf` function conducts the augmented Dickey-Fuller test. The number `1` that follows the `adf` command is the number of lags to use in the augmented version, in this case only one. Then, list the series name and any options you wish to invoke. Here, the `--c` option is used, indicating that we want a constant term included in the Dickey-Fuller regression. The `--verbose` statement is included so that **gretl** will print the results from the Dickey-Fuller regression itself. I think this makes interpreting the result much easier, so I always include it.

Other options in the example include `-nc` which directs the Dickey-Fuller regression to omit the constant altogether. The `--difference` option tells **gretl** to run the augmented Dickey-Fuller regressions under the assumption that the first difference of the series is nonstationary.

Finally, the `coint` command conducts the Engle-Granger test for the cointegration of the two series that follow. Again, the number `1` that follows `coint` is actually for the first step of the procedure, which tells **gretl** how many lags to include in the initial augmented Dickey-Fuller regressions.

The output generated from the simple command `coint 1 Bonds FedFunds` is shown below.

```
# Engle-Granger test of cointegration
? coint 1 Bonds FedFunds
Step 1: testing for a unit root in Bonds

Augmented Dickey-Fuller test, order 1, for Bonds
sample size 79
unit-root null hypothesis: a = 1

   test with constant
   estimated value of (a - 1): -0.0562195
   test statistic: tau_c(1) = -1.97643
   asymptotic p-value 0.2975

Step 2: testing for a unit root in FedFunds

Augmented Dickey-Fuller test, order 1, for FedFunds
sample size 79
unit-root null hypothesis: a = 1

   test with constant
   estimated value of (a - 1): -0.0370668
   test statistic: tau_c(1) = -2.0903
   asymptotic p-value 0.2487

Step 3: cointegrating regression

Cointegrating regression -
OLS estimates using the 81 observations 1985:1-2005:1
Dependent variable: Bonds

   VARIABLE  COEFFICIENT   STDERROR   T STAT   P-VALUE

  const        1.64373     0.19482     8.437  <0.00001 ***
  FedFunds     0.832505    0.03448    24.147  <0.00001 ***
  Unadjusted R-squared = 0.880682
  Adjusted R-squared = 0.879172
  Durbin-Watson statistic = 0.413856
  First-order autocorrelation coeff. = 0.743828
  Akaike information criterion (AIC) = 175.927
  Schwarz Bayesian criterion (BIC) = 180.716
  Hannan-Quinn criterion (HQC) = 177.848

Step 4: Dickey-Fuller test on residuals

lag order 1
sample size 79
unit-root null hypothesis: a = 1

   estimated value of (a - 1): -0.31432
   test statistic: tau_c(2) = -4.54282
   asymptotic p-value 0.0009968

P-values based on MacKinnon (JAE, 1996)

There is evidence for a cointegrating relationship if:
(a) The unit-root hypothesis is not rejected for the individual
    variables.
(b) The unit-root hypothesis is rejected for the residuals
    (uhat) from the cointegrating regression.
```

Notice that at the bottom of the output **gretl** gives you some useful advice on interpreting the outcome of the test. Cointegration requires both series to be I(1)–not rejecting nonstationarity in the initial Dickey-Fuller regressions and then rejecting nonstationarity in the Dickey-Fuller regression using the residuals. Nice!

12.6 Script

```
open C:\userdata\gretl\data\poe\usa.gdt
```

```
# Difference each variable
diff gdp inflation FedFunds Bonds

# Augmented Dickey Fuller regressions
ols d_FedFunds const FedFunds(-1) d_FedFunds(-1)
ols d_Bonds const Bonds(-1) d_Bonds(-1)

# Augmented Dickey Fuller regressions using built in functions
adf 1 FedFunds --c --verbose
adf 1 Bonds --c --verbose

# Dickey-Fuller regressions for first differences
adf 0 FedFunds --nc --verbose --difference
adf 0 Bonds --nc --verbose --difference

# Summary Statistics
smpl 1985:1 1994:4
summary
smpl full
smpl 1995:1 2004:4
summary
smpl full

#Spurious Regressions
open C:\userdata\gretl\data\poe\spurious.gdt

gnuplot rw1 rw2 --with-lines --time-series
gnuplot rw1 rw2
ols rw1 rw2 const

# Engle-Granger test of cointegration
open C:\userdata\gretl\data\poe\usa.gdt
coint 1 Bonds FedFunds
```

Chapter 13

Vector Error Correction and Vector Autoregressive Models: Introduction to Macroeconometrics

The vector autoregression model is a general framework used to describe the dynamic interrelationship between stationary variables. So, the first step in your analysis should be to determine whether the levels of your data are stationary. If not, take the first differences of your data and try again. Usually, if the levels (or log-levels) of your time series are not stationary, the first differences will be.

13.1 Vector Error Correction

If the time series are not stationary then we need to modify the vector autoregressive (VAR) framework to allow consistent estimation of the relationships between the series. The vector error correction model (VECM) is just a special case of the VAR for variables that are stationary in their differences (i.e., I(1)) and cointegrated.

In the first example, we use quarterly data on the Gross Domestic Product of Australia and the U.S. to estimate a VEC model. We decide to use the vector error correction model because (1) the time series are not stationary in their levels but are in their differences (2) the variables are cointegrated.

In an effort to keep the discussion moving, the authors of *POE* opted to avoid discussing how they actually determined the series were nonstationary in levels, but stationary in differences. This is an important step and I will take some time here to explain how one could approach this. There are several ways to do this and I'll show you two ways to do it in **gretl**.

13.1.1 Series Plots–constant and trends

Our initial impressions of the data are gained from looking at plots of the two series. The data plots are obtained in the usual way after importing the dataset. The data on U.S. and Australian GDP are found in the *gdp.gdt* file and were collected from 1970:1 - 2004:4.[1] Open the data and set the data structure to quarterly time-series using the `setobs 4` command, start the series at 1970:1, and use the `--time-series` option.

```
open C:\userdata\gretl\data\poe\gdp.gdt
setobs 4 1970:1 --time-series
```

One purpose of the plots is to help you determine whether the Dickey-Fuller regressions should contain constants, trends or squared trends. The simplest way to do this is from the console using the `scatters` command.

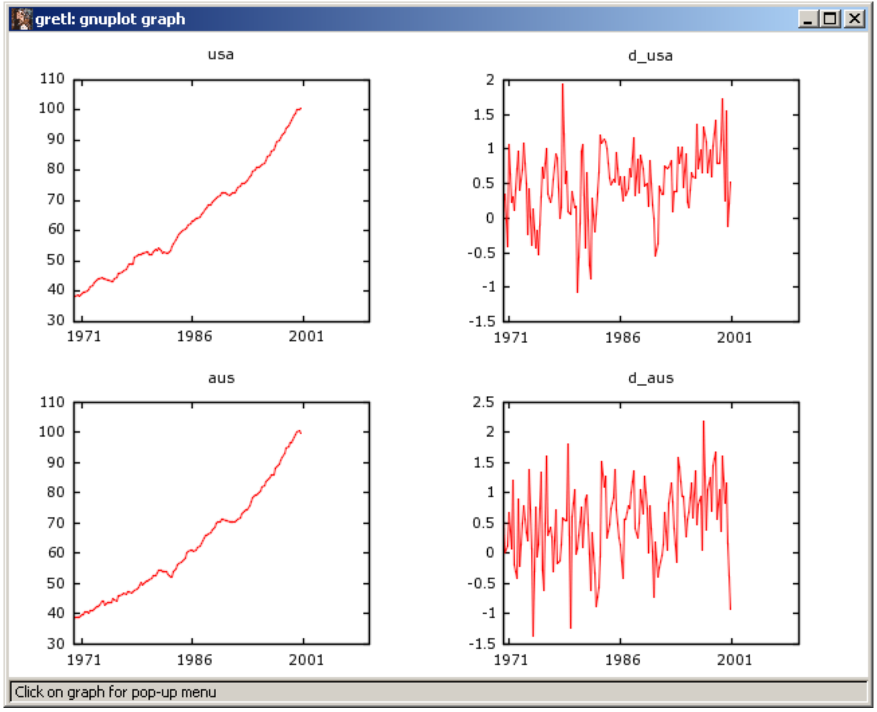
```
scatters usa diff(usa) aus diff(aus)
```

The `scatters` command produces multiple graphs, each containing one of the listed series. The `diff()` function is used to take the differences of `usa` and `aus`, which appear in the graphs featured in Figure 13.1 below.

This takes two steps from the pull-down menu. First, use the mouse to highlight the two series and then create the differences using `Add>First`

[1] *POE* refers to these variables as U and A, respectively.

Figure 13.1: The levels of Australian and U.S. GDP appear to be nonstationary and cointegrated. The difference plots have a nonzero mean, indicating a constant in their ADF regressions.



`differences of selected variables`. Then, select `View>Multiple graphs>Time series`. Add the variables to the selected list box to produce Figure 13.1.

From the time series plots it appears that the levels are mildly parabolic in time. The differences have a small trend. This means that the augmented Dickey-Fuller (ADF) regressions need to contain these elements.

13.1.2 Selecting Lag Length

The second consideration is the specification of lags for the ADF regressions. There are several ways to select lags and **gretl** automates one of these. The basic concept is to include enough lags in the ADF regressions to make the residuals white noise. These will be discussed presently.

Testing Down

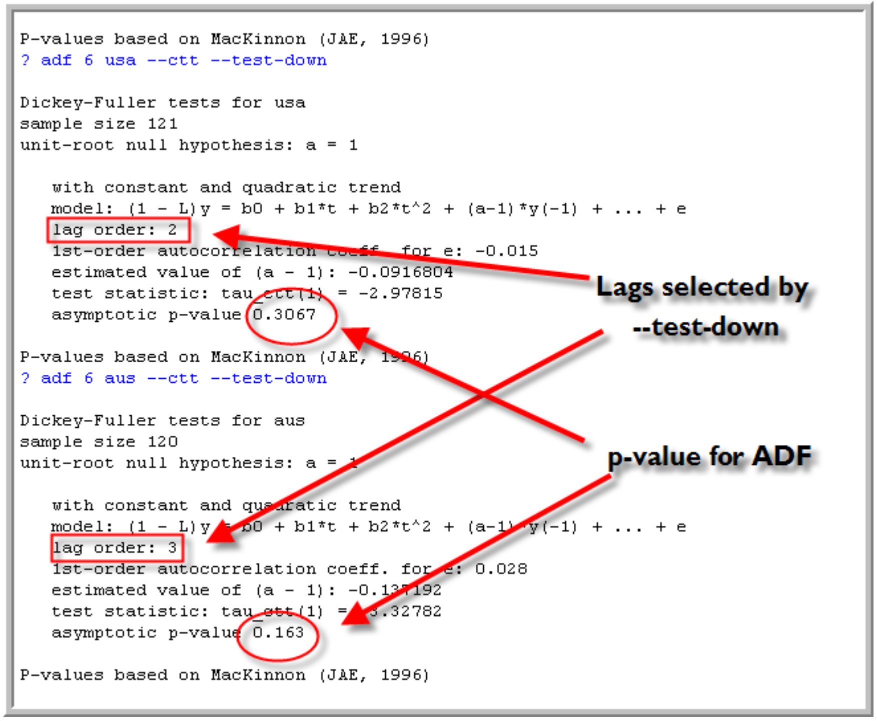
The first strategy is to include just enough lags so that the last one is statistically significant. **Gretl** automates this using the `--test-down` option for the augmented Dickey-Fuller regressions. Start the ADF regressions with a generous number of lags and **gretl** automatically reduces that number until the t-ratio on the longest remaining lag is significant at the 10 percent level. For the levels series we start with a maximum lag of 6, include a constant, trend, and trend squared (`--ctt option`), and use the `--test-down` option.

```
adf 6 usa --ctt --test-down
adf 6 aus --ctt --test-down
```

The result is shown in Figure 13.2. The `--test-down` option selected two lags for the `usa` series and three for `aus`. Both ADF statistics are insignificant at the 5% or 10% level, indicating they are nonstationary. This is repeated for the differenced series using the commands:

```
adf 6 diff(usa) --ct --test-down
adf 6 diff(aus) --ct --test-down
```

Figure 13.2: Based on ADF tests, the levels of Australian and U.S. GDP are nonstationary.



```
P-values based on MacKinnon (JAE, 1996)
? adf 6 usa --ctt --test-down

Dickey-Fuller tests for usa
sample size 121
unit-root null hypothesis: a = 1

   with constant and quadratic trend
   model: (1 - L)y = b0 + b1*t + b2*t^2 + (a-1)*y(-1) + ... + e
   lag order: 2
   1st-order autocorrelation coeff. for e: -0.015
   estimated value of (a - 1): -0.0916804
   test statistic: tau_ctt(1) = -2.97815
   asymptotic p-value 0.3067

P-values based on MacKinnon (JAE, 1996)
? adf 6 aus --ctt --test-down

Dickey-Fuller tests for aus
sample size 120
unit-root null hypothesis: a = 1

   with constant and quadratic trend
   model: (1 - L)y = b0 + b1*t + b2*t^2 + (a-1)*y(-1) + ... + e
   lag order: 3
   1st-order autocorrelation coeff. for e: 0.028
   estimated value of (a - 1): -0.137192
   test statistic: tau_ctt(1) = -3.32782
   asymptotic p-value 0.163

P-values based on MacKinnon (JAE, 1996)
```

The selected lags for the U.S. and Australia are one and three, respectively. Both ADF statistics are significant at the 5% level and we conclude that the differences are stationary.

Testing Up

The other strategy is to test the residuals from the augmented Dickey-Fuller regressions for autocorrelation. In this strategy you can start with a small model, and test the residuals of the Dickey-Fuller regression for autocorrelation using an LM test. If the residuals are autocorrelated, add another lagged difference of the series to the ADF regression and test the residuals again. Once the LM statistic is insignificant, you quit you are done. This is referred to as **testing-up**.

To employ this strategy in **gretl**, you'll have to estimate the ADF equations manually using the `ols` command. Since the data series has a constant and quadratic trend, you have to define a time trend (`genr time`) and trend squared (`genr t2 = time*time`) to include in the regressions. You will also need to generate the differences to use in a new function called `lags`. The script to do this follows:

```
genr time
genr t2 = time*time
genr d_usa = diff(usa)
```

Now, estimate a series of augmented Dickey-Fuller regressions using ols. Follow each regression with the LM test for autocorrelation of the residuals discussed in Chapter 9.

```
ols diff(usa) usa(-1) lags(1,d_usa) const time t2 --quiet
lmtest 1 --autocorr
ols diff(usa) usa(-1) lags(2,d_usa) const time t2 --quiet
lmtest 1 --autocorr
```

The first `ols` regression is the ADF(1). It includes 1 lagged value of the `d_usa` as a regressor in addition to the lagged value of `usa`, a constant, a trend, and a squared trend. **Gretl**'s `lags(q,variable)` function creates a series of lags from `1` through `q` of `variable`. So in the first regression, `lags(1,d_usa)` creates a single lagged value of `d_usa`. After the regression, use the `lmtest 1 --autocorr` to conduct the LM test of first order autocorrelation discussed in Chapter 9. If the p-value is greater than .10 then this is your model. If not, add another lag of `d_usa` using `lags(2,d_usa)` and repeat the test. In this example, the ADF(2) produces residuals that are not autocorrelated and 'wins' the derby.

In this code example we chose to suppress the results from the first regression so that the output from the tests would fit on one page (Figure 13.3). In practice, you could skip this option and read the Dickey-Fuller t-ratio directly from the output. The only disadvantage of this is that the proper p-value for it is not computed using the manual approach.

If you repeat this exercise for `aus` (as we have done in the script at the end of the chapter) you will find that testing up determines ***zero*** lags of `d_aus` are required in the Dickey-Fuller regression; testing down revealed ***three*** lags were needed. The incongruency occurs because we did a poor job of testing up, failing to include enough autocorrelation terms in in the LM test. This illustrates a danger of testing up. When we conducted the LM test using only a single autocorrelation term, we had not searched far enough in the past to detect significant autocorrelations that lie further back in time. Adding terms to the autocorrelation test using `lmtest 3 --autocorr` resolves this.

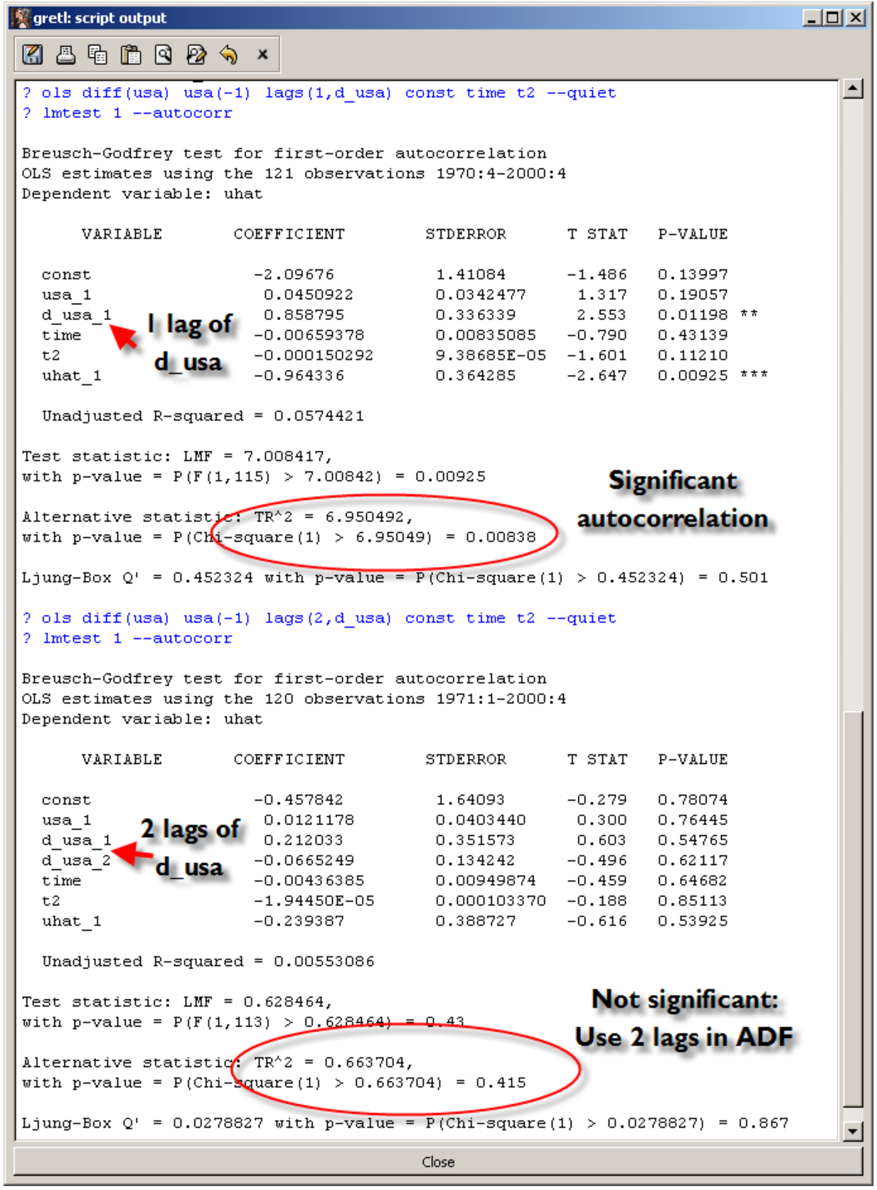
So which is better, testing down or testing up? I think the econometric consensus is that testing down is safer. We'll leave it for future study!

13.1.3 Cointegration Test

Given that the two series are stationary in their differences (i.e., both are I(1)), the next step is to test whether they are cointegrated. In the discussion that follows, we return to reproducing results from *POE*. To do this, use least squares to estimate the following regression.

$$aus_t = \beta usa_t + e_t \tag{13.1}$$

Figure 13.3: Manually estimate the ADF regressions and use LM tests for autocorrelation to determine the proper lag length.


gretl: script output
1 lag of d_usa
Significant autocorrelation
2 lags of d_usa
Not significant: Use 2 lags in ADF
Close

```
? ols diff(usa) usa(-1) lags(1,d_usa) const time t2 --quiet
? lmtest 1 --autocorr

Breusch-Godfrey test for first-order autocorrelation
OLS estimates using the 121 observations 1970:4-2000:4
Dependent variable: uhat

      VARIABLE       COEFFICIENT        STDERROR      T STAT    P-VALUE

  const                -2.09676          1.41084      -1.486    0.13997
  usa_1                 0.0450922        0.0342477     1.317    0.19057
  d_usa_1               0.858795         0.336339      2.553    0.01198 **
  time                 -0.00659378       0.00835085   -0.790    0.43139
  t2                   -0.000150292      9.38685E-05  -1.601    0.11210
  uhat_1               -0.964336         0.364285     -2.647    0.00925 ***

  Unadjusted R-squared = 0.0574421

Test statistic: LMF = 7.008417,
with p-value = P(F(1,115) > 7.00842) = 0.00925

Alternative statistic: TR^2 = 6.950492,
with p-value = P(Chi-square(1) > 6.95049) = 0.00838

Ljung-Box Q' = 0.452324 with p-value = P(Chi-square(1) > 0.452324) = 0.501

? ols diff(usa) usa(-1) lags(2,d_usa) const time t2 --quiet
? lmtest 1 --autocorr

Breusch-Godfrey test for first-order autocorrelation
OLS estimates using the 120 observations 1971:1-2000:4
Dependent variable: uhat

      VARIABLE       COEFFICIENT        STDERROR      T STAT    P-VALUE

  const                -0.457842         1.64093      -0.279    0.78074
  usa_1                 0.0121178        0.0403440     0.300    0.76445
  d_usa_1               0.212033         0.351573      0.603    0.54765
  d_usa_2              -0.0665249        0.134242     -0.496    0.62117
  time                 -0.00436385       0.00949874   -0.459    0.64682
  t2                   -1.94450E-05      0.000103370  -0.188    0.85113
  uhat_1               -0.239387         0.388727     -0.616    0.53925

  Unadjusted R-squared = 0.00553086

Test statistic: LMF = 0.628464,
with p-value = P(F(1,113) > 0.628464) = 0.43

Alternative statistic: TR^2 = 0.663704,
with p-value = P(Chi-square(1) > 0.663704) = 0.415

Ljung-Box Q' = 0.0278827 with p-value = P(Chi-square(1) > 0.0278827) = 0.867
```

obtain the residuals, $\hat{e}_t$, and then estimate

$$\Delta\hat{e}_t = \gamma\hat{e}_{t-1} + u_t \tag{13.2}$$

This is the "case 1 test" from Chapter 12 of Hill et al. (2008) and the 5% critical value for the t-ratio is -2.76. The following script estimates the model cointegrating regression, saves the residuals, and estimates the regression required for the unit root test.

```
ols aus usa
genr uhat = $uhat
ols diff(uhat) uhat(-1)
```

The result is:

$$\widehat{\Delta e_t} = \underset{(0.044279)}{-0.127937}\hat{e}_{t-1} \tag{13.3}$$

$$T = 123 \quad \bar{R}^2 = 0.0640 \quad F(1,122) = 8.3482 \quad \hat{\sigma} = 0.5985$$
$$\text{(standard errors in parentheses)}$$

The t-ratio is $-0.1279/.0443 = -2.889$ which lies in the rejection region for this test. Therefore, you reject the null hypothesis of no cointegration.

13.1.4 VECM

You have two difference stationary series that are cointegrated. Consequently, an error correction model of the short-run dynamics can be estimated using least squares. The error correction model is:

$$\begin{aligned} \Delta\text{aus}_t &= \beta_{11} + \beta_{12}\hat{e}_{t-1} + v_{1t} & (13.4) \\ \Delta\text{usa}_t &= \beta_{21} + \beta_{22}\hat{e}_{t-1} + v_{2t} & (13.5) \end{aligned}$$

and the estimates

$$\begin{aligned} \Delta\widehat{aus}_t &= \underset{(8.491)}{0.491706} + \underset{(-2.077)}{-0.0987029}\hat{e}_{t-1} \\ \Delta\widehat{usa}_t &= \underset{(10.924)}{0.509884} + \underset{(0.790)}{+0.0302501}\hat{e}_{t-1} \end{aligned}$$

$$\text{(t-statistics in parentheses)}$$

which is produced using

```
ols diff(aus) const uhat(-1)
ols diff(usa) const uhat(-1)
```

13.2 Vector Autoregression

The vector autoregression model (VAR) is actually a little simpler to estimate than the VEC model. It is used when there is no cointegration among the variables and it is estimated using time series that have been transformed to their stationary values.

In the example from *POE*, we have macroeconomic data on GDP and the CPI for the United States. The data are found in the *growth.gdt* dataset and have already been transformed into their natural logarithms. In the dataset, $\ln(GDP)$ is referred to as G and $\ln(CPI)$ as P. As in the previous example, the step is to determine whether the variables are stationary. If they are not, then you transform them into stationary time series and test for cointegration.

The data need to be analyzed in the same way as the GDP example. Examine the plots to determine possible trends and use the ADF tests to determine which form of the data are stationary. These data are nonstationary in levels, but stationary in differences. Then, estimate the cointegrating vector and test the stationarity of its residuals. If stationary, the series are cointegrated and you estimate a VECM. If not, then a VAR treatment is sufficient.

Open the data and take a look at the time series plots.

```
open C:\userdata\gretl\data\poe\growth.gdt
scatters G diff(G) P diff(P)
```

Next, estimate

$$ln(GDP)_t = \beta_1 + \beta_2 ln(CPI) + u_t \tag{13.6}$$

using least squares and obtain the residuals, $\hat{u}_t$. Then, difference the least squares residuals and estimate

$$\Delta\hat{u}_t = \alpha_1 + \alpha_2\hat{u}_{t-1} + redidual, \tag{13.7}$$

again, using least squares. The t-ratio on α_2 is computed and compared to the 5% critical value tabled in *POE* (Table 12.3), which is -3.37. The computed value of the statistic is -.97, which is not in the rejection region of the test; we conclude that the residuals are stationary which means that G and P are not cointegrated. The script that accomplishes this is

```
ols G const P
series uhat = $uhat
ols diff(uhat) uhat(-1)
```

You could use the built-in command for the augmented Dickey-Fuller regressions `adf` to obtain the t-ratio on the lagged residual. Unfortunately, the critical values produced by the automatic routine does not take into account that the regressors are estimated residuals and they are not valid for the Engle-Granger test of cointegration. If you choose to use the `adf` command, be sure to use the the `--nc` no constant option in this case.

```
adf 0 uhat --nc --verbose
```

The `--verbose` option tells **gretl** to print the actual regression results from the Dickey-Fuller test. The regression results will match those you obtained using the manual method above. Ignore the p-value for the Dickey-Fuller since the regressors are residuals. Since G and P are not cointegrated, a vector autoregression model can be estimated using the differences.

The script to estimate a first order VAR appears below:

```
var 1 diff(P) diff(G)
```

The `diff()` function is used to take the first differences of the time series and can be used in the `var` command. The command `var 1` tells **gretl** to estimate a VAR of order 1, which means lag the right-hand-side variable one period. Then list the variables to include on the right-hand side.

In practice, you might want to explore whether the order of the VAR (number of lags on the right-hand side) are sufficient. This can be done using the --lagselect option in the var statement. You start the VAR with a relatively long lag length and **gretl** estimates each successively smaller version, computing various goodness-of-fit measures. **Gretl** then tells you which is the optimal lag length based on each criterion. For instance, starting the VAR at 8 lags and using --lagselect is accomplished by:

```
var 8 diff(G) diff(P) --lagselect
```

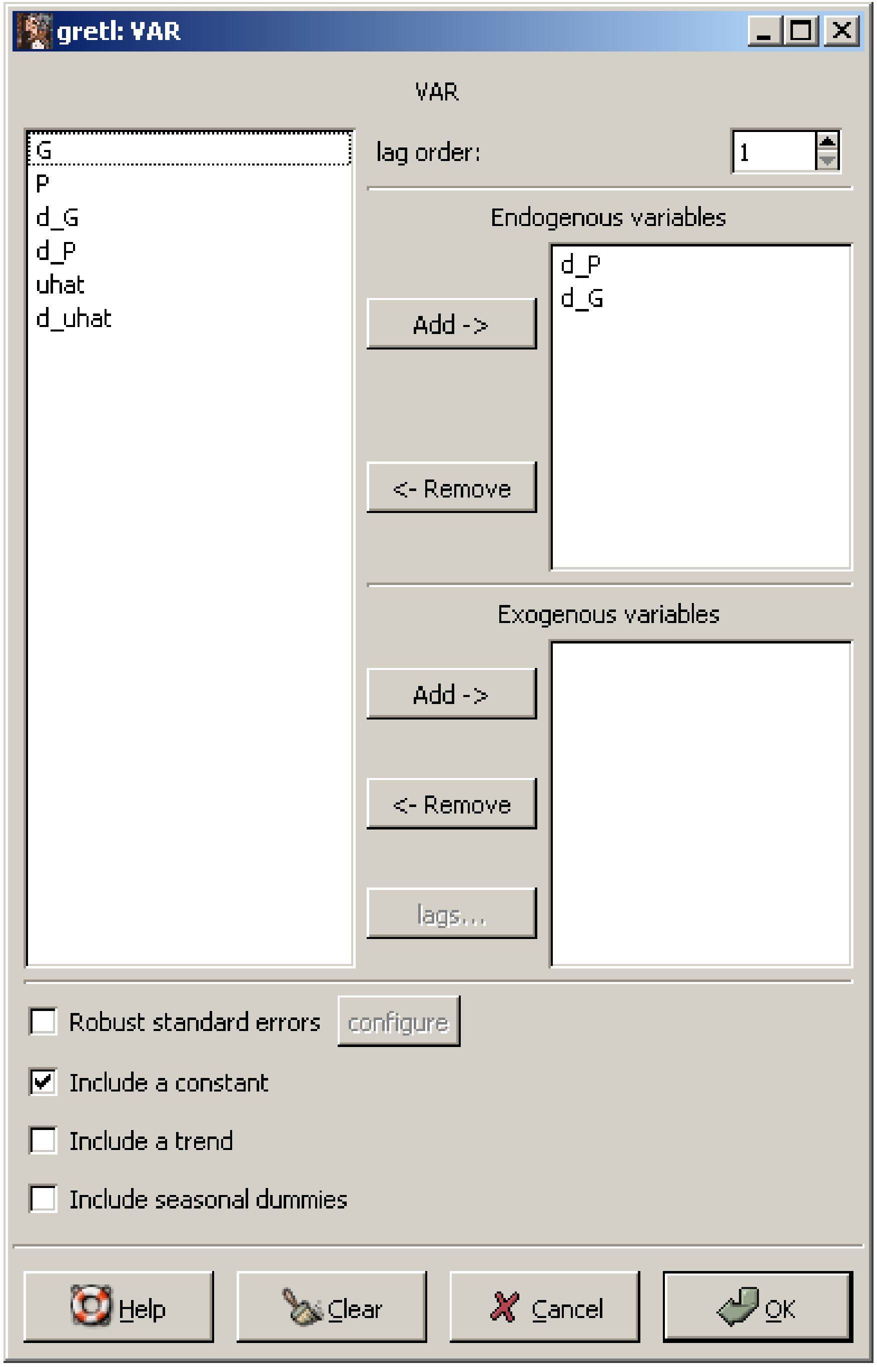
You can also get **gretl** to generate this command through the dialogs. Select Model>Time series>VAR lag selection from the pull-down menu. This reveals the VAR lag selection dialog box. You can choose the maximum lag to consider, the variables to include in the model, and whether the model should contain constant, trend, or seasonal dummies. The output is:

```
? var 8 diff(G) diff(P) --lagselect
VAR system, maximum lag order 8

The asterisks below indicate the best (that is, minimized) values
of the respective information criteria, AIC = Akaike criterion,
BIC = Schwartz Bayesian criterion and HQC = Hannan-Quinn criterion.

lags        loglik    p(LR)       AIC          BIC          HQC

   1    1273.78393            -14.827882   -14.717648   -14.783154
   2    1277.61696  0.10461  -14.825929   -14.642206   -14.751382
   3    1300.25039  0.00000  -15.043864*  -14.786652*  -14.939498*
   4    1303.30981  0.19045  -15.032863   -14.702162   -14.898679
   5    1306.32104  0.19748  -15.021299   -14.617108   -14.857295
   6    1311.53702  0.03375  -15.035521   -14.557841   -14.841699
   7    1313.40649  0.44249  -15.010602   -14.459433   -14.786961
   8    1315.11728  0.48990  -14.983828   -14.359170   -14.730368
```

The AIC, BIC, and HQC criteria each select a VAR with 3 lags.

Obtaining the impulse responses is easy in **gretl**. The first step is to estimate the VAR. From the main **gretl** window choose Model>Time series>Vector Autoregression. This brings up the dialog, shown in Figure 13.4. Set the lag order to 1, and add the differenced variables to the box

Figure 13.4: From the main **gretl** window, choose `Model>Time series>Vector Autogregression` to bring up this VAR dialog box.



gretl: VAR

VAR

G
P
d_G
d_P
uhat
d_uhat

lag order: 1

Endogenous variables

d_P
d_G

Exogenous variables

☐ Robust standard errors

☑ Include a constant

☐ Include a trend

☐ Include seasonal dummies

labeled `Endogenous Variables`. Make sure the 'Include a constant' box is checked and click OK.

The results are shown in Figure 13.5. You can generate impulse responses by selecting `Analysis>Impulse responses` from the results window. This will produce the results shown in Figure 13.6.

These can be graphed for easier interpretation from the results window by selecting `Graphs>Impulse responses (combined)` from the pull-down menu. The graph is shown in Figure 13.7. This yields the graph shown in Figure 13.8.

The forecast error variance decompositions (FEVD) are obtained similarly. Select `Analysis>Forecast variance decomposition` from the vector autoregression model window to obtain the result shown in Figure 13.9.

Figure 13.5: VAR results

VAR system, lag order 1
OLS estimates, observations 1960:3–2004:4 ($T = 178$)

Equation 1: d_P

Variable	Coefficient	Std. Error	t-statistic	p-value
const	0.00143280	0.000710432	2.0168	0.0452
d_P$_{t-1}$	0.826816	0.0447068	18.4942	0.0000
d_G$_{t-1}$	0.0464420	0.0398581	1.1652	0.2455

Sum of squared residuals	0.00347709
Standard error of residuals ($\hat{\sigma}$)	0.00445747
Unadjusted R^2	0.667250
Adjusted $\bar{R}^2$	0.663447
$F(2, 175)$	175.460

Equation 2: d_G

Variable	Coefficient	Std. Error	t-statistic	p-value
const	0.00981441	0.00125091	7.8458	0.0000
d_P$_{t-1}$	−0.326952	0.0787188	−4.1534	0.0001
d_G$_{t-1}$	0.228505	0.0701813	3.2559	0.0014

Sum of squared residuals	0.0107802
Standard error of residuals ($\hat{\sigma}$)	0.00784863
Unadjusted R^2	0.168769
Adjusted $\bar{R}^2$	0.159269
$F(2, 175)$	17.7656

Figure 13.6: Impulse Response Functions

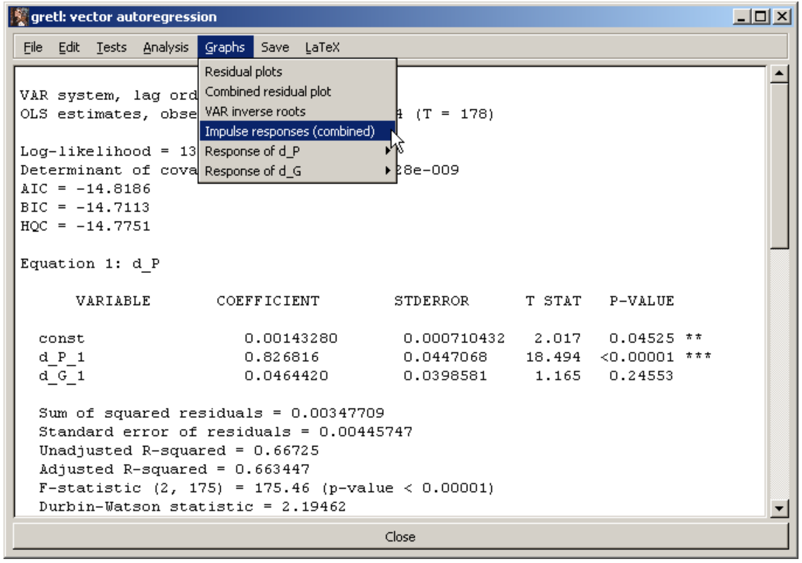
Responses to a one-standard error shock in d_G

period	d_G	d_P
1	0.00778221	0.000358053
2	0.00166121	0.000657465
3	0.000164635	0.000620753
4	−0.000165336	0.000520894
5	−0.000208088	0.000423005
6	−0.000185852	0.000340084
7	−0.000153659	0.000272555
8	−0.000124224	0.000218217
9	−9.97324e-005	0.000174656
10	−7.98936e-005	0.000139777
11	−6.39564e-005	0.000111859
12	−5.11870e-005	8.95168e-005

Responses to a one-standard error shock in d_P

period	d_G	d_P
1	0.000000	0.00440523
2	−0.00144030	0.00364231
3	−0.00151998	0.00294463
4	−0.00131008	0.00236408
5	−0.00107230	0.00189382
6	−0.000864213	0.00151604
7	−0.000693149	0.00121335
8	−0.000555095	0.000971026
9	−0.000444321	0.000777080
10	−0.000355598	0.000621867
11	−0.000284577	0.000497655
12	−0.000227737	0.000398253

Figure 13.7: Select `Graphs>Impulse responses (combined)` from the VAR results window.

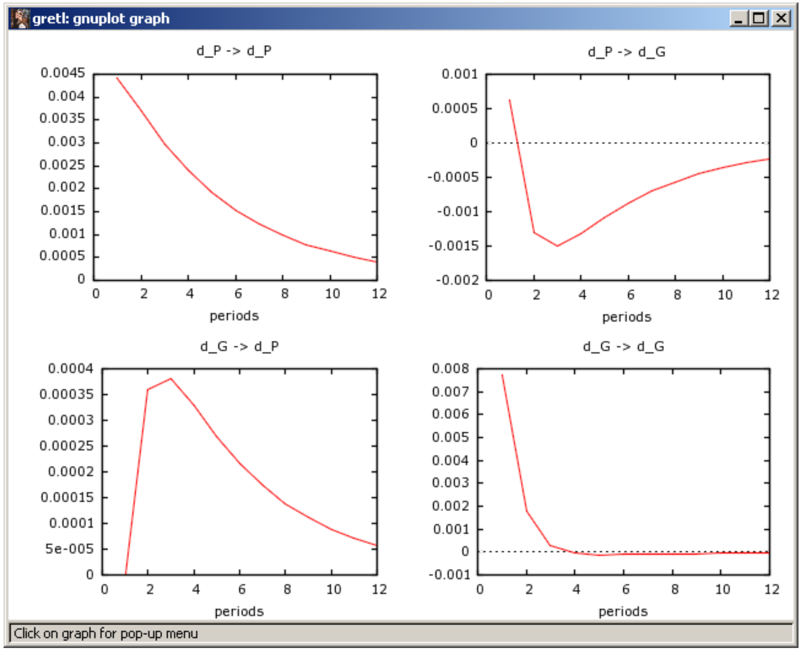


```
VAR system, lag ord                         
OLS estimates, obse                        4 (T = 178)

Log-likelihood = 13
Determinant of cova                        28e-009
AIC = -14.8186
BIC = -14.7113
HQC = -14.7751

Equation 1: d_P

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                 0.00143280       0.000710432   2.017   0.04525 **
  d_P_1                 0.826816         0.0447068    18.494  <0.00001 ***
  d_G_1                 0.0464420        0.0398581     1.165   0.24553

  Sum of squared residuals = 0.00347709
  Standard error of residuals = 0.00445747
  Unadjusted R-squared = 0.66725
  Adjusted R-squared = 0.663447
  F-statistic (2, 175) = 175.46 (p-value < 0.00001)
  Durbin-Watson statistic = 2.19462
```

13.3 Script

```
#VECM example
open C:\userdata\gretl\data\poe\gdp.gdt

#Declare the data to be time series
setobs 4 1970:1 --time-series

#Analyze the plots for constants and trends
scatters usa diff(usa) aus diff(aus)

#Testing down with ADF
adf 6 usa --ctt --test-down
adf 6 aus --ctt --test-down

adf 6 usa --difference --ct --test-down
adf 6 aus --difference --ct --test-down
```

Figure 13.8: U.S. ln(GDP) and ln(CPI) impulse responses



```
#Testing up (manually for usa)
genr time
genr t2 = time*time
genr d_usa = diff(usa)
genr d_aus = diff(aus)

ols diff(usa) usa(-1) const time t2
lmtest 1 --autocorr
ols diff(usa) usa(-1) lags(1,d_usa) const time t2 --quiet
lmtest 1 --autocorr
ols diff(usa) usa(-1) lags(2,d_usa) const time t2 --quiet
lmtest 1 --autocorr

#This test can be misleading: not enough AR terms in LM test
ols diff(aus) aus(-1) const time t2
lmtest 1 --autocorr

#Be sure to test for enough AR terms in the LM test!
ols diff(aus) aus(-1) const time t2
```

Figure 13.9: Forecast Error Variance Decompositions

Decomposition of variance for d_G

period	std. error	d_G	d_P
1	0.00778221	100.0000	0.0000
2	0.00808683	96.8279	3.1721
3	0.00823008	93.5265	6.4735
4	0.00833534	91.2187	8.7813
5	0.0084066	89.7399	10.2601
6	0.00845295	88.8068	11.1932
7	0.00848272	88.2175	11.7825
8	0.00850177	87.8440	12.1560
9	0.00851395	87.6064	12.3936
10	0.00852175	87.4550	12.5450
11	0.00852674	87.3582	12.6418
12	0.00852994	87.2964	12.7036

Decomposition of variance for d_P

period	std. error	d_G	d_P
1	0.00441975	0.6563	99.3437
2	0.0057648	1.6865	98.3135
3	0.00650301	2.2365	97.7635
4	0.00693897	2.5278	97.4722
5	0.00720519	2.6891	97.3109
6	0.00737081	2.7825	97.2175
7	0.00747498	2.8385	97.1615
8	0.00754094	2.8728	97.1272
9	0.00758289	2.8941	97.1059
10	0.00760963	2.9076	97.0924
11	0.0076267	2.9161	97.0839
12	0.00763762	2.9215	97.0785

```
lmtest 3 --autocorr
ols diff(aus) aus(-1) lags(1,d_aus) const time t2 --quiet
lmtest 3 --autocorr
ols diff(aus) aus(-1) lags(2,d_aus) const time t2 --quiet
lmtest 3 --autocorr
ols diff(aus) aus(-1) lags(3,d_aus) const time t2 --quiet
lmtest 3 --autocorr

#Cointegration test
ols aus usa
genr uhat = $uhat
ols diff(uhat) uhat(-1)
adf 0 uhat --nc

#VECM
ols diff(aus) const uhat(-1)
ols diff(usa) const uhat(-1)

#Growth example
open C:\userdata\gretl\data\poe\growth.gdt

#Analyze the plots
scatters G diff(G) P diff(P)

#Cointegration test
ols G const P
series uhat = $uhat
ols diff(uhat) uhat(-1)

adf 3 uhat --nc --test-down --verbose
adf 0 uhat --nc --verbose

#VAR
var 1 diff(P) diff(G)

#Using lagselect
var 8 diff(G) diff(P) --lagselect

#Estimate the VAR with IRFs and FEVDs
var 1 diff(P) diff(G) --impulse-responses --variance-decomp
```

Chapter 14

Time-Varying Volatility and ARCH Models: Introduction to Financial Econometrics

In this chapter we'll estimate several models in which the variance of the dependent variable changes over time. These are broadly referred to as ARCH (autoregressive conditional heteroskedasticity) models and there are many variations upon the theme.

14.1 ARCH and GARCH

The basic ARCH(1) model can be expressed as:

$$
\begin{aligned}
y_t &= \beta + e_t && (14.1)\\
e_t|I_{t-1} &\sim N(0, h_t) && (14.2)\\
h_t &= \alpha_0 + \alpha_1 e_{t-1}^2 \qquad \alpha_0 > 0, \qquad 0 \le \alpha_1 < 1 && (14.3)
\end{aligned}
$$

The first equation describes the behavior of the mean of your time series. In this case, equation (14.1) indicates that we expect the time series to vary randomly about its mean, β. If the mean of your time series drifts over time or is explained by other variables, you'd add them to this equation just as you would a regular regression model. The second equation indicates that the error of the regression, e_t, are normally distributed and heteroskedastic. The variance of the current period's error depends on information that is revealed in the preceding period, i.e., I_{t-1}. The variance of e_t is given the symbol h_t. The final equation describes how the variance behaves. Notice that h_t depends on the error in the preceding time period. The parameters in this equation have to be positive to ensure that the variance, h_t, is positive.

The ARCH(1) model can be extended to include more lags of the errors, e_{t-q}. In this case, q refers to the order of the ARCH model. For example, ARCH(2) replaces (14.3) with $h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2$. When estimating regression models that have ARCH errors in **gretl**, you'll have to specify this order.

ARCH is treated as a special case of a more general model in **gretl** called GARCH. GARCH stands for generalized autoregressive conditional heteroskedasticity and it adds lagged values of the variance itself, h_{t-p}, to (14.3). The GARCH(1,1) model is:

$$\begin{aligned} y_t &= \beta + e_t && (14.4)\\ e_t|I_{t-1} &\sim N(0,h_t) && (14.5)\\ h_t &= \delta + \alpha_1 e_{t-1}^2 + \beta_1 h_{t-1} && (14.6) \end{aligned}$$

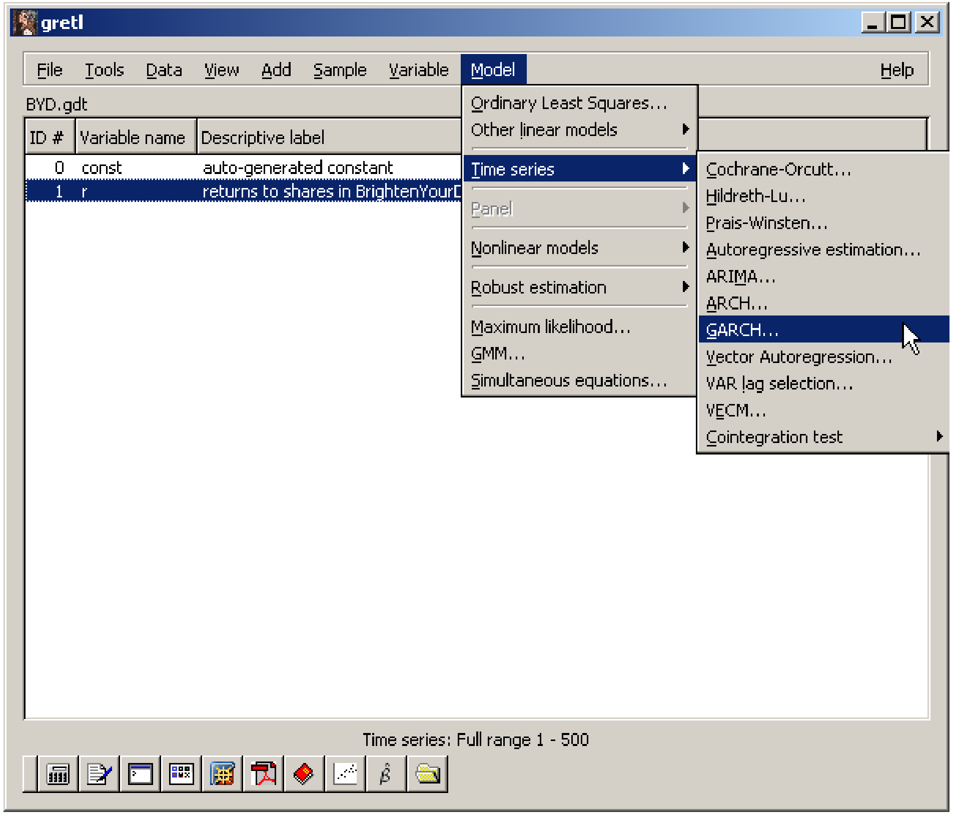
The difference between ARCH (14.3) and its generalization (14.6) is a term $\beta_1 h_{t-1}$, a function of the lagged variance. In higher order GARCH(p,q) model's, q refers to the number of lags of e_t and p refers to the number of lags of h_t to include in the model of the regression's variance.

To open the dialog for estimating ARCH and GARCH in **gretl** choose `Model>Time series>GARCH` from the main **gretl** window as shown in Figure 14.1 below.[1]

To estimate the ARCH(1) model, you'll place the time series `r` into the dependent variable box and set q=1 and p=0 as shown in Figure (14.2) This

[1] In a later version of **gretl** , and ARCH option has been added. You can use this as well, but the answer you get will be slightly different due to differences in the method used to estimate the model.

Figure 14.1: Choose `Model>Time series>GARCH` from the main **gretl** window.

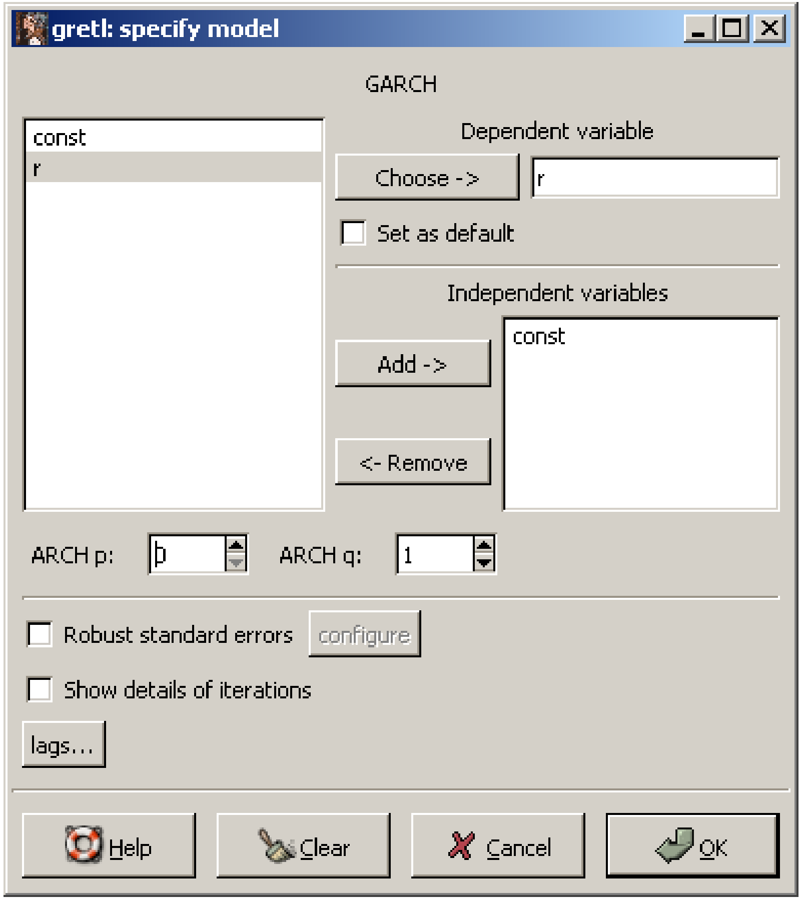


BYD.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated constant
1	r	returns to shares in BrightenYourD

Time series: Full range 1 - 500

Figure 14.2: Estimating ARCH using the dialog box in **gretl** .



Dependent variable: r

□ Set as default

Independent variables: const

ARCH p: 0 ARCH q: 1

□ Robust standard errors

□ Show details of iterations

yields the results:

$$\hat{r} = \underset{(26.886)}{1.06394}$$

$$\hat{h}_t = \underset{(9.914)}{0.642139} + \underset{(6.247)}{0.569347}\, e_{t-1}^2$$

$$T = 500 \quad \ln L = -740.7932 \quad \hat{\sigma} = 1.2211$$

$$\text{(t-statistics in parentheses)}$$

To estimate the GARCH(1,1) model, set p=1 and q=1 to obtain:

$$\widehat{\mathrm{r}} = \underset{(0.040465)}{1.04987}$$

$$\hat{\sigma}_t^2 = \underset{(0.089941)}{0.40105} + \underset{(0.10157)}{0.491028}\, \varepsilon_{t-1}^2 + \underset{(0.1115)}{0.237999}\, \sigma_{t-1}^2$$

$$T = 500 \quad \ln L = -736.0281 \quad \hat{\sigma} = 1.2166$$

$$\text{(standard errors in parentheses)}$$

You will notice that the coefficient estimates and standard errors for the ARCH(1) and GARCH(1,1) models are quite close to those in Chapter 14 of your textbook. To obtain these, you will have to change the default variance-covariance computation using `set garch_vcv op` before running the script. Although this gets you close the the results in *POE*, using the `garch_vcv op` is not usually recommended; just use the **gretl** default, `set garch_vcv unset`.

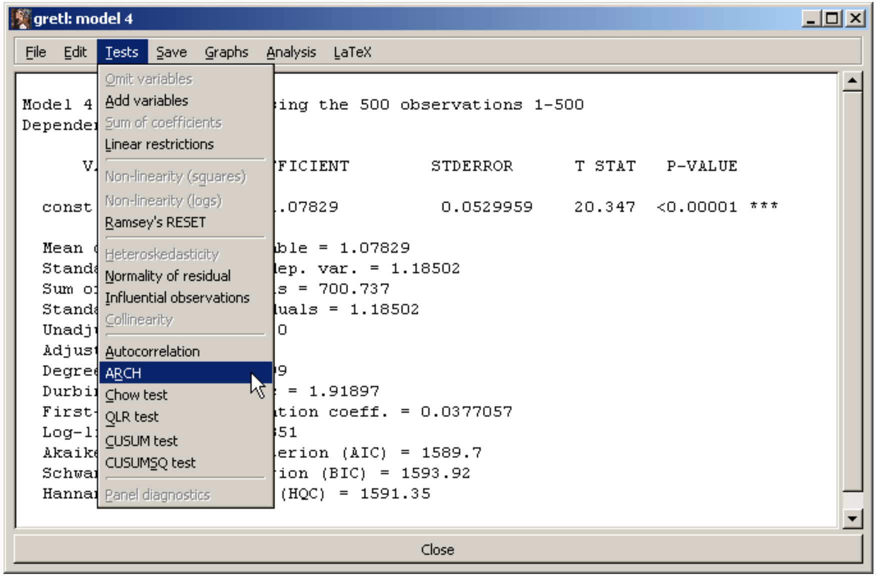
The standard errors and t-ratios often vary a bit, depending on which software and numerical techniques are used. This is the nature of maximum likelihood estimation of the model's parameters. With maximum likelihood estimation the model's parameters are estimated using numerical optimization techniques. All of the techniques usually get you to the same parameter estimates, i.e., those that maximize the likelihood function; but, they do so in different ways. Each numerical algorithm arrives at the solution iteratively based on reasonable starting values and the method used to measure the curvature of the likelihood function at each round of estimates. Once the algorithm finds the maximum of the function, the curvature measure is reused as an estimate of the variance covariance matrix. Since curvature can be measured in slightly different ways, the routine will produce slightly different estimates of standard errors.

Gretl gives you a way to choose which method you like use for estimating the variance-covariance matrix. And, as expected, this choice will produce different standard errors and t-ratios. The `set garch_vcv` command allows you to choose among five alternatives: `unset`–which restores the default, `hessian`, `im` (information matrix) , `op` (outer product matrix), `qml` (QML estimator), or `bw` (Bollerslev-Wooldridge). If unset is given the default is restored, which in this case is the Hessian; if the ”robust” option is given for the garch command, QML is used.

14.2 Testing for ARCH

Testing for the presence of ARCH in the errors of your model is straightforward. In fact, there are at least two ways to proceed. The first is to estimate the regression portion of your model using least squares. Then choose the `Tests>ARCH` from the model's pull-down menu. This is illustrated in Figure 14.3 below.

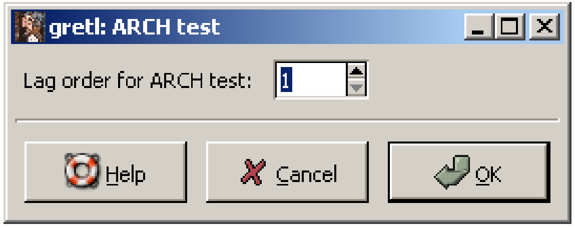
Figure 14.3: Choose `Tests>ARCH` from the model's pull-down menu.



```
Model 4                  ing the 500 observations 1-500
Depende

      V                  FICIENT        STDERROR       T STAT   P-VALUE

  const                  .07829          0.0529959    20.347  <0.00001 ***

  Mean                   ble = 1.07829
  Standa                 ep. var. = 1.18502
  Sum o                  s = 700.737
  Standa                 uals = 1.18502
  Unadj                  0
  Adjus
  Degre                  9
  Durbi                  = 1.91897
  First-                 tion coeff. = 0.0377057
  Log-l                  51
  Akaik                  erion (AIC) = 1589.7
  Schwa                  ion (BIC) = 1593.92
  Hanna                  (HQC) = 1591.35
```

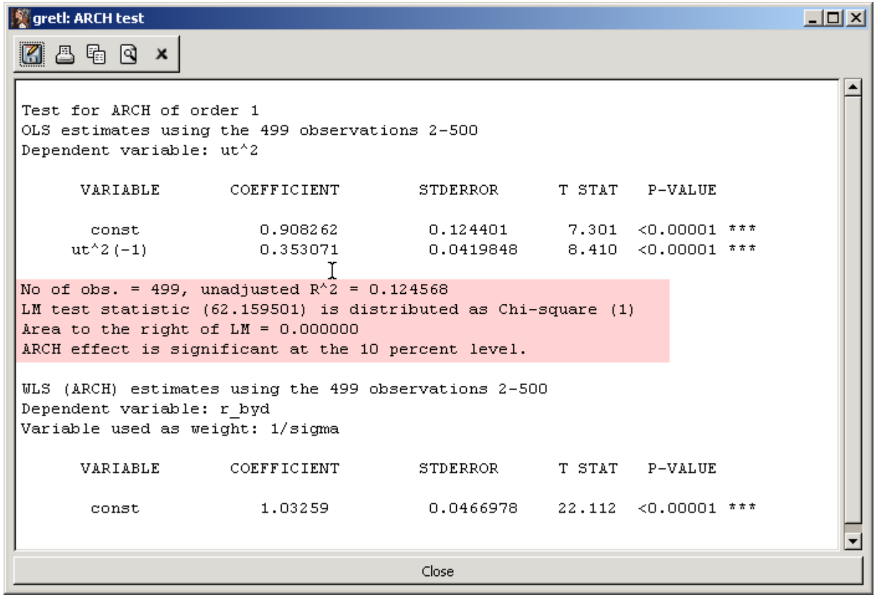
This brings up the box where you tell **gretl** what order of ARCH(q) you want as your alternative hypothesis. In the example, $q = 1$ which leads to the result obtained in the text. The output is shown below in Figure 14.5. **Gretl** produces the LM statistic discussed in your text; the relevant part is highlighted in red.

Figure 14.4: Testing ARCH dialog box



Lag order for ARCH test: 1

Figure 14.5: ARCH test results



```
Test for ARCH of order 1
OLS estimates using the 499 observations 2-500
Dependent variable: ut^2

      VARIABLE       COEFFICIENT        STDERROR       T STAT   P-VALUE

       const           0.908262          0.124401       7.301   <0.00001 ***
     ut^2(-1)          0.353071          0.0419848      8.410   <0.00001 ***

No of obs. = 499, unadjusted R^2 = 0.124568
LM test statistic (62.159501) is distributed as Chi-square (1)
Area to the right of LM = 0.000000
ARCH effect is significant at the 10 percent level.

WLS (ARCH) estimates using the 499 observations 2-500
Dependent variable: r_byd
Variable used as weight: 1/sigma

      VARIABLE       COEFFICIENT        STDERROR       T STAT   P-VALUE

       const           1.03259           0.0466978     22.112   <0.00001 ***
```

The other way to conduct this test is manually. The first step is to estimate the regression (14.1) using **gretl** . Save the squared residuals and then regress these on their lagged value. Take TR^2 from this regression as your test statistic. The script for this appears below:

```
open c:\userdata\gretl\data\poe\BYD.gdt

ols r const
series ehat = $uhat
series ehat2 = ehat*ehat
ols ehat2 const ehat2(-1)
scalar tr2 = $trsq
```

The first line estimates the regression

$$r_t = \beta + e_t \tag{14.7}$$

The residuals are saved in `ehat` and then squared as `ehat2`. The next line estimates the regression

$$\hat{e}_t = \alpha_1 + \alpha_2 \hat{e}_{t-1} + u_t \tag{14.8}$$

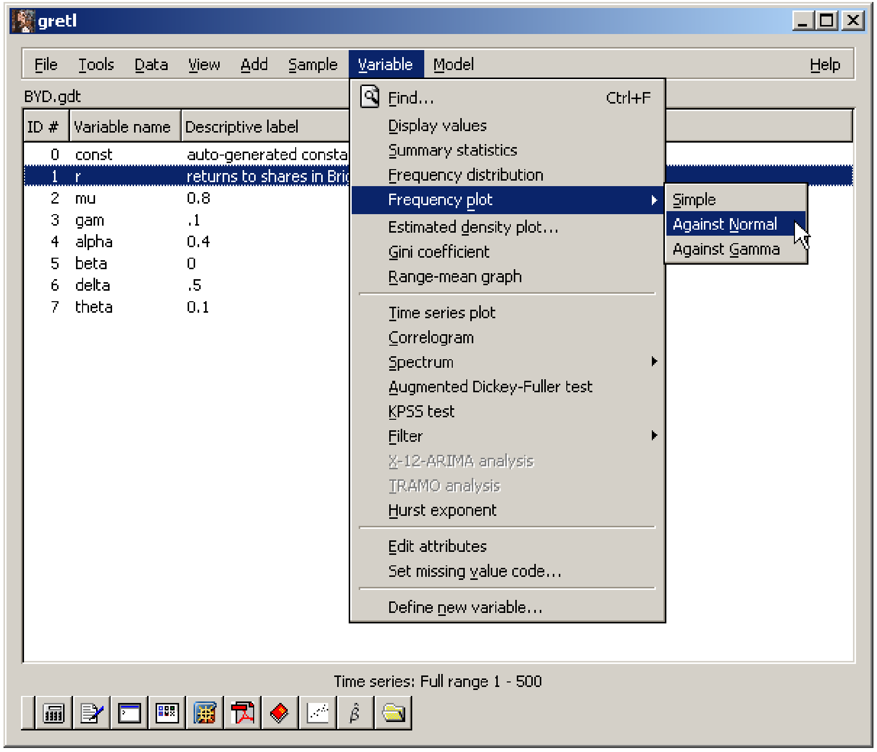
The notation `ehat2(-1)` takes the variable `ehat2` and offsets it in the dataset by the amount in parentheses. In this case, `ehat2(-1)` puts a minus one period lag of `ehat2` into your regression. The final line computes TR^2 from the regression.

14.3 Simple Graphs

There are several figures that you can produce with **gretl** and **gnuplot** . One useful graph is a histogram of the time series you are studying. The easiest way to get this is through the pull-down menus. In Figure 14.6 you'll find a histogram of the Brighten Your Day returns. A normal density is superimposed on the series. Selecting `Variable>Frequency plot>against Normal` from the pull-down menu opens a small dialog box that allows you to control how the histogram looks. You can choose the number of bins, which in this case has been set to 23 (Figure 14.7). Click OK and the result appears in Figure 14.8.

Once you've estimated your ARCH or GARCH model, you can graph the behavior of the variance as done in the textbook. After estimating ARCH or GARCH, you can save the predicted variances using the command `genr ht = $h`. Then plot them using `gnuplot ht time`. The result is shown in Figure 14.9. A prettier plot can be obtained using the pull-down menus or by editing the plot yourself. To modify the graph, right click on the graph and choose `edit`. From here you can add labels, titles or replace the crosses with lines. That's what I have done to produce the result in Figure 14.10.

Figure 14.6: Highlight the desired series using your cursor, then choose `Variable>Frequency plot>against Normal` from the pull-down menu

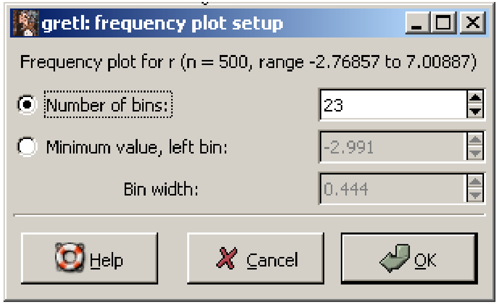


BYD.gdt

ID #	Variable name	Descriptive label
0	const	auto-generated consta
1	r	returns to shares in Bri
2	mu	0.8
3	gam	.1
4	alpha	0.4
5	beta	0
6	delta	.5
7	theta	0.1

Time series: Full range 1 - 500

Figure 14.7: Choosing `Variable>Frequency plot>against Normal` from the pull-down menu reveals this dialog box.



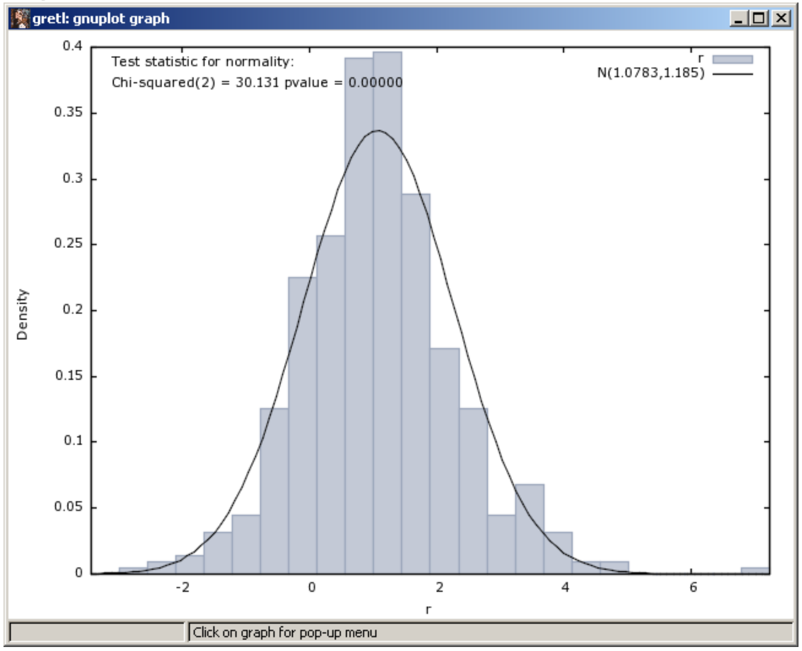
Frequency plot for r (n = 500, range -2.76857 to 7.00887)

● Number of bins: 23

○ Minimum value, left bin: -2.991

Bin width: 0.444

Figure 14.8: The histogram produced using the dialogs from the pull-down menu in **gretl**.



Test statistic for normality:
Chi-squared(2) = 30.131 pvalue = 0.00000

Figure 14.9: Plot of the variances after estimating the GARCH(1,1) using the BrightenYourDay returns. Right click on the graph to bring up the menu shown. Then choose `edit` to modify your graph.

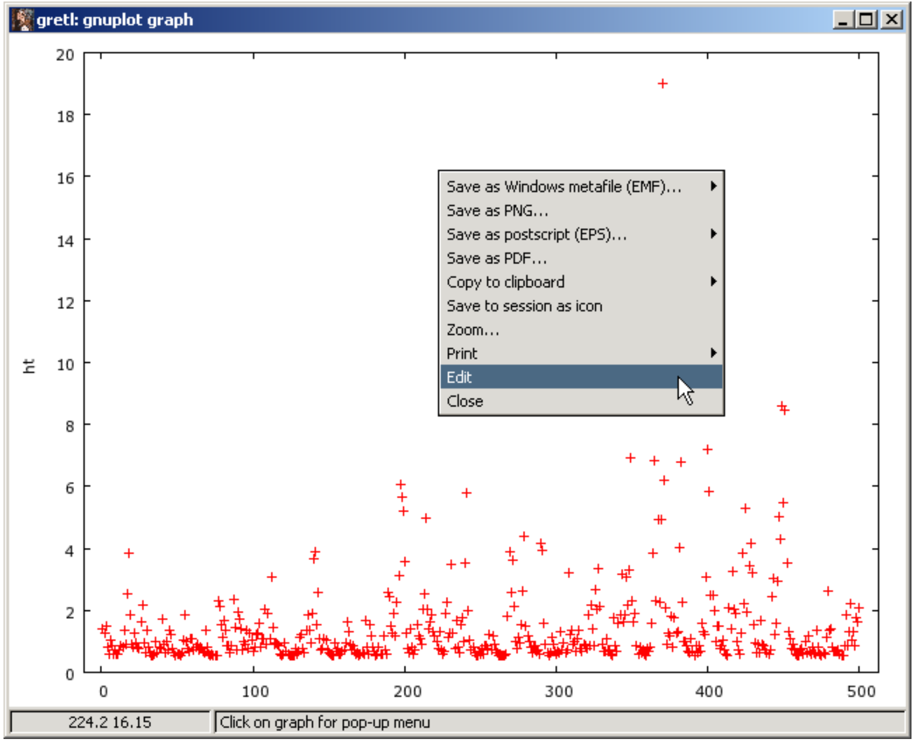
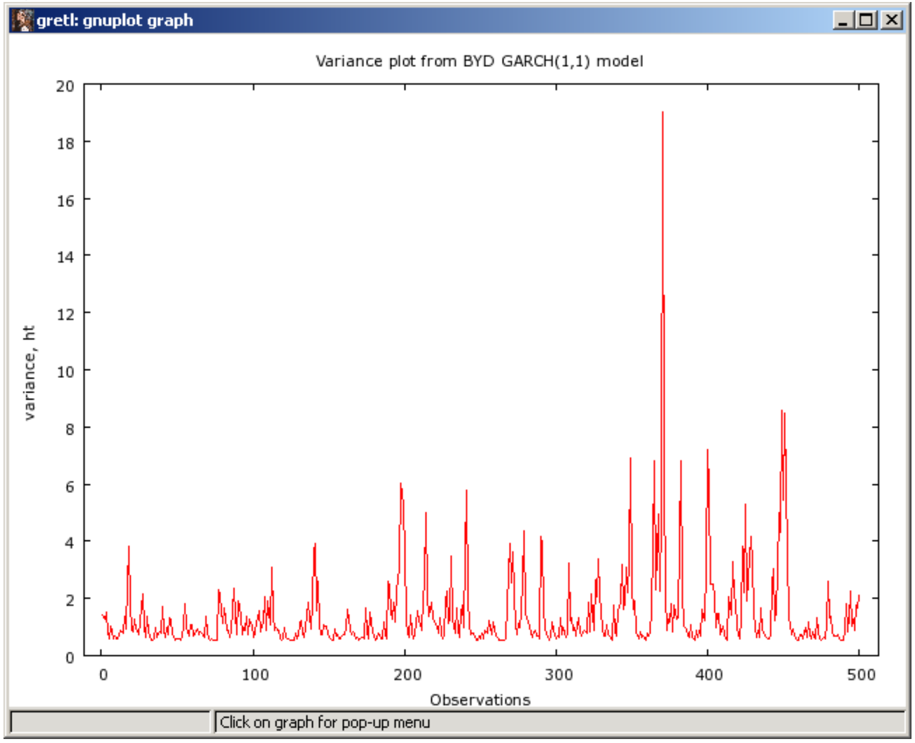


Figure 14.10: Plot of the variances after estimating the GARCH(1,1) using Brighten Your Day's returns



14.4 Threshold ARCH

Threshold ARCH (TARCH) can also be estimated in **gretl**, though it requires a little programming; there aren't any pull-down menus for this estimator. Instead, we'll introduce **gretl**'s powerful `mle` command that allows user defined (log) likelihood functions to be maximized.

The threshold ARCH model replaces the variance equation (14.3) with

$$h_t = \delta + \alpha_1 e_{t-1}^2 + \gamma d_{t-1} e_{t-1}^2 + \beta_1 h_{t-1} \tag{14.9}$$

$$d_t = \begin{cases} 1 & if\, e_t < 0 \\ 0 & otherwise \end{cases} \tag{14.10}$$

The model's parameters are estimated by finding the values that maximize its likelihood. Maximum likelihood estimators are discussed in appendix C of Hill et al. (2008).

Gretl provides a fairly easy way to estimate via maximum likelihood that can be used for a wide range of estimation problems (see Chapter 16 for other examples). To use **gretl**'s `mle` command, you must specify the log-likelihood function that is to be maximized. Any parameters contained in the function must be given reasonable starting values for the routine to work properly. Parameters can be declared and given starting values (using the genr command).

Numerical optimization routines use the partial derivatives of the objective function to iteratively find the minimum or maximum of the function. If you want, you can specify the analytical derivatives of the log-likelihood function with respect to each of the parameters in **gretl**; if analytical derivatives are not supplied, **gretl** tries to compute a numerical approximation. The actual results you obtain will depend on many things, including whether analytical derivatives are used and the starting values.

For the threshold GARCH model, open a new script file and type in the program that appears in Figure 14.11.

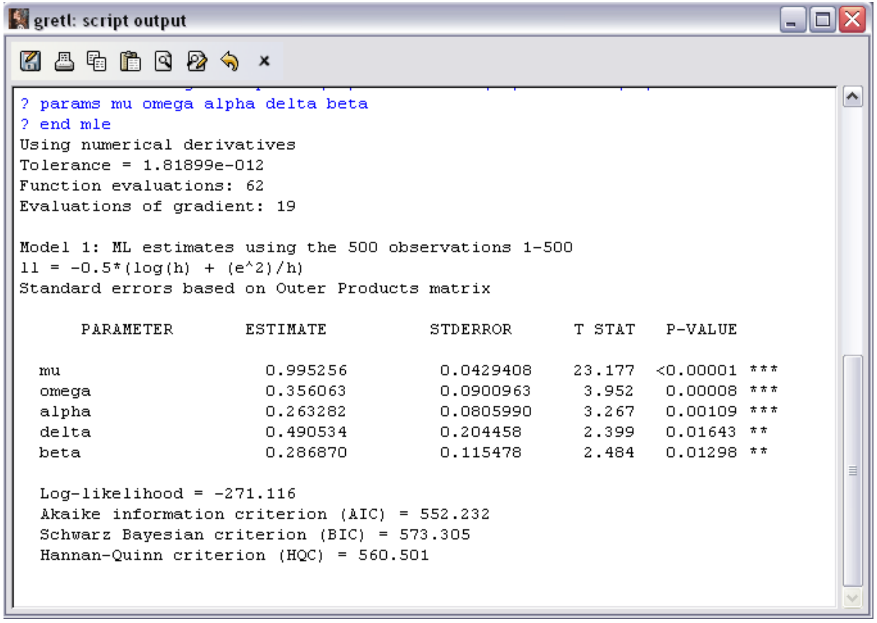
The first few lines of the script gives starting values for the parameters. The second part of the script contains the the algebraic expression of the likelihood function. The first line `ll = -0.5*(log(h) + (eˆ2)/h)` is what

Figure 14.11: Threshold GARCH script

```
open c:\userdata\gretl\data\poe\BYD.gdt

scalar mu = 0.5
scalar omega = .5
scalar alpha = 0.4
scalar delta = 0.1
scalar beta = 0

mle ll = -0.5*(log(h) + (e^2)/h)
   series h = var(r)
   series e = r - mu
   series e2 = e^2
   series e2m = e2 * (e<0)
   series h = omega + alpha*e2(-1) + delta*e2m(-1) + beta*h(-1)
   params mu omega alpha delta beta
end mle
```

is called the *kernel* of the normal probability density function. Recall that the errors of the ARCH model are assumed to be normally distributed and this is reflected in the kernel. Next, we have to specify an initial guess for the variances of the model, and these are set using `var(r)`. Then, the errors are generated, squared, and the threshold term is created using `series e2m = e2 * (e<0)`; the expression `(e<0)` takes the value of 1 for negative errors, `e`, and is zero otherwise. Then, the heteroskedastic function h_t is specified. The parameters of the model are given at the end, which also tells **gretl** to print the estimates out once it has finished the numerical optimization. The mle loop is ended with `end mle`. The output appears in Figure 14.12. The coefficient estimates are very close to those printed in your text, but the standard errors and corresponding t-ratios are quite a bit different. This is not that unusual since different pieces of software that no doubt use different algorithms were used to numerically maximize the log-likelihood function.

Figure 14.12: Threshold ARCH results



```
? params mu omega alpha delta beta
? end mle
Using numerical derivatives
Tolerance = 1.81899e-012
Function evaluations: 62
Evaluations of gradient: 19

Model 1: ML estimates using the 500 observations 1-500
ll = -0.5*(log(h) + (e^2)/h)
Standard errors based on Outer Products matrix

      PARAMETER       ESTIMATE          STDERROR      T STAT   P-VALUE

  mu                    0.995256         0.0429408    23.177  <0.00001 ***
  omega                 0.356063         0.0900963     3.952   0.00008 ***
  alpha                 0.263282         0.0805990     3.267   0.00109 ***
  delta                 0.490534         0.204458      2.399   0.01643 **
  beta                  0.286870         0.115478      2.484   0.01298 **

  Log-likelihood = -271.116
  Akaike information criterion (AIC) = 552.232
  Schwarz Bayesian criterion (BIC) = 573.305
  Hannan-Quinn criterion (HQC) = 560.501
```

14.5 Garch-in-Mean

The Garch-in-mean (MGARCH) model adds the equation's variance to the regression function. This allows the average value of the dependent variable to depend on volatility of the underlying asset. In this way, more risk (volatility) can lead to higher average return. The equations are listed below:

$$y_t = \beta_0 + \theta h_t + e_t \tag{14.11}$$

$$h_t = \delta + \alpha_1 e_{t-1}^2 + \gamma d_{t-1} e_{t-1}^2 + \beta_1 h_{t-1} \tag{14.12}$$

Notice that in this formulation we left the threshold term in the model. The errors are normally distributed with zero mean and variance h_t.

The parameters of this model can be estimated using **gretl**, though the recursive nature of the likelihood function makes it a bit more difficult. In the script below (Figure 14.13) you will notice that we've defined a function to compute the log-likelihood.[2] The function is called `gim_filter` and it

[2] Actually, **gretl** genius Professor 'Jack' Lucchetti wrote the function and I'm very

contains eight arguments. The first argument is the time series, y. Then, each of the parameters is listed (mu, theta, delta, alpha, gam, and beta) as scalars. The final argument is a placeholder for the variance, h, that is computed within the function.

Once the function is named and its arguments defined, you need to initiate series for the variances and the errors; these have been called lh and le, respectively. The log-likelihood function is computed using a loop that runs from the second observation through the last. The length of the series can be obtained using the saved result $nobs, which is assigned to the variable T.

Gretl's loop syntax is fairly simple, though there are several variations. In this example the loop is controlled using the special index variable, i. In this case you specify starting and ending values for i, which is incremented by one each time round the loop. In the TGARCH example the loop syntax looks like this:

```
loop for i=2..T --quiet
.
.
.
end loop
```

The first line start the loop using an index variable named i. The first value of i is set to 2. The index i will increment by 1 until it reaches T, which has already been defined as being equal to $nobs. The end loop statement tells **gretl** the point at which to return to the top of the loop and advance the increment i. The --quiet option just reduces the amount of output that is written to the screen.

Within the loop itself, you'll want to lag the index and create an indicator variable that will take the value of 1 when the news is bad ($e_{t-1} < 0$). The next line squares the residual. lh[i] uses the loop index to place the variance computation from the iteration into the i^{th} row of lh. The line that begins le[i]= works similarly for the errors of the mean equation.

The variances are collected in h and the residuals in le, the latter of

grateful!

which is returned when the function is called. The function is closed using `end function`.

If this looks too complicated, you can simply highlight the code with your cursor, copy it using Ctrl-C, and paste it into a **gretl** script file (or use the scripts provided with this book).

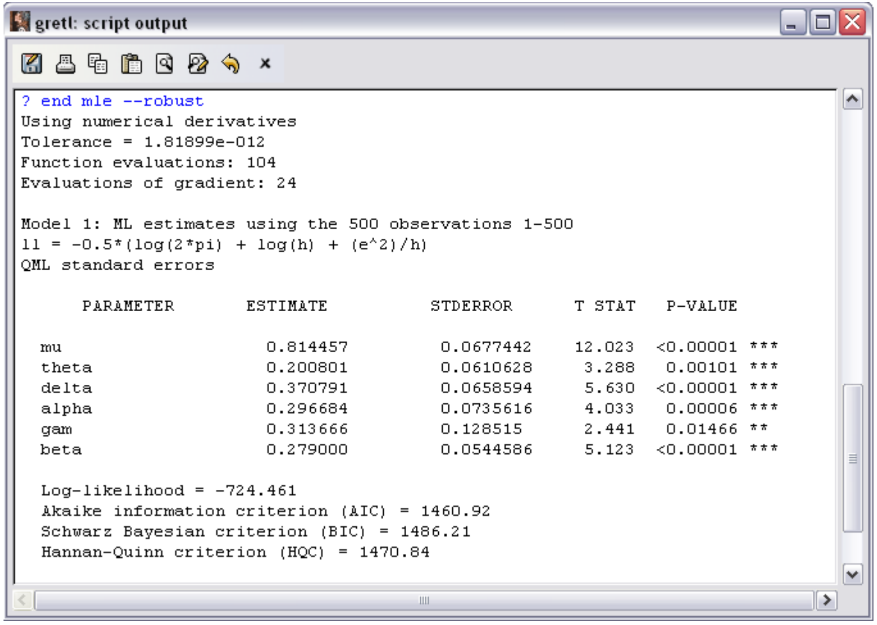
Once the function is defined, you need to initialize each parameter just as you did in TGARCH. The series that will eventually hold the variances also must be initialized. The latter is done using `series h = NA`, which creates the series `h` and fills it with missing values (NA). The missing values for observations 2 through T are replaced as the function loops.

Next, the built-in `mle` command is issued and the normal density kernel is specified just as it was in the TGARCH example. Then, use the predefined `e=gim_filter( )` function, putting in the variable `r` for the time series, the initialized parameters, and `&h` as a pointer to the variances that will be computed within the function. Issue the `params` statement to identify the parameters and have them print to the screen. Close the loop and run the script. The results appear in Figure 14.14 below. This is a difficult likelihood to maximize and **gretl** may take some time to compute the estimates. Still, it is quite remarkable that we get so close using a free piece of software and the numerical derivatives that it computes for us. I'm impressed!

Figure 14.13: The MGARCH script includes a function to compute the log-likelihood.

```
function gim_filter(series y, \
        scalar mu, scalar theta, scalar delta, scalar alpha, \
        scalar gam, scalar beta, series *h)

    series lh = var(y)
    series le = y - mu
    scalar T = $nobs
    loop for i=2..T --quiet
        scalar ilag = $i - 1
        scalar d = (le[ilag]<0)
        scalar e2lag = le[ilag]^2
        lh[i] = delta + alpha*e2lag + gam*e2lag*d + beta*lh[ilag]
        le[i] = le[i] - theta*lh[i]
    end loop

    series h = lh
    return series le

end function

open c:\userdata\gretl\data\poe\BYD.gdt

scalar mu = 0.8
scalar gam = .1
scalar alpha = 0.4
scalar beta = 0
scalar delta = .5
scalar theta = 0.1

series h = NA

mle ll = -0.5*(log(2*pi) + log(h) + (e^2)/h)
   e = gim_filter(r, mu, theta, delta, alpha, gam, beta, &h)
   params mu theta delta alpha gam beta
end mle --robust
```

Figure 14.14: Garch-in-mean results



```
? end mle --robust
Using numerical derivatives
Tolerance = 1.81899e-012
Function evaluations: 104
Evaluations of gradient: 24

Model 1: ML estimates using the 500 observations 1-500
ll = -0.5*(log(2*pi) + log(h) + (e^2)/h)
QML standard errors

      PARAMETER       ESTIMATE          STDERROR      T STAT   P-VALUE

  mu                    0.814457         0.0677442     12.023   <0.00001 ***
  theta                 0.200801         0.0610628      3.288    0.00101 ***
  delta                 0.370791         0.0658594      5.630   <0.00001 ***
  alpha                 0.296684         0.0735616      4.033    0.00006 ***
  gam                   0.313666         0.128515       2.441    0.01466 **
  beta                  0.279000         0.0544586      5.123   <0.00001 ***

  Log-likelihood = -724.461
  Akaike information criterion (AIC) = 1460.92
  Schwarz Bayesian criterion (BIC) = 1486.21
  Hannan-Quinn criterion (HQC) = 1470.84
```

14.6 Script

```
open c:\userdata\gretl\data\poe\BYD.gdt

# ARCH(1) Using built in command for ARCH
arch 1 r const

# GARCH(0,1)=ARCH(1)
garch 0 1 ; r const

# GARCH(1,1)
garch 1 1 ; r const

#LM test for ARCH
ols r const
lmtest 1 --arch

#LM test manually
ols r const
```

```
series ehat = $uhat
series ehat2 = ehat*ehat
ols ehat2 const ehat2(-1)
scalar tr2 = $trsq

#Plotting
garch 1 1 ; r const
genr ht = $h
gnuplot ht time

#Threshold Garch
open c:\userdata\gretl\data\poe\BYD.gdt

scalar mu = 0.5
scalar omega = .5
scalar alpha = 0.4
scalar delta = 0.1
scalar beta = 0

mle ll = -0.5*(log(h) + (e^2)/h)
   series h = var(r)
   series e = r - mu
   series e2 = e^2
   series e2m = e2 * (e<0)
   series h = omega + alpha*e2(-1) + delta*e2m(-1) + beta*h(-1)
   params mu omega alpha delta beta
end mle


#Garch in Mean
function gim_filter(series y, \
        scalar mu, scalar theta, scalar delta, scalar alpha, \
        scalar gam, scalar beta, series *h)

    series lh = var(y)
    series le = y - mu
    scalar T = $nobs
    loop for i=2..T --quiet
        scalar ilag = $i - 1
        scalar d = (le[ilag]<0)
        scalar e2lag = le[ilag]^2
        lh[i] = delta + alpha*e2lag + gam*e2lag*d + beta*lh[ilag]
        le[i] = le[i] - theta*lh[i]
    end loop

    series h = lh
    return series le

end function

open c:\userdata\gretl\data\poe\BYD.gdt

scalar mu = 0.8
scalar gam = .1
scalar alpha = 0.4
scalar beta = 0
scalar delta = .5
scalar theta = 0.1

series h = NA

mle ll = -0.5*(log(2*pi) + log(h) + (e^2)/h)
   e = gim_filter(r, mu, theta, delta, alpha, gam, beta, &h)
   params mu theta delta alpha gam beta
end mle --robust
```

Chapter 15

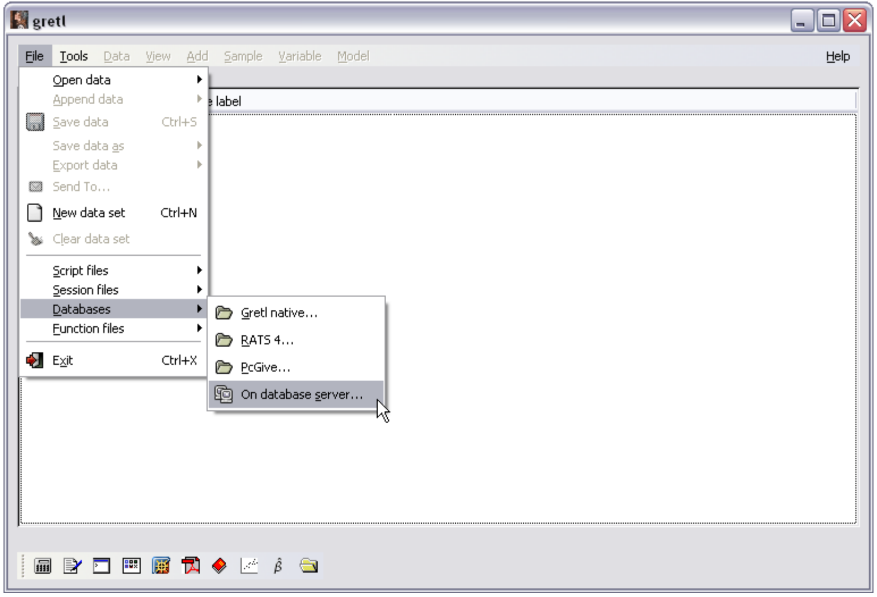
Pooling Time-Series and Cross-Sectional Data

A panel of data consists of a group of cross-sectional units (people, firms, states or countries) that are observed over time. Following Hill et al. (2008) we will denote the number of cross-sectional units by N and the number of time periods we observe them as T.

Gretl gives you easy access to a number of useful panel data sets via its database server.[1] These include the Penn World Table and the Barro and Lee (1996) data on international educational attainment. These data can be installed using `File>Databases>On database server` from the menu bar as shown in Figure 15.1 below. From here, select a database you want. In Figure 15.2 the entry for the Penn World Table is highlighted. To its right, you are given information about whether that dataset is installed on your computer. Double click on `pwtna` and a listing of the series in this database will appear in a new window. From that window you can search for a particular series, display observations, graph a series, or import it. This is a VERY useful utility, both for teaching and research and I encourage you to explore what is available on the **gretl** server.

[1] Your computer must have access to the internet to use this.

Figure 15.1: Accessing data from the database server via the pull-down menus



15.1 A Basic Model

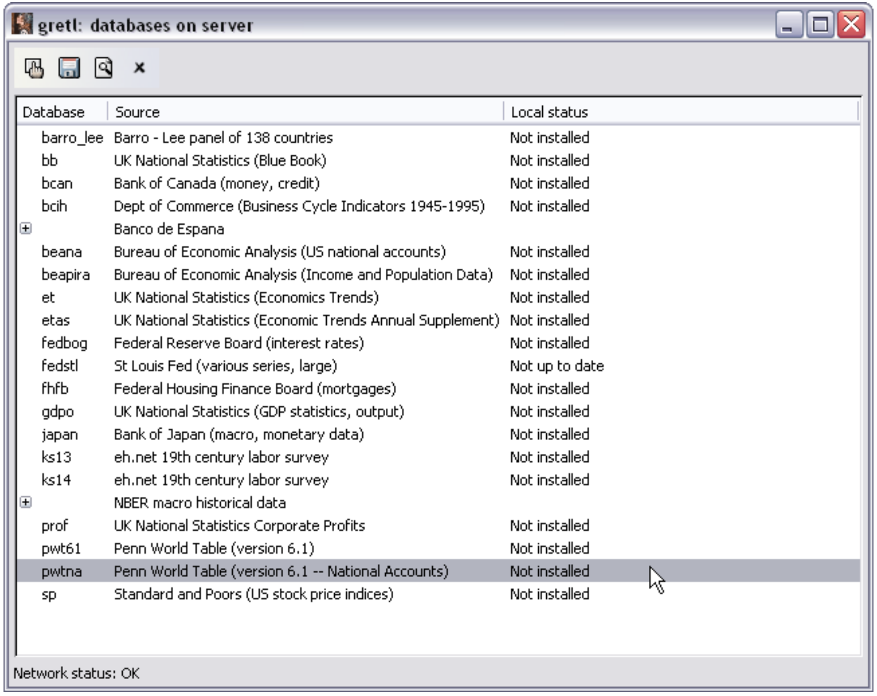
The most general expression of linear regression models that have both time and unit dimensions is seen in equation 15.1 below.

$$y_{it} = \beta_{1it} + \beta_{2it}x_{2it} + \beta_{3it}x_{3it} + e_{it} \tag{15.1}$$

where $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$. If we have a full set of time observations for every individual then there will be NT total observations in the sample. The panel is said to be **balanced** in this case. It is not unusual to have some missing time observations for one or more individuals. When this happens, the total number of observation is less than NT and the panel is **unbalanced**.

The biggest problem with equation (15.1) is that even if the panel is complete (balanced), the model contains 3 times as many parameters as observations (NT)! To be able to estimate the model, some assumptions have to be made in order to reduce the number of parameters. One of the most common assumptions is that the slopes are constant for each individual and

Figure 15.2: Installing a data from the database server via the pull-down menus



Database	Source	Local status
barro_lee	Barro - Lee panel of 138 countries	Not installed
bb	UK National Statistics (Blue Book)	Not installed
bcan	Bank of Canada (money, credit)	Not installed
bcih	Dept of Commerce (Business Cycle Indicators 1945-1995)	Not installed
⊞	Banco de Espana	
beana	Bureau of Economic Analysis (US national accounts)	Not installed
beapira	Bureau of Economic Analysis (Income and Population Data)	Not installed
et	UK National Statistics (Economics Trends)	Not installed
etas	UK National Statistics (Economic Trends Annual Supplement)	Not installed
fedbog	Federal Reserve Board (interest rates)	Not installed
fedstl	St Louis Fed (various series, large)	Not up to date
fhfb	Federal Housing Finance Board (mortgages)	Not installed
gdpo	UK National Statistics (GDP statistics, output)	Not installed
japan	Bank of Japan (macro, monetary data)	Not installed
ks13	eh.net 19th century labor survey	Not installed
ks14	eh.net 19th century labor survey	Not installed
⊞	NBER macro historical data	
prof	UK National Statistics Corporate Profits	Not installed
pwt61	Penn World Table (version 6.1)	Not installed
pwtna	Penn World Table (version 6.1 -- National Accounts)	Not installed
sp	Standard and Poors (US stock price indices)	Not installed

Network status: OK

every time period; also, the intercepts vary only by individual. This model is shown in equation (15.2).

$$y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it} \tag{15.2}$$

This specification, which includes $N + 2$ parameters, includes dummy variables that allow the intercept to shift for each individual. By using such a model you are saying that over short time periods there are no substantive changes in the regression function. Obviously, the longer your time dimension, the more likely this assumption will be false.

In equation (15.2) the parameters that vary by individual are called **individual fixed effects** and the model is referred to as **one-way fixed effects**. The model is suitable when the individuals in the sample differ from one another in a way that does not vary over time. It is a useful way to avoid unobserved differences among the individuals in your sample that would otherwise have to be omitted from consideration. Remember, omitting relevant variables may cause least squares to be biased and inconsistent; a one-way fixed effects model, which requires the use of panel data, can be very useful in mitigating the bias associated with time invariant, unobservable effects.

If you have a longer panel and are concerned that the regression function is shifting over time, you can add $T - 1$ time dummy variables to the model. The model becomes

$$y_{it} = \beta_{1i} + \beta_{1t} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it} \tag{15.3}$$

where either β_{1i} or β_{1t} have to be omitted in order to avoid perfect collinearity. This model contains $N + (T - 1) + 2$ parameters which is generally fewer than the NT observations in the sample. Equation (15.3) is called the **two-way fixed effects** model because it contains parameters that will be estimated for each individual and each time period.

15.2 Estimation

Estimating models using panel data is straightforward in **gretl** . There are several built in functions to estimate fixed effects, random effects, and seemingly related regression models. In this section the **gretl** commands for each will be discussed using the examples in Hill et al. (2008).

In order to use the predefined procedures for estimating models using panel data in **gretl** you have to first make sure that your data have been properly structured in the program. The dialog boxes for assigning dataset structure in **gretl** are shown in Figures 7.2 and 7.3. The data have to include variables that identify each individual and time period. Select the `Panel` option using the radio button and **gretl** will then be able to work behind the scenes to accurately account for the time and individual dimensions. The datasets that come with this manual have already been setup this way, but if you are using your own data you'll want to to assign the proper dataset structure to it so that all of the appropriate panel data procedures are available for use.

Now consider the investment model suggested by Grunfeld (1958). Considering investment decisions of only two firms, General Electric (GE) and Westinghouse (W), we have

$$INV_{GE,t} = \beta_{1,GE} + \beta_{2,GE}V_{GE,t} + \beta_{3,GE}K_{GE,t} + e_{GE,t} \tag{15.4}$$

$$INV_{W,t} = \beta_{1,W} + \beta_{2,W}V_{W,t} + \beta_{3,W}K_{W,t} + e_{W,t} \tag{15.5}$$

where $t = 1, 2, \ldots, 20$.

How one proceeds at this point depends on the nature of the two firms and the behavior of all the omitted factors affecting investment. There are a number of modeling options and *POE* suggests several tests to explore whether the modeling decision we make is an appropriate one. These are considered in the following sections.

15.2.1 Pooled Least Squares

Suppose that that the two firms behave identically and that the other factors influencing investment also have similar effects. In this case, you could simply pool the observations together and estimate a single equation via least squares. This simple model implies that the intercepts and each of the slopes for the two equations are the same and that the omitted factors are not correlated with one another and that they have the same variability. In other words, there is no autocorrelation and the variances are homoscedastic; when the data are actually generated in this way, least squares is efficient.

In terms of the parameters of the model, $\beta_{i,GE} = \beta_{i,W}$ for $i = 1, 2, 3$; $E[e_{GE,t}] = E[e_{W,t}] = 0$; $Var[e_{GE,t}] = Var[e_{W,t}] = \sigma^2$; $Cov(e_{GE,t}, e_{W,t}) = 0$ for all time periods; and $Cov(e_{i,t}, e_{i,s}) = 0$ for $t \neq s$ for each firm, $i = GE, W$. It should be clear that in this case,

$$INV_{i,t} = \beta_1 + \beta_1 + \beta_2 V_{i,t} + \beta_3 K_{i,t} + e_{i,t} \tag{15.6}$$

for observations $i = GE, W$ and $t = 1, 2, \ldots, 10$. The **gretl** script for estimating this model using *grunfeld.gdt* is

```
open c:\userdata\gretl\data\poe\grunfeld.gdt
smpl firm = 3 | firm = 8 --restrict
ols Inv const V K
lmtest --panel
```

The results are

$$\widehat{\text{Inv}} = \underset{(7.02)}{17.87} + \underset{(0.0062)}{0.015}\ \text{V} + \underset{(0.0186)}{0.144}\ \text{K}$$
$$T = 40 \quad \bar{R}^2 = 0.7995 \quad F(2, 37) = 78.752 \quad \hat{\sigma} = 21.158$$
$$\text{(standard errors in parentheses)}$$

Using the robust option would yield consistent standard errors even if the two firms have different variances. The final line in the script performs a test of the equal variance null hypothesis against the alternative that the variances of the two firms differ $\sigma^2_{GE,t} = \sigma^2_{GE} \neq \sigma^2_{W} = \sigma^2_{W,t}$. Note, in this test the *errors within each group are homoscedastic.* The output from this test is

```
Likelihood ratio test for groupwise heteroskedasticity -
  Null hypothesis: the units have a common error variance
  Test statistic: Chi-square(1) = 13.1346
  with p-value = 0.000289899
```

which allows us to reject homoscedasticity in favor of groupwise heteroskedasticity at any reasonable level of significance.

The scenario that leads us to use this model seems unlikely, though. At a minimum the variances of the two conglomerate firms will differ due to their differences in size or diversity. Further, since the two firms share output in at least one industry, omitted factors like macroeconomic or market conditions, might reasonably affect the firms similarly. Finally, there is no reason to believe that the coefficients of the two firms will be similar.

15.2.2 Fixed Effects

In the fixed effects model, the intercepts for each firm (or individual) are allowed to vary, but the slopes for each firm are equal. It is particularly useful when each individual has unique characteristics that are both unmeasurable and constant over time (also known by the fancy sounding phrase, 'unobserved time-invariant heterogeneity'). The general form of this model is found in equation (15.2). The **gretl** command to estimate this model is extremely simple. Once your data set is structured within **gretl** as a panel, the fixed effect model is estimated using the `panel` command as shown below in the script.

```
open c:\userdata\gretl\data\poe\grunfeld.gdt
smpl full
panel Inv const V K
```

The results are:

Model 2: Fixed-effects estimates using 200 observations
Dependent variable: Inv

Variable	Coefficient	Std. Error	*t*-statistic	p-value
V	0.109771	0.0118549	9.2596	0.0000
K	0.310644	0.0173704	17.8835	0.0000

Sum of squared residuals	522855.
Standard error of residuals ($\hat{\sigma}$)	52.7366
Unadjusted R^2	0.944144
Adjusted $\bar{R}^2$	0.940876
$F(11, 188)$	288.893
Durbin–Watson statistic	0.667695
Log-likelihood	-1070.6

Test for differing group intercepts –

  Null hypothesis: The groups have a common intercept

  Test statistic: $F(9, 188) = 48.9915$

  with p-value = $P(F(9, 188) > 48.9915) = 1.11131\text{e-}044$

By default, **gretl** will test the hypothesis that the fixed effects are the same for each individual. If you do not reject this hypothesis, then you can estimate the model using pooled least squares as discussed in the previous section. The test statistic has an F(9,188) sampling distribution if the pooled least squares model is the correct one. The computed value is 48.99 and the p-value is less than 5%, therefore we would reject the pooled least squares formulation in favor of the fixed effect model in this example.

In this formulation you are assuming that the errors of your model are homoscedastic within each firm and across firms, and that there is no contemporaneous correlation across firms. **Gretl** allows you to compute standard errors that are robust to the homoscedasticity assumption. Simply use the `--robust` option in the panel regression. i.e., `panel Inv const V K --robust`. This option computes the *cluster* standard errors that are discussed in Chapter 15 of Hill et al. (2008).

15.2.3 Random Effects

Gretl also estimates random effects models using the `panel` command. In the random effects model, the individual firm differences are thought to represent random variation about some average intercept for the individual in the sample. Rather than estimate a separate fixed effect for each firm, you estimate an overall intercept that represents this average. Implicitly, the regression function for the sample firms vary randomly around this average.

The variability of the individual effects is captured by a new parameter, σ_u^2. The larger this parameter is, the more variation you find in the implicit regression functions for the firms.

Once again, the model is based on equation (15.2). The difference is that $\beta_{1i} = \bar{\beta}_1 + u_i$ where u_i represents random variation. The model becomes:

$$y_{it} = \bar{\beta}_1 + u_i + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it} \tag{15.7}$$

The new parameter, σ_u^2, is just the variance of the random effect, u_i. If $\sigma_u^2 = 0$ then the effects are "fixed" and you can use the fixed effects estimator if the effects are indeed different across firms or the pooled estimator if they are not.

To estimate the model, using the Grunfeld data use the script

```
open c:\userdata\gretl\data\poe\grunfeld.gdt
smpl full
panel Inv const V K --random-effects
```

This yields

Model 3: Random-effects (GLS) estimates using 200 observations
Dependent variable: Inv

Variable	Coefficient	Std. Error	t-statistic	p-value
const	−57.872	28.8747	−2.0043	0.0464
V	0.109478	0.0104895	10.4369	0.0000
K	0.308694	0.0171938	17.9538	0.0000

Mean of dependent variable	145.907
S.D. of dependent variable	216.886
Sum of squared residuals	4.28309e+08
Standard error of residuals ($\hat{\sigma}$)	1470.77
$\hat{\sigma}_{\varepsilon}^2$	2781.14
$\hat{\sigma}_u^2$	7218.23
θ	0.861203
Akaike information criterion	3488.98
Schwarz Bayesian criterion	3498.88
Hannan–Quinn criterion	3492.99

Breusch-Pagan test –
  Null hypothesis: Variance of the unit-specific error = 0
  Asymptotic test statistic: $\chi_1^2 = 797.781$
  with p-value = 1.63899e-175

Hausman test –
  Null hypothesis: GLS estimates are consistent
  Asymptotic test statistic: $\chi_2^2 = 2.2155$
  with p-value = 0.330301

Gretl tests the null hypothesis $\sigma_u^2 = 0$ against the alternative $\sigma_u^2 > 0$ by default and is referred to as the **Breusch-Pagan test**.

The **Hausman test** is a test of the null hypothesis that the random effects are indeed random. If they are random then they should not be correlated with any of your other regressors. If they are correlated with other regressors, then you should use the fixed effects estimator to obtain consistent parameter estimates of your slopes.

In the Grunfeld data, a p-value less than 5% indicates that the Breusch-Pagan test rejects the hypothesis that the effects are not random (in other words, the effects **are** random). For the Hausman test, the p-value is greater than 5%. The random effects do not appear to be correlated with the regressors and random effects can be used.

15.2.4 SUR

The acronym SUR stands for **seemingly unrelated regression equations**. SUR is another way of estimating panel data models that are long (large T) but not wide (small N). More generally though, it is used to estimate systems of equations that do not necessarily have any parameters in common and are hence unrelated. In the SUR framework, each firm in your sample is parametrically different; each firm has its own regression function, i.e., different intercept and slopes. Firms are not totally unrelated, however. In this model the firms are linked by what is **not** included in the regression rather than by what is. The firms are thus related by unobserved factors and SUR requires us to specify how these omitted factors are linked in the system's error structure.

In the basic SUR model, the errors are assumed to be homoscedastic and linearly independent within each equation, or in our case, each firm. The error of each equation may have its own variance. Most importantly, each equation (firm) is correlated with the others in the same time period. The latter assumption is called **contemporaneous correlation**, and it is this property that sets SUR apart from other models.

In the context of the two firm Grunfeld model in (15.4) this would mean that $Var[e_{GE,t}] = \sigma^2_{GE}; Var[e_{W,t}] = \sigma^2_W$; $Cov(e_{GE,t}, e_{W,t}) = \sigma_{GE,W}$ for all time periods; and $Cov(e_{i,t}, e_{i,s}) = 0$ for $t \neq s$ for each firm, $i = GE, W$. So in the SUR model you essentially have to estimate a variance for each individual and a covariance between each pair of individuals. These are then used to construct a generalized least squares estimator of the equations parameters.

Even though SUR requires a T and an N dimemsion, it is not specifically a panel technique. This is because the equations in an SUR system may be modeling different behaviors for a single individual rather than the same behavior for several individuals. As mentioned before, it is best used when panels are long and narrow since this gives you more observations to estimate the equations variances and the cross equation covariances. More time observations reduces the sampling variation associated with these estimates, which in turn improves the performance of the feasible generalized least squares estimator. If your panel dataset has a very large number of individuals and only a few years, then FGLS may not perform very well in a statistical sense. In the two firm Grunfeld example, N=2 and T=20 so we needn't worry about this warning too much, although the asymptotic inferences are based on T (and not N) being infinite.

When estimating an SUR model, the data have to be arranged in a slightly different way than in the preceding panel examples. Basically, they need to be arranged as a time series (not a panel) with different firms variables listed separately. Hill et al. (2008) have done this for us in the *grunfeld2.gdt* data set. The **gretl** script to estimate the two firm SUR model using this data is

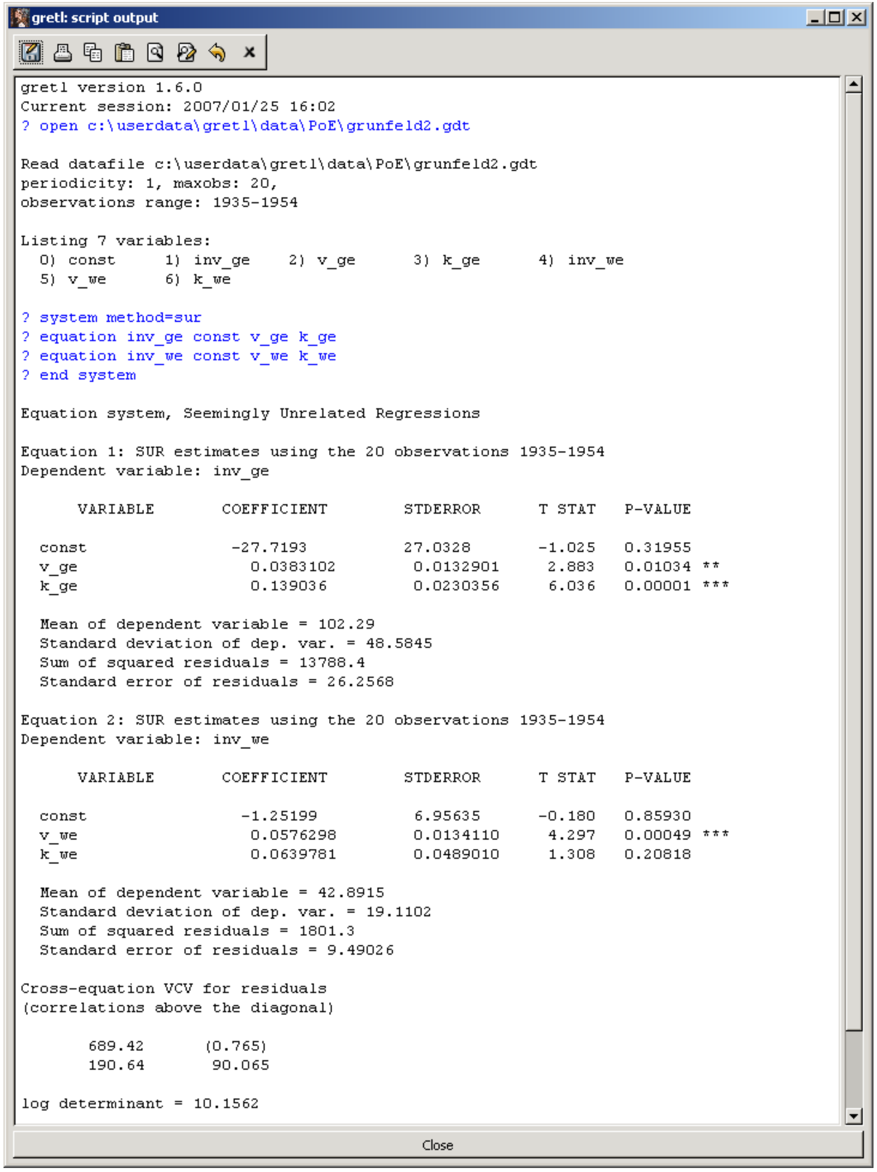
```
open c:\userdata\gretl\data\poe\grunfeld2.gdt

system name="Grunfeld"
equation inv_ge const v_ge k_ge
equation inv_we const v_we k_we
end system

estimate "Grunfeld" method=sur
```

Since SUR is a method of estimating a system of equations (just like you did in chapter 11), the same syntax is used here. It consists of a block of code that starts with the `system name="Grunfeld"` line. One advantage naming your system is that results are attached to it and you can perform subsequent computations based on them. For instance, with a saved set of equations you can impose restrictions on a single equation in the model or impose restrictions across equations. This is accomplished using the `restrict` statement.

Following the system name, each equation is put on a separate line. Notice that each equation is identified using `equation` which is followed by the dependent variable and then the independent variables which include a constant. Close the system block using the `end system` command. The system is then estimated using the line `estimate "Grunfeld" method=sur`. Executing this script yields Figure 15.3 below.

The test to determine whether there is sufficient contemporaneous correlation is simple to do from the standard output. Recall from *POE* that

Figure 15.3: The results from the two firm model estimated as seemingly unrelated regression equations



```
gretl version 1.6.0
Current session: 2007/01/25 16:02
? open c:\userdata\gretl\data\PoE\grunfeld2.gdt

Read datafile c:\userdata\gretl\data\PoE\grunfeld2.gdt
periodicity: 1, maxobs: 20,
observations range: 1935-1954

Listing 7 variables:
  0) const      1) inv_ge     2) v_ge       3) k_ge       4) inv_we
  5) v_we       6) k_we

? system method=sur
? equation inv_ge const v_ge k_ge
? equation inv_we const v_we k_we
? end system

Equation system, Seemingly Unrelated Regressions

Equation 1: SUR estimates using the 20 observations 1935-1954
Dependent variable: inv_ge

      VARIABLE       COEFFICIENT        STDERROR      T STAT    P-VALUE

  const               -27.7193          27.0328      -1.025    0.31955
  v_ge                  0.0383102        0.0132901     2.883    0.01034 **
  k_ge                  0.139036         0.0230356     6.036    0.00001 ***

  Mean of dependent variable = 102.29
  Standard deviation of dep. var. = 48.5845
  Sum of squared residuals = 13788.4
  Standard error of residuals = 26.2568

Equation 2: SUR estimates using the 20 observations 1935-1954
Dependent variable: inv_we

      VARIABLE       COEFFICIENT        STDERROR      T STAT    P-VALUE

  const                -1.25199          6.95635      -0.180    0.85930
  v_we                  0.0576298        0.0134110     4.297    0.00049 ***
  k_we                  0.0639781        0.0489010     1.308    0.20818

  Mean of dependent variable = 42.8915
  Standard deviation of dep. var. = 19.1102
  Sum of squared residuals = 1801.3
  Standard error of residuals = 9.49026

Cross-equation VCV for residuals
(correlations above the diagonal)

       689.42       (0.765)
       190.64        90.065

log determinant = 10.1562
```

the test is based on the squared correlation

$$r^2_{GE,W} = \frac{\hat{\sigma^2}_{GE,W}}{\hat{\sigma^2}_{GE}\hat{\sigma^2}_{W}} \tag{15.8}$$

A little caution is required here. The squared correlations are supposed to be computed based on the residuals from the *least squares estimator*, not SUR. The "Cross-equation VCV for residuals" in the output in Figure 15.3 is computed based on SUR residuals. So, you'll need to change the estimation method to `ols` and rerun the script to get the right inputs for this statistic. The new script is:

```
open c:\userdata\gretl\data\poe\grunfeld2.gdt

system name="Grunfeld"
equation inv_ge const v_ge k_ge
equation inv_we const v_we k_we
end system

estimate "Grunfeld" method=ols
```

and the resulting cross-equation variance covariance for the residuals is

```
Cross-equation VCV for residuals
(correlations above the diagonal)

       777.45      (0.729)
       207.59       104.31
```

Then you compute

$$r^2_{GE,W} = \frac{207.59^2}{(777.45)(104.31)} = 0.729 \tag{15.9}$$

Notice that **gretl** produces this number for you in the upper diagonal of the matrix and places it in parentheses. Using the given computation the test statistic is

$$LM = Tr^2_{GE,W} \xrightarrow{d} \chi^2_{(1)} \tag{15.10}$$

Table 15.1: Script for imposing cross-equation restrictions in an SUR model

```
system name="Grunfeld"
equation inv_ge const v_ge k_ge
equation inv_we const v_we k_we
end system

restrict "Grunfeld"
b[1,1]-b[2,1]=0
b[1,2]-b[2,2]=0
b[1,3]-b[2,3]=0
end restrict

estimate "Grunfeld" method=sur --geomean
```

provided the null hypothesis of no correlation is true. The arithmetic is $(20 * 0.729) = 14.58$

The `restrict` command can be used to impose the cross-equation restrictions on a system of equations that has been previously defined and named. The set of restrictions is started with the keyword `restrict` and terminated with `end restrict`. Some additional details and examples of how to use the `restrict` command are given in section 6.1. Each restriction in the set is expressed as an equation. Put the linear combination of parameters to be tested on the left-hand-side of the equality and a numeric value on the right. Parameters are referenced using `b[i,j]` where i refers to the equation number in the system, and j the parameter number. So, to equate the intercepts in equations one and two use the statement

$$b[1,1] - b[2,1] = 0 \tag{15.11}$$

The full syntax for testing the full set of cross-equation restrictions

$$\beta_{1,GE} = \beta_{1,W}, \quad \beta_{2,GE} = \beta_{2,W}, \quad \beta_{3,GE} = \beta_{3,W} \tag{15.12}$$

on equation 15.4 is shown in Table 15.1: **Gretl** estimates the two equation SUR subject to the restrictions. Then it computes an F-statistic of the null hypothesis that the restrictions are true versus the alternative that at least one of them is not true. It returns the computed F-statistic and its p-value.

A p-value less than the desired level of significance leads to a rejection of the hypothesis.

The **gretl** output from this test procedure is

```
F test for the specified restrictions:
  F(3,34) = 2.92224 with p-value 0.0478934
```

which matches the results in the text. At the 5% level of significance, the equality of the two equations is rejected.

15.3 NLS Example

Hill et al. (2008) provides a subset of National Longitudinal Survey which is conducted by the US Department of Labor. The database includes observations on women, who in 1968, were between the ages of 14 and 24. It then follows them through time, recording various aspects of their lives annually until 1973 and bi-annually afterwards. Our sample consists of 716 women observed in 5 years (1982, 1983, 1985, 1987 and 1988). The panel is balanced and there are 3580 total observations.

Two models are considered in equations (15.13) and (15.14) below.

$$\begin{aligned} ln(WAGE)_{it} = \beta_{1i} + \beta_2 exper_{it} + \beta_3 exper^2_{it} + \beta_4 tenure_{it} \\ + \beta_5 tenure^2_{it} + \beta_6 south_{it} + \beta_7 union_{it} + e_{it} \end{aligned} \tag{15.13}$$

$$\begin{aligned} ln(WAGE)_{it} = \beta_{1i} + \beta_2 exper_{it} + \beta_3 exper^2_{it} + \beta_4 tenure_{it} \\ + \beta_5 tenure^2_{it} + \beta_6 south_{it} + \beta_7 union_{it} + \\ + \beta_8 black_{it} + \beta_9 educ_{it} + e_{it} \end{aligned} \tag{15.14}$$

The first model (15.13) is estimated using fixed effects. Race (`black`) and education (`educ`) are added to form the model in (15.14). Since these variables do not change for individuals in the sample, their influences cannot be estimated using fixed effects. So, this equation is estimated using random effects using the script below:

```
open c:\userdata\gretl\data\poe\nels_panel.gdt
panel lwage const exper exper2 tenure tenure2 south union
panel lwage const exper exper2 tenure tenure2 south union \
  black educ --random-effects
```

Notice that in the random effects line a backslash follows the variable `union`. This is the continuation command, which tells **gretl** that the command continues on the next line. The results, in tabular form, are in Table 15.2 below. Wisely, **gretl** has omitted the R^2 for the random effects model. Recall that R^2 is only suitable for linear models estimated using OLS, which is the case for one-way fixed effects.

The complete set of results of random effects estimation is shown in the table 15.3 below. The estimate of $\hat{\sigma}_\varepsilon = \sqrt{0.0380681} = 0.1951$. Also, the result of the LM test for the randomness of the individual effects ($\sigma_u^2 > 0$) and the Hausman test of the independence of the random effects from the regressors matches that of your text.

The conclusion from these tests is that even though there is evidence of random effects (LM rejects), the random effects are not independent of the regressors; GLS estimator will be inconsistent and you'll have to use the fixed effects estimator of the smaller model. As a result, you will be unable to determine the effects of education and race on wages.

There is one difference between the **gretl** results and those from *POE*, namely $\hat{\sigma}_u = \sqrt{0.115887} = 0.3404$ from **gretl** is slightly larger than that obtained by Hill et al. (2008) using Stata. This is not too surprising since there are several ways to compute σ_u^2. The difference apparently has little effect on the computation of the coefficients and standard errors since these are fairly close matches to those in the text.

Table 15.2: Fixed Effects and Random Effects estimates for equations (15.13) and (15.14), respectively.

Model Estimates
Dependent variable: lwage

	Fixed Effects	Random Effects
exper	0.04108^{**}	0.04362^{**}
	(0.006620)	(0.006358)
exper2	-0.0004091	-0.0005610^{**}
	(0.0002733)	(0.0002626)
tenure	0.01391^{**}	0.01415^{**}
	(0.003278)	(0.003167)
tenure2	-0.0008962^{**}	-0.0007553^{**}
	(0.0002059)	(0.0001947)
south	-0.01632	-0.08181^{**}
	(0.03615)	(0.02241)
union	0.06370^{**}	0.08024^{**}
	(0.01425)	(0.01321)
const		0.5339^{**}
		(0.07988)
black		-0.1167^{**}
		(0.03021)
educ		0.07325^{**}
		(0.005331)
n	3580	3580
$\bar{R}^2$	0.8236	
ℓ	1173.78	-6999.08

Standard errors in parentheses
* indicates significance at the 10 percent level
** indicates significance at the 5 percent level

Table 15.3: Random-effects (GLS) estimates using 3580 observations

Dependent variable: lwage

Variable	Coefficient	Std. Error	t-statistic	p-value
const	0.533929	0.0798828	6.6839	0.0000
exper	0.0436170	0.00635758	6.8606	0.0000
exper2	−0.000560959	0.000262607	−2.1361	0.0327
tenure	0.0141541	0.00316656	4.4699	0.0000
tenure2	−0.000755342	0.000194726	−3.8790	0.0001
south	−0.0818117	0.0224109	−3.6505	0.0003
union	0.0802353	0.0132132	6.0724	0.0000
black	−0.116737	0.0302087	−3.8643	0.0001
educ	0.0732536	0.00533076	13.7417	0.0000

Mean of dependent variable	1.91824
S.D. of dependent variable	0.464607
Sum of squared residuals	10460.3
Standard error of residuals ($\hat{\sigma}$)	1.71126
$\hat{\sigma}^2_{\varepsilon}$	0.0380681
$\hat{\sigma}^2_{u}$	0.115887
θ	0.743683
Akaike information criterion	14016.2
Schwarz Bayesian criterion	14071.8
Hannan–Quinn criterion	14036.0

Breusch-Pagan test –
Null hypothesis: Variance of the unit-specific error = 0
Asymptotic test statistic: $\chi^2_1 = 3859.28$
with p-value = 0

Hausman test –
Null hypothesis: GLS estimates are consistent
Asymptotic test statistic: $\chi^2_6 = 20.7252$
with p-value = 0.00205521

15.4 Script

```
open c:\userdata\gretl\data\poe\grunfeld.gdt
smpl firm = 3 | firm = 8 --restrict
ols Inv const V K
lmtest --panel

open c:\userdata\gretl\data\poe\grunfeld.gdt
smpl full
panel Inv const V K

open c:\userdata\gretl\data\poe\grunfeld.gdt
smpl full
panel Inv const V K --random-effects


open c:\userdata\gretl\data\poe\grunfeld2.gdt
system name="Grunfeld"
equation inv_ge const v_ge k_ge
equation inv_we const v_we k_we
end system

estimate "Grunfeld" method=sur --geomean

restrict "Grunfeld"
b[1,1]-b[2,1]=0
b[1,2]-b[2,2]=0
b[1,3]-b[2,3]=0
end restrict

estimate "Grunfeld" method=sur --geomean

system name="Grunfeld"
equation inv_ge const v_ge k_ge
equation inv_we const v_we k_we
end system

estimate "Grunfeld" method=ols --geomean
```

Chapter 16

Qualitative and Limited Dependent Variable Models

16.1 Probit

There are many things in economics that cannot be meaningfully quantified. How you vote in an election, whether you go to graduate school, whether you work for pay, or what major you choose has no natural way of being quantified. Each of these expresses a *quality* or *condition* you possess. Models of how these decisions are determined by other variables are called **qualitative choice** or **qualitative variable** models.

In a **binary choice** model, the decision you wish to model has only two possible outcomes. You assign artificial numbers to each outcome so that you can do further analysis. In a binary choice model it is conventional to assign '1' to the variable if it possesses a particular quality or if a condition exists and '0' otherwise. Thus, your dependent variable is

$$y_t = \begin{cases} 1 & \text{if individual t has the quality} \\ 0 & \text{if not.} \end{cases}$$

The probit statistical model expresses the probability p that your dependent variable takes the value 1 as a function of your independent variables.

$$P[(y_t|x_t) = 1] = \Phi(\beta_1 + \beta_2 x_t) \tag{16.1}$$

where Φ is the cumulative normal probability distribution (cdf). Estimating this model using maximum likelihood is very simple since the MLE of the probit model is already programmed into **gretl**. The syntax for a script is the same as for linear regression except you use the `probit` command in place of `ols`. The following script estimates how the difference in travel time between bus and auto affects the probability of driving a car. The dependent variable (*auto*) is equal to 1 if travel is by car, and *dtime* is (bus time - auto time).

```
open c:\userdata\gretl\data\poe\transport.gdt

probit auto const dtime
genr p1 = $coeff(const)+$coeff(dtime)*20
genr dt = dnorm(p1)*$coeff(dtime)
genr p2 = cnorm($coeff(const)+$coeff(dtime)*30)
```

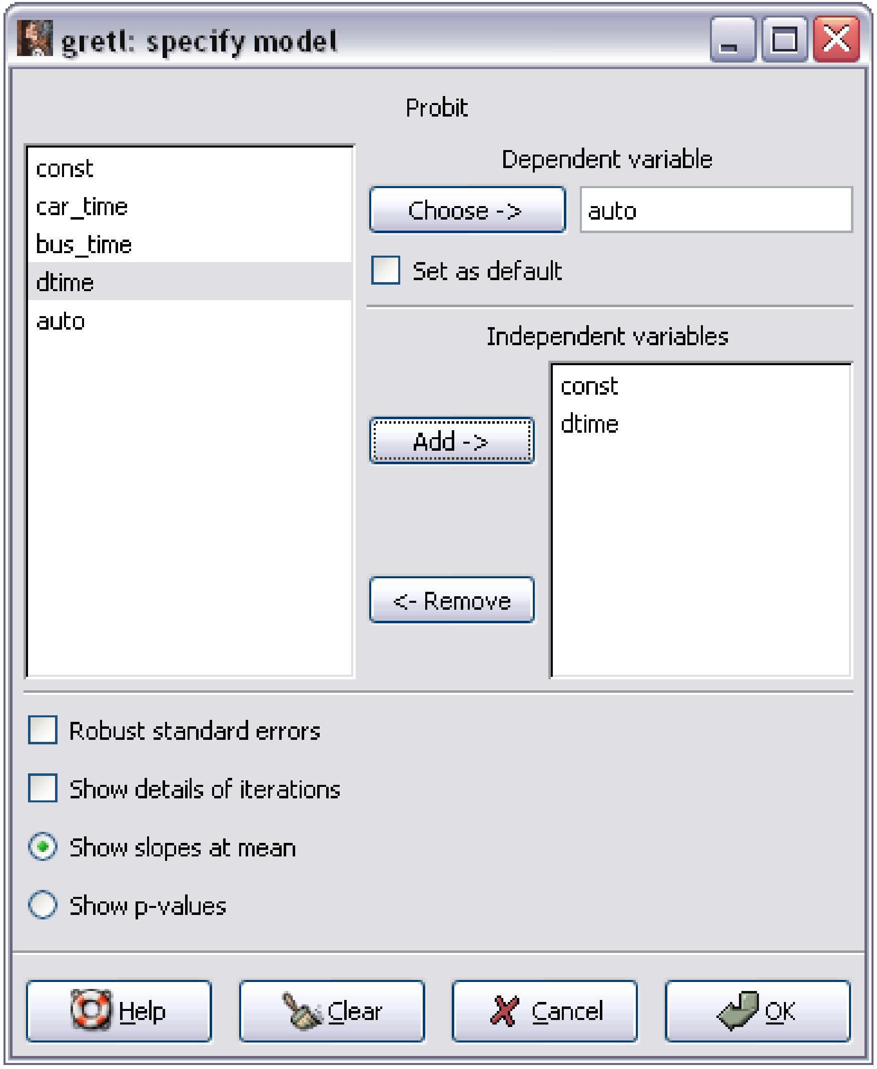
The second line computes the predicted value of the index ($\beta_1 + \beta_2 dtime$) when $dtime = 20$ using the estimates from the probit MLE. The next line computes the marginal affect on the probability of driving if you increase the difference in travel time by one minute when $dtime = 20$, i.e., $\phi(\beta_1 + \beta_2 dtime)\beta_2$. The `dnorm` function in **gretl** computes $\phi()$, the normal pdf evaluated at the argument in parentheses. The last line computes the estimated probability of driving, given that it takes 30 minutes longer to ride the bus. This computation requires `cnorm`, which computes the cumulative normal cdf, $\Phi()$.

The results are:

```
        p1 =  0.535545
        dt =  0.0103690
        p2 =  0.798292
```

Of course, you can also access the probit estimator from the pull-down menus using `Model>Nonlinear models>Probit`. The dialog box (Figure

Figure 16.1: Use `Model>Nonlinear models>Probit` to open the Probit model's dialog box.



gretl: specify model

Probit

const
car_time
bus_time
dtime
auto

Dependent variable: auto

□ Set as default

Independent variables:
const
dtime

□ Robust standard errors

□ Show details of iterations

● Show slopes at mean

○ Show p-values

16.1 looks very similar to the one for linear regression, except it gives you a new option to view the details of the iterations.

Whether you use the script or the dialog box, you will get the following results:

Model 2: Probit estimates using the 21 observations 1–21
Dependent variable: auto

Variable	Coefficient	Std. Error	t-statistic	p-value
const	−0.0644342	0.399244	−0.1614	0.8718
dtime	0.0299989	0.0102867	2.9163	0.0035

Mean of auto = 0.476
Number of cases 'correctly predicted' = 19 (90.5 percent)
McFadden's pseudo-R^2 = 0.575761
$f(\beta' x)$ at mean of independent vars = 0.397
Log-likelihood = −6.16516
Likelihood ratio test: $\chi^2_1 = 16.734$ (p-value 0.000043)

Several other statistics are computed. They include a measure of fit (McFadden's pseudo-R^2), the value of $f(\beta' x)$ at mean of independent variables, and a test statistic for the null hypothesis that the coefficients on the independent variables (but not the constant) are jointly zero; this corresponds to the overall F-statistic of regression significance in Chapter 6.

16.2 Multinomial Logit

Gretl doesn't include a routine to estimate multinomial logit (MNL) using maximum likelihood, so the alternatives are either (1) use **gretl**'s maximum likelihood module to estimate your own or (2) use another piece of software! In this section I'll give you an idea of how to estimate this model using **gretl** and in section 16.9 I'll show you how to estimate the model in another free software called *R*.

Although **Gretl** doesn't include a specific function for estimating MNL, it can do it with a little effort. **Gretl** contains two things that make this reasonably easy to do. First, it includes a module for maximizing likelihood functions (see Chapter 14 for other examples). To use the `mle` function, the user has to write a program using **gretl**'s language to compute a model's log-likelihood given the data. The parameters of the log-likelihood must be declared and given starting values (using the `genr` command). If you want, you can specify the derivatives of the log-likelihood function with respect to each of the parameters; if analytical derivatives are not supplied, a numerical approximation is computed. In many instances, the numerical approximations work quite well. In the event that the computations based on numerical derivatives fail, you may have to specify analytical ones to make the program work.

Gretl also includes a way for users to define new functions. These are placed in a script that can be run from the script editor. Once a function is written, it can often be reused with ease. Functions can also be published and shared via **gretl**'s database server. The **Gretl** Users Guide will have the most up-to-date information on the use of functions and I suggest you look there for further information. What appears below is taken from the **gretl** Users Guide. The example for MNL for *POE* requires only a slight modification in order for the program to run with our dataset.

Functions must be defined before they are called (used). The syntax for defining a function looks like this

```
function name(inputs)
    function body
end function
```

You select a name to give your function. Keep it under 32 characters and start the name with a character. The inputs usually include the data and any parameters included in the log-likelihood. The parameters can be in matrix or scalar form.

The multinomial logit function, which can be found in the **Gretl** User's Guide, is defined

```
function mlogitlogprobs(series y, matrix X, matrix theta)
```

```
  scalar n = max(y)
  scalar k = cols(X)
  matrix b = mshape(theta,k,n)
  matrix tmp = X*b
  series ret = -ln(1 + sumr(exp(tmp)))
  loop for i=1..n --quiet
    series x = tmp[,i]
    ret += (y=$i) ? x : 0
  end loop
return series ret
end function
```

The function is named `mlogitlogprobs` and has three arguments. The first is the dependent variable, `series y`, the second is set of independent variables contained in `matrix X`, and the last is the matrix of parameters, called `theta`. Scalars in the function are defined for sample size, number of regressors, and the coefficients are placed in an nxk array in order to match the dimensionality of the data. The index `tmp=X*b` is created and `ret` returns the log-likelihood function. Don't worry if you can't make sense of this because you should not have to change any of this to estimate MNL with another dataset. That is one of the beauties of defining and using a function.

To use the `mlogitlogprobs` function, you need to know a little about how it works. You will have to get your data into the right form in order for the function to work properly. After loading the data, make sure that the dependent choice variable is in the correct format for the function. The function requires the choices to start at 0. If you list the data, you'll find that `psechoice` is coded 1, 2, 3 instead of the required 0, 1, 2. So the next step is to subtract 1 from `psechoice`.

Create the matrix of regressors, define the number of regressors and use these to initialize the matrix of coefficients, `theta`. Then list the dependent variable, matrix of independent variables, and the initialized parameter matrix in the function. Click the run button and wait for the result.

```
open c:\userdata\gretl\data\poe\nels_small.gdt

# dep. var. must be 0-based
```

```
psechoice = psechoice-1

#put regressors into a matrix called X
smpl full matrix X = { grades const }

scalar k = cols(X)
matrix theta = zeros(2*k, 1)
mle loglik = mlogitlogprobs(psechoice,X,theta)
params theta
end mle --verbose --hessian
```

The only changes I had to make to the original example in the **Gretl** User Guide are (1) change the dataset (2) change the dependent variable to `psechoice` (3) put the desired regressors into X and (4) make sure the function contains the desired variables.

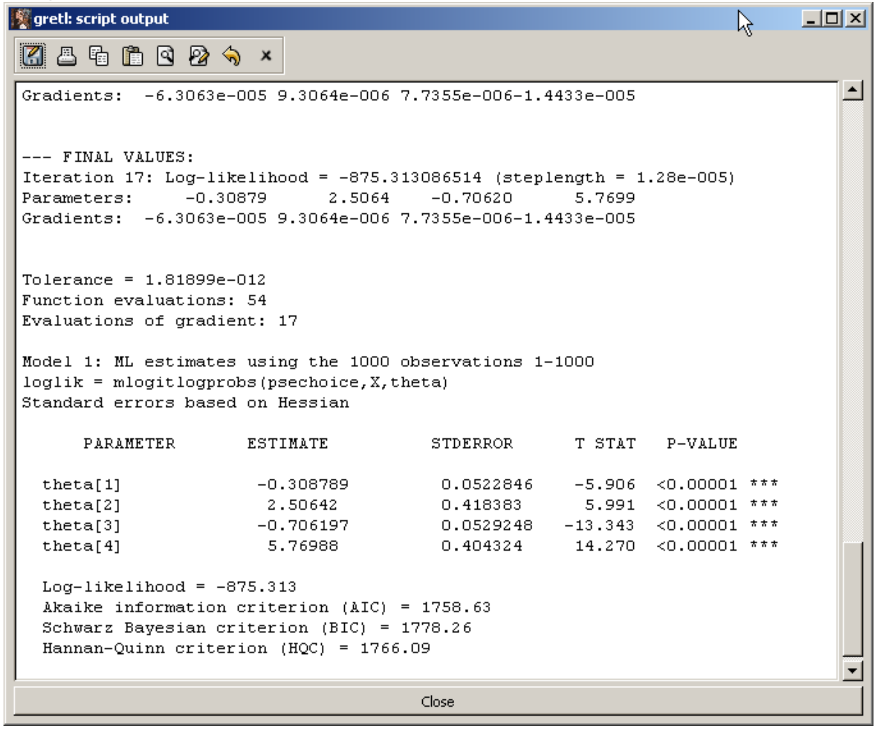
The results from the program appear below in Figure 16.2. Wow! They match those in *POE* and are dirt simple to obtain![1] Finally, if you want to produce the probabilities and marginal effects, you can use the estimates **gretl** has stored in the 4x1 vector called `theta`. They are available for further computations using the scalar or matrix functions. The script at the end of this chapter computes the predicted probabilities and marginal effects in Table 16.3 or *POE*. Although the predictions are close to those in *POE*, the marginal effects are somewhat different.

16.3 Conditional Logit

Gretl doesn't include a routine to estimate conditional logit, so you'll want to use *R* to estimate this model. See sections 16.9 and 16.9.2 for details.

[1] Thanks to Jack Lucchetti for pointing this out to me.

Figure 16.2: These results are from a **gretl** function taken from the **Gretl** Users Guide.



```
Gradients:  -6.3063e-005 9.3064e-006 7.7355e-006-1.4433e-005


--- FINAL VALUES:
Iteration 17: Log-likelihood = -875.313086514 (steplength = 1.28e-005)
Parameters:     -0.30879      2.5064    -0.70620      5.7699
Gradients:  -6.3063e-005 9.3064e-006 7.7355e-006-1.4433e-005


Tolerance = 1.81899e-012
Function evaluations: 54
Evaluations of gradient: 17

Model 1: ML estimates using the 1000 observations 1-1000
loglik = mlogitlogprobs(psechoice,X,theta)
Standard errors based on Hessian

      PARAMETER       ESTIMATE          STDERROR      T STAT   P-VALUE

  theta[1]             -0.308789         0.0522846    -5.906  <0.00001 ***
  theta[2]              2.50642          0.418383      5.991  <0.00001 ***
  theta[3]             -0.706197         0.0529248   -13.343  <0.00001 ***
  theta[4]              5.76988          0.404324     14.270  <0.00001 ***

  Log-likelihood = -875.313
  Akaike information criterion (AIC) = 1758.63
  Schwarz Bayesian criterion (BIC) = 1778.26
  Hannan-Quinn criterion (HQC) = 1766.09
```

16.4 Ordered Probit

In this example, the probability of attending no college, a 2 year college, and a 4 year college are modeled as a function of a student's grades. In principle, we would expect that those with higher grades to be more likely to attend a 4 year college and less likely to skip college altogether. In the dataset, grades are measured on a scale of 1 to 13, with 1 being the highest. That means that if higher grades increase the probability of going to a 4 year college, the coefficient on grades will be *negative*. The probabilities are modeled using the normal distribution in this model where the outcomes represent increasing levels of difficulty.

We can use **gretl** to estimate the ordered probit model because its `probit` command actually handles multinomial choice as well as binomial choice. Open the `nels_small.gdt` data

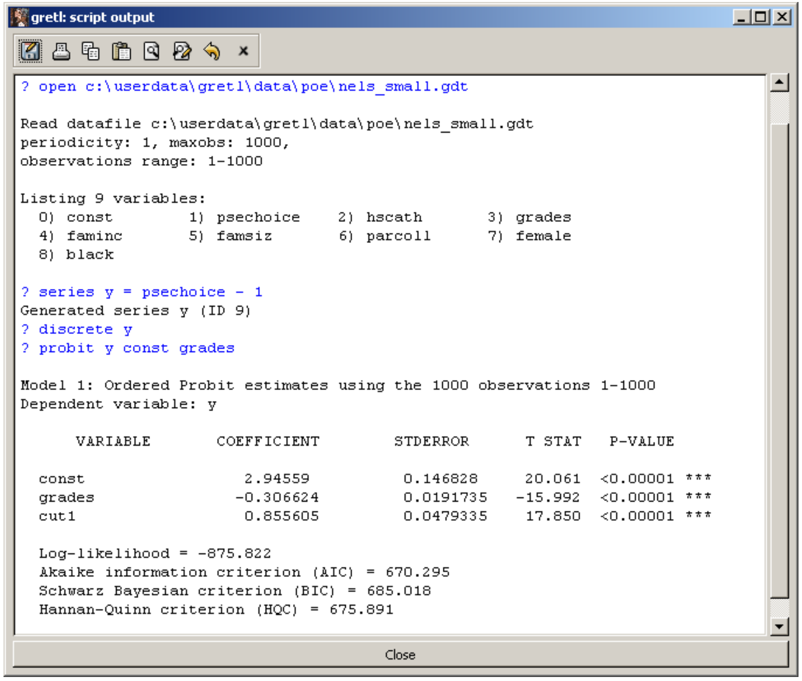
```
open c:\userdata\gretl\data\poe\nels_small.gdt
```

The minimum choice must be coded as a zero. Once `psechoice` is recoded in this way, the usual probit syntax works.

```
series y = psechoice - 1
discrete y probit y const grades
```

The results in Figure 16.3 are very much like the ones in *POE* and produced by MCMCpack below. **Gretl** reports the estimates in a different way, but the results are equivalent. In **gretl** the intercept is equal to the parameter $-\mu_1$ in the 'no-intercept' model computed by default in Stata and shown in *POE*. The second cut-off in the no-intercept model is $\mu_2 = -(Intercept{-}\gamma_2)$, where γ_2 is the single threshold in **gretl**. So, these results are nearly identical to those in *POE*.

Figure 16.3: Ordered probit results from the **gretl**'s probit command



```
? open c:\userdata\gretl\data\poe\nels_small.gdt

Read datafile c:\userdata\gretl\data\poe\nels_small.gdt
periodicity: 1, maxobs: 1000,
observations range: 1-1000

Listing 9 variables:
  0) const         1) psechoice    2) hscath       3) grades
  4) faminc        5) famsiz       6) parcoll      7) female
  8) black

? series y = psechoice - 1
Generated series y (ID 9)
? discrete y
? probit y const grades

Model 1: Ordered Probit estimates using the 1000 observations 1-1000
Dependent variable: y

      VARIABLE       COEFFICIENT        STDERROR      T STAT   P-VALUE

  const                 2.94559          0.146828      20.061  <0.00001 ***
  grades               -0.306624         0.0191735    -15.992  <0.00001 ***
  cut1                  0.855605         0.0479335     17.850  <0.00001 ***

  Log-likelihood = -875.822
  Akaike information criterion (AIC) = 670.295
  Schwarz Bayesian criterion (BIC) = 685.018
  Hannan-Quinn criterion (HQC) = 675.891
```

16.5 Poisson Regression

When the dependent variable in a regression model is a count of the number of occurrences of an event you'll want to use the poisson regression model. In these models, the dependent variable is a nonnegative integer, (i.e., $y = 0, 1, \ldots$), which represent the number of occurrences of a particular event. The probability of a given number of occurrences is modeled as a function of independent variables.

$$P(Y = y|x) = \frac{e^{-\lambda}\lambda^y}{y!} \; y = 0, 1, 2, \ldots \tag{16.2}$$

where $\lambda = \beta_1 + \beta_2 x$ is the regression function.

Estimating this model using maximum likelihood is very simple since the MLE of the poisson regression model is already programmed into **gretl**. The syntax for a script is the same as for linear regression except you use the `possion` command in place of `ols`. This is shown in the following script which replicates the example from your textbook.

A country's total number of medals (medaltot) in the 1988 olympics is modeled as a function of *ln*(*gdp*) and *ln*(*pop*). Of course, you can also access the poisson regression estimator from the pull-down menus using `Model>Nonlinear models>Possion`. To replicate the example in *POE* be sure to restrict the sample to 1988 before estimating the model.

```
open c:\userdata\gretl\data\poe\olympics.gdt

smpl year = 88 --restrict
genr lpop = log(pop)
genr lgdp = log(gdp)
poisson medaltot const lpop lgdp
genr mft = exp($coeff(const)+$coeff(lpop)*median(lpop) \
                  +$coeff(lgdp)*median(lgdp))*$coeff(lgdp)
```

The results for poisson model are:

Model 1: Poisson estimates using the 151 observations 29–179

Dependent variable: medaltot

Variable	Coefficient	Std. Error	t-statistic	p-value
const	−15.887	0.511805	−31.0420	0.0000
lpop	0.180038	0.0322801	5.5773	0.0000
lgdp	0.576603	0.0247217	23.3238	0.0000

Mean of dependent variable	4.88742
S.D. of dependent variable	16.6267
McFadden's pseudo-$R^2 = 0.544658$	
Log-likelihood	−722.33

The marginal effect computation yields:

```
? genr mft = exp($coeff(const)+$coeff(lpop)*median(lpop) \
  +$coeff(lgdp)*median(lgdp))*$coeff(lgdp)
Generated scalar mft (ID 14) = 0.497753
```

16.6 Tobit

The tobit model is essentially just a linear regression where some of the observations on your dependent variable have been censored. A **censored** variable is one that, once it reaches a limit, it is recorded at that limit no matter what it's actual value might be. For instance, anyone earning $1 million or more per year might be recorded in your dataset at the upper limit of $1 million. That means that Bill Gates and the authors of your textbook earn the same amount in the eyes of your dataset (just kidding, fellas). Least squares can be seriously biased in this case and it is wise to use a censored regression model (tobit) to estimate the parameters of the regression when a portion of your sample is censored.

Hill et al. (2008) use tobit to estimate a model of hours worked shown in equation (16.3).

$$hours_i = \beta_1 + \beta_2 * educ_i + \beta_3 exper_i + \beta_4 * age_i + \beta_5 * kidsl6_i + e_i \tag{16.3}$$

using the *mroz.gdt* data. A number of individuals in the sample do not work and report zero hours worked. Estimation of this model in **gretl** is shown in the following script which replicates the example from *POE*. The script estimates a tobit model of hours worked and generates the marginal effect of another year of schooling on the average hours worked.

```
open "c:\userdata\gretl\data\poe\mroz.gdt"
tobit hours const educ exper age kidsl6
```

The results from the basic tobit estimation of the hours worked equation are:

Model 2: Tobit estimates using the 753 observations 1–753
Dependent variable: hours

Variable	Coefficient	Std. Error	*t*-statistic	p-value
const	1349.88	382.729	3.5270	0.0004
educ	73.2910	20.7496	3.5322	0.0004
exper	80.5353	6.58247	12.2348	0.0000
age	−60.767	7.27480	−8.3532	0.0000
kidsl6	−918.91	113.036	−8.1294	0.0000

Mean of dependent variable	740.576
S.D. of dependent variable	871.314
Censored observations	43.2%
$\hat{\sigma}$	1133.70
Log-likelihood	−3827.1

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2_2 = 6.31677$
with p-value = 0.0424944

The marginal effect of another year of schooling on hours worked is

$$\frac{\partial E(Hours_i)}{\partial Educ_i} = \Phi(\widehat{Hours_i})\hat{\beta}_2, \tag{16.4}$$

where $\widehat{Hours}_i$ is the estimated regression function evaluated at the mean levels of education, experience, and age for a person with one child under the age of six. Then, the `cnorm` function is used to compute the normal CDF, $\Phi(\widehat{Hours}_i)$, evaluated at the prediction.

```
genr H_hat = $coeff(const)+$coeff(educ)*mean(educ) \
                          +$coeff(exper)*mean(exper) \
                          +$coeff(age)*mean(age)+$coeff(kidsl6)*1
genr z = cnorm(H_hat/$sigma)
genr pred = z*$coeff(educ)
```

Note, the backward slashes (\) used at the end of the first two lines in the generation of `H_hat` are continuation commands. The backslash at the end of a line tells **gretl** that the next line is a continuation of the current line. This helps keep your programs looking good (and in this case, fitting within the margins of the page!).

Finally, estimates of the restricted sample using least squares and the full sample that includes the zeros for hours worked follow.

```
smpl hours > 0 --restrict
ols hours const educ exper age kidsl6

smpl --full
ols hours const educ exper age kidsl6
```

Notice that the sample is restricted to the positive observations using the `smpl hours > 0 --restrict` statement. To estimate the model using the entire sample the full range is restored using `smpl full`.

16.7 Simulation

You can use **gretl** to show that least squares is biased when the sample is censored using a Monte Carlo simulation. The simulated data are generated

$$y_i^* = -9 + 1x_i + e_i \tag{16.5}$$

where $e_i \sim N(0, 16)$. Then,

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* > 0 \\ 0 & \text{if } y_i^* \leq 0 \end{cases}$$

The $x_i \sim U(0, 20)$, which are held constant in the simulated samples.

The following script demonstrates that least squares is indeed biased when all observations, including the zero ones, are included in the sample. The line `genr yi = y > 0` is a logical statement that generates '1' or '0' depending on whether the statement to the right of the equal sign is true. Thus, a new variable, `yi`, is created that takes the value 1 if `y >0` and is zero if not. When multiplied by `y` in the next statement, the result is a sample, `yc`, censored from below at zero.

```
open c:\userdata\gretl\data\poe\tobit.gdt
smpl 1 200
genr xs = 20*uniform()
loop 1000 --progressive
   genr y = -9 + 1*xs  + 4*normal()
   genr yi = y > 0
   genr yc = y*yi
   ols yc const xs
   genr b1s = $coeff(const)
   genr b2s = $coeff(xs)
   store coeffs.gdt b1s b2s
endloop
```

To repeat the exercise using least squares on only the positive observations use

```
open c:\userdata\gretl\data\poe\tobit.gdt
genr xs = 20*uniform()
genr idx = 1
matrix A = zeros(1000,3)
loop 1000
   smpl --full
   genr y = -9 + 1*xs + 4*normal()
   smpl y > 0 --restrict
   ols y const xs --quiet
   genr b1s = $coeff(const)
   genr b2s = $coeff(xs)
   matrix A[idx,1]=idx
   matrix A[idx,2]=b1s
   matrix A[idx,3]=b2s
   genr idx = idx + 1
endloop

A matrix bb = meanc(A) bb
```

In this case, we are not able to use the `--progressive` loop construct in **gretl**. Without it, **gretl** generates a lot of output to the screen, but it can't be avoided in this case. Using the regular loop function, store each round's estimates in a matrix called `A`. Then, after the loop is finished, `matrix bb = meanc(A)` returns the column means of your matrix. These are the average values of the parameters in the Monte Carlo.

16.8 Selection Bias

Selection bias occurs when your sample is truncated and the cause of that truncation is correlated with your dependent variable. Ignoring the correlation, the model could be estimated using least squares or truncated least squares. In either case, obtaining consistent estimates of the regression parameters is not possible. In this section the basic features of the this model will be presented.

Consider a model consisting of two equations. The first is the **selection equation**, defined

$$z_i^* = \gamma_1 + \gamma_2 w_i + u_i, \quad i = 1, \ldots, N \tag{16.6}$$

where z_i^* is a latent variable, γ_1 and γ_2 are parameters, w_i is an explanatory variable, and u_i is a random disturbance. The latent variable is unobservable, but we do observe the dichotomous variable

$$z_i = \begin{cases} 1 & z_i^* > 0 \\ 0 & \text{otherwise} \end{cases} \tag{16.7}$$

The second equation, called the **regression equation**, is the linear model of interest. It is

$$y_i = \beta_1 + \beta_2 x_i + e_i, \quad i = 1, \ldots, n, \quad N > n \tag{16.8}$$

where y_i is an observable random variable, β_1 and β_2 are parameters, x_i is an exogenous variable, and e_i is a random disturbance. It is assumed that the random disturbances of the two equations are distributed as

$$\begin{bmatrix} u_i \\ e_i \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & \sigma_e^2 \end{pmatrix}\right] \tag{16.9}$$

A selectivity problem arises when y_i is observed only when $z_i = 1$ and $\rho \neq 0$. In this case the ordinary least squares estimator of β in (16.8) is biased and inconsistent. A consistent estimator has been suggested by Heckman (1979) and is commonly referred to as Heckman's two-step estimator, or more simply, *Heckit*. Because the errors are normally distributed, there is also a maximum likelihood estimator of the parameters. **Gretl** includes routines for both.

The two-step (Heckit) estimator is based on conditional mean of y_i given that it is observed

$$E[y_i|z_i > 0] = \beta_1 + \beta_2 x_i + \beta_\lambda \lambda_i \tag{16.10}$$

where

$$\lambda_i = \frac{\phi(\gamma_1 + \gamma_2 w_i)}{\Phi(\gamma_1 + \gamma_2 w_i)} \tag{16.11}$$

is the *inverse Mill's ratio*; $\phi(\cdot)$ is the standard normal probability density function evaluated at the argument, and $\Phi(\cdot)$ is the cumulative density function of the standard normal random variable evaluated at the argument $(\gamma_1 + \gamma_2 w_i)$. The argument $(\gamma_1 + \gamma_2 w_i)$ is commonly referred to as the **index function**. Adding a random disturbance yields:

$$y_i = \beta_1 + \beta_2 x_i + \beta_\lambda \lambda_i + v_i \tag{16.12}$$

It can be shown that (16.12) is heteroskedastic and if λ_i were known (and nonstochastic), then the selectivity corrected model (16.12) could be estimated by generalized least squares. Alternately, the heteroskedastic model (16.12) could be estimated by ordinary least squares, using White's heteroskedasticity consistent covariance estimator (HCCME) for hypothesis testing and the construction of confidence intervals. Unfortunately, λ_i is not known and must be estimated using the sample. The stochastic nature of λ_i in (16.12) makes the automatic use of HCCME in this context inappropriate.

The two-steps of the Heckit estimator consist of

1. estimate the selection equation to obtain $\hat{\gamma}_1$ and $\hat{\gamma}_2$. Use these in equation (16.11) to estimate the inverse Mill's ratio, $\hat{\lambda}_i$.
2. Add $\hat{\lambda}_i$ to the regression model as in equation (16.12) and estimate it using least squares.

The example from *POE* uses the *mroz.gdt* data. The first thing we'll do is to estimate the model ignoring selection bias using least squares on the nonzero observations. Load the data and generate the natural logarithm of wages. Since wages are zero for a portion of the sample, **gretl** will generate an error when you take the natural logs. You can safely ignore it as **gretl** will simply create missing values for the variables that cannot be transformed. Then use the `ols` command to estimate a linear regression on the truncated subset.

```
open c:\userdata\gretl\data\poe\mroz.gdt
genr lwage = log(wage)
regress lwage
const educ exper
```

The results are:

OLS estimates using the 428 observations 1–428
Dependent variable: lwage

Variable	Coefficient	Std. Error	t-statistic	p-value
const	−0.400174	0.190368	−2.1021	0.0361
educ	0.109489	0.0141672	7.7283	0.0000
exper	0.0156736	0.00401907	3.8998	0.0001

Sum of squared residuals	190.195
Standard error of residuals ($\hat{\sigma}$)	0.668968
Unadjusted R^2	0.148358
Adjusted $\bar{R}^2$	0.144350
$F(2, 425)$	37.0180

Notice that the sample has been truncated to include only 428 observations for which hour worked are actually observed. The estimated return to education is about 11%, and the estimated coefficients of both education and experience are statistically significant.

The Heckit procedure takes into account that the decision to work for pay may be correlated with the wage a person earns. It starts by modeling the decision to work and estimating the resulting selection equation using a probit model. The model can contain more than one explanatory variable, w_i, and in this example we have four: a womans age, her years of education, a dummy variable for whether she has children and the marginal tax rate that she would pay upon earnings if employed. Generate a new variable `kids` which is a dummy variable indicating the presence of any kids in the household.

```
generate kids = (kidsl6+kids618>0)
```

Estimate the probit model, generate the index function, and use it to compute the inverse Mill's ratio variable. Finally, estimate the regression including the IMR as an explanatory variable.

```
list X = const educ exper
list W = const mtr age kids educ
probit lfp W
genr ind = $coeff(const) + $coeff(age)*age + \
           $coeff(educ)*educ + $coeff(kids)*kids + $coeff(mtr)*mtr
genr lambda = dnorm(ind)/cnorm(ind)
ols lwage X lambda

heckit lwage X ; lfp W --two-step
```

This script uses a convenient way to accumulate variables into a set using the `list` command first encountered in section 10.3.3. The command `list X = const educ exper` puts the variables contained in `const`, `educ`, and `exper` into a set called `X`. Once defined, the set of variables can be referred to as `X` rather than listing them individually as we've done up to this point. Similarly, we've put the variables from the selection equation into a set called `W`. The `dnorm` and `cnorm` functions return the normal density and normal cumulative density evaluated at the argument, respectively. The results are:

OLS estimates using the 428 observations 1–428
Dependent variable: lwage

Variable	Coefficient	Std. Error	t-statistic	p-value
const	0.810542	0.494472	1.6392	0.1019
educ	0.0584579	0.0238495	2.4511	0.0146
exper	0.0163202	0.00399836	4.0817	0.0001
lamda	−0.866439	0.326986	−2.6498	0.0084

Sum of squared residuals	187.097
Standard error of residuals ($\hat{\sigma}$)	0.664278
Unadjusted R^2	0.162231
Adjusted $\bar{R}^2$	0.156304
$F(3, 424)$	27.3688

Notice that the estimated coefficient of the inverse Mill's ratio is statistically significant, implying that there is a selection bias in the least squares estimator. Also, the estimated return to education has fallen from approximately 11% (which is inconsistently estimated) to approximately 6%. Unfortunately, the usual standard errors do not account for the fact that the inverse Mills ratio is itself an estimated value and so they are not technically correct. To obtain the correct standard errors, you will use **gretl**'s built-in `heckit` command.

The `heckit` command syntax is

```
heckit y const x2 x3 ... xk; z const w2 w3 ... ws --options
```

where `const x2 ... xk` are the k independent variables for the regression and `const w2 ....  ws` are the s independent variables for the selection equation. In this example, we've used the two-step option which mimics the manual procedure employed above, but returns the correct standard errors. If you don't specify the option, **gretl** will estimate the model using maximum likelihood. For the Mroz data the **gretl** command is

```
heckit lwage X ; lfp W --two-step
```

Again, we've used the results from the `list` function, which put the independent variables for the regression into `X` and the variables for the selection equation into `W`.

The results appear below:

Two-step Heckit estimates using the 428 observations 1–428
Dependent variable: lwage

Variable	Coefficient	Std. Error	*t*-statistic	p-value
const	0.810542	0.610798	1.3270	0.1845
educ	0.0584579	0.0296354	1.9726	0.0485
exper	0.0163202	0.00420215	3.8838	0.0001
lambda	−0.866439	0.399284	−2.1700	0.0300
const	1.19230	0.720544	1.6547	0.0980
educ	0.0837753	0.0232050	3.6102	0.0003
kids	−0.313885	0.123711	−2.5372	0.0112
mtr	−1.3938	0.616575	−2.2606	0.0238
age	−0.0206155	0.00704470	−2.9264	0.0034

Mean of dependent variable	1.19017
S.D. of dependent variable	0.723198
Total observations	753
Censored observations	43.2%
$\hat{\sigma}$	0.932559
$\hat{\rho}$	−0.929098

To use the pull-down menus, select `Model>Nonlinear models>Heckit` from **gretl**'s main window. This will reveal the dialog shown in figure 16.4. Enter `lwage` as the dependent variable and the 0/1 variable `lfp` as the selection variable. Then enter the desired independent variables for the regression and selections equations. Finally, select the *2-step estimation* button at the bottom of the dialog box and click OK.

You will notice that the coefficient estimates are identical to the ones produced manually above. However, the standard errors, which are now consistently estimated, have changed. The t-ratio of the coefficient on the inverse Mills ratio, $\hat{\lambda}$, has fallen to -2.17, but it is still significant at the 5% level. **Gretl** also produces the estimates of the selection equation, which appear directly below those for the regression.

16.9 Using R for Qualitative Choice Models

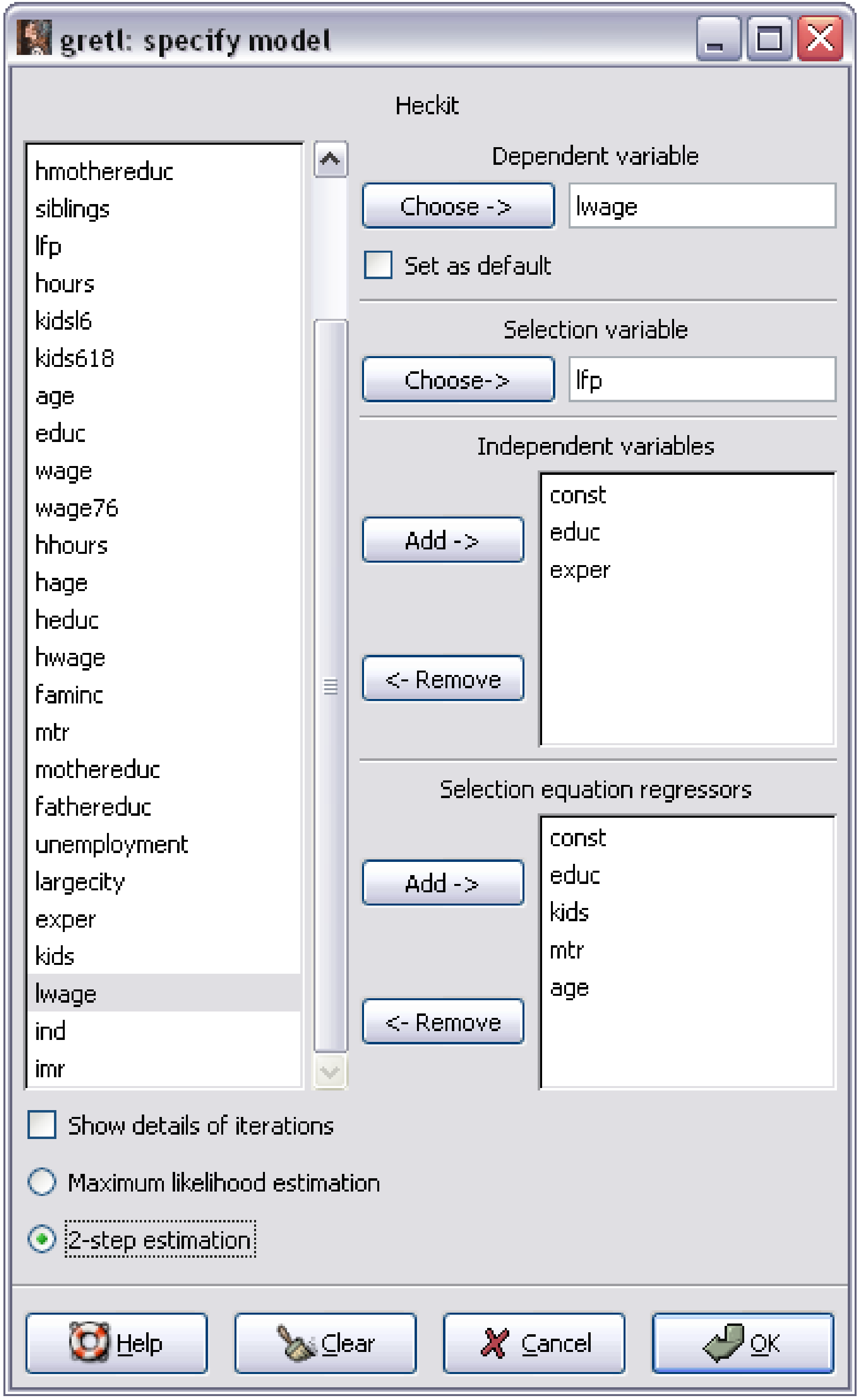
R is a programming language that can be very useful for estimating sophisticated econometric models. In fact, many statistical procedures have been written for *R*. Although **gretl** is reasonably powerful, there are still many things that it won't do. The ability to export **gretl** data into *R* makes it possible to do some fancy econometrics with relative ease.

To do some of these, you'll need a copy of *R* and access to its packages. A package is just a collection of programs written in *R* that make it easier to use for specific tasks. Below, we use a package to read in data saved in Stata's format and another to estimate qualitative choice models.

The *R* software package that is used to estimate qualitative choice models is called `MCMCpack`. MCMCpack stands for Markov Chain Monte Carlo package and it can be used to estimate every qualitative choice model in this chapter. We will just use it to estimate multinomial logit, conditional logit, and ordered probit. So, let's take a quick look at MCMCpack and what it does.

The Markov chain Monte Carlo (MCMC) methods are basic numerical tools that are often used to compute Bayesian estimators. In Bayesian analysis one combines what one already knows (called the *prior*) with what is observed through the sample (the likelihood function) to estimate the pa-

Figure 16.4: Choose `Model>Nonlinear models>Heckit` from **gretl**'s main window to reveal the dialog box for Heckit.



gretl: specify model

Heckit

Dependent variable: lwage

□ Set as default

Selection variable: lfp

Independent variables:

const
educ
exper

Selection equation regressors:

const
educ
kids
mtr
age

□ Show details of iterations

○ Maximum likelihood estimation

● 2-step estimation

rameters of a model. The information available from the sample information is contained in the likelihood function; this is the same likelihood function discussed in your book. If we tell the Bayesian estimator that everything we know is contained in the sample, then the two estimators are essentially the same. That is what happens with MCMCpack under its defaults. The biggest difference is in how the two estimators are computed. The MLE is computed using numerical optimization of the likelihood function, whereas MCMCpack uses simulation to accomplish virtually the same thing. See Lancaster (2004) or Koop (2003) for an introduction to Bayesian methods and its relationship to maximum likelihood.

The MCMC creates a series of estimates–called a (Markov) chain–and that series of estimates has a probability distribution. Under the proper circumstances the probability distribution of the chain will mimic that of the MLE. Various features of the chain can be used as estimates. For instance, the sample mean is used by MCMCpack to estimate the parameters of the multinomial logit model. MCMCpack uses variation within the chain to compute the MLE variance covariance matrix, which is produced using the `summary` command.

One piece of information that you must give to MCMCpack is the desired length of your Markov chain. In the examples here, I chose 20,000, which is the number used in the sample programs included in MCMCpack. Longer chains tend to be more accurate, but take longer to compute. This number gets us pretty close to the MLEs produced by Stata.

16.9.1 Multinomial Logit

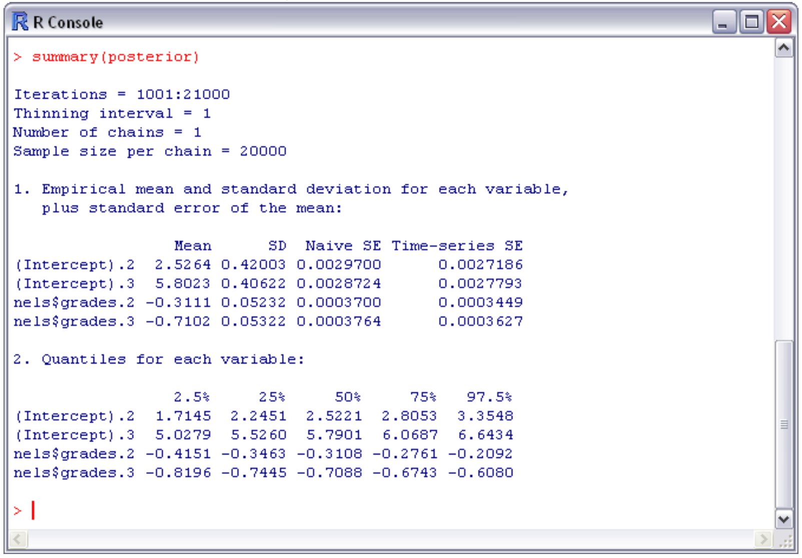
The program code to estimate the multinomial logit example is shown below:

```
library(foreign)
nels <- read.dta("C:/Data/Stata/nels_small.dta")

library(MCMCpack)
posterior <- MCMCmnl(nels$psechoice ~
                        nels$grades, mcmc=20000)
summary(posterior)
```

First, read the Stata dataset `nels_small.dta`[2] into an object we will call `nels`. This requires you to first load the *foreign* library in *R* using the command `library(foreign)`. The `read.dta( )` command reads data in Stata's format; its argument points to the location on your computer where the Stata dataset resides. Refer to sections D.1 and D.2 for a brief introduction to packages and reading Stata datasets in *R*.

Then load `MCMCpack` library into *R*. The next line calls the multinomial logit estimator (`MCMCmnl`). The first argument of `MCMCmnl` is the dependent variable `nels$psechoice`, followed by a ∼, and then the independent variable `nels$grades`. The last argument tells *R* how many simulated values to compute, in this case 20,000. The results of the simulation are stored in the object called `posterior`. Posterior is the name given in the Bayesian literature to the probability distribution of the estimates. The mean or median of this distribution is used as a point estimate (vis-a-vis the MLE). The last line of the program requests the summary statistics from the Markov chain. The results appear in Figure 16.5 In the MNL model, the estimates from MCMCpack are a little different from those produced by Stata, but they are reasonably close.

Figure 16.5: Multinomial logit results from the MCMCmnl estimator in *R*



```
> summary(posterior)

Iterations = 1001:21000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 20000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

                  Mean      SD  Naive SE Time-series SE
(Intercept).2   2.5264 0.42003 0.0029700      0.0027186
(Intercept).3   5.8023 0.40622 0.0028724      0.0027793
nels$grades.2 -0.3111 0.05232 0.0003700      0.0003449
nels$grades.3 -0.7102 0.05322 0.0003764      0.0003627

2. Quantiles for each variable:

                  2.5%     25%     50%     75%   97.5%
(Intercept).2   1.7145  2.2451  2.5221  2.8053  3.3548
(Intercept).3   5.0279  5.5260  5.7901  6.0687  6.6434
nels$grades.2 -0.4151 -0.3463 -0.3108 -0.2761 -0.2092
nels$grades.3 -0.8196 -0.7445 -0.7088 -0.6743 -0.6080

> 
```

[2] This should be available from the *POE* website.

To compute predicted probabilities and marginal effects, you can use the following script for inspiration:

```
library(foreign)
nels <- read.dta("C:/Data/Stata/nels_small.dta")
library(MCMCpack)
posterior <- MCMCmnl(nels$psechoice ~
                         nels$grades, mcmc=20000)
summary(posterior)
summary(nels$grades)

q5 <- quantile(nels$grades,.05)
q50 <- quantile(nels$grades,.5)

b12 <- mean(posterior[,1])
b13 <- mean(posterior[,2])
b22 <- mean(posterior[,3])
b23 <- mean(posterior[,4])

"No College probabilities"
p1_50 <- 1/(1+exp(b12+b22*q50)+exp(b13+b23*q50))
p1_5 <- 1/(1+exp(b12+b22*q5)+exp(b13+b23*q5))
p1_50
p1_5

"Marginal effects, No College"
p2_50 <- 1/(1+exp(b12+b22*(q50+1))+exp(b13+b23*(q50+1)))
p2_5 <- 1/(1+exp(b12+b22*(q5+1))+exp(b13+b23*(q5+1)))
p2_50-p1_50
p2_5-p1_5

"2 Year college probabilities"
p1_50 <-  exp(b12+b22*q50)/(1+exp(b12+b22*q50)+exp(b13+b23*q50))
p1_5 <- exp(b12+b22*q5)/(1+exp(b12+b22*q5)+exp(b13+b23*q5))
p1_50
p1_5

"Marginal effects, 2 Year College"
p2_50 <- exp(b12+b22*(q50+1))/
```

```
            (1+exp(b12+b22*(q50+1))+exp(b13+b23*(q50+1)))
p2_5 <- exp(b12+b22*(q5+1))/
            (1+exp(b12+b22*(q5+1))+exp(b13+b23*(q5+1)))
p2_50-p1_50
p2_5-p1_5

"4 Year college probabilities"
p1_50 <- exp(b13+b23*q50)/(1+exp(b12+b22*q50)+exp(b13+b23*q50))
p1_5 <- exp(b13+b23*q5)/(1+exp(b12+b22*q5)+exp(b13+b23*q5))
p1_50
p1_5

"Marginal effects, 4 Year college"
p2_50 <- exp(b13+b23*(q50+1))/
            (1+exp(b12+b22*(q50+1))+exp(b13+b23*(q50+1)))
p2_5 <- exp(b13+b23*(q5+1))/
            (1+exp(b12+b22*(q5+1))+exp(b13+b23*(q5+1)))
p2_50-p1_50
p2_5-p1_5
```

16.9.2 Conditional Logit

In this example I'll show you how to use MCMCpack in *R* to estimate the conditional logit model.

The first order of business is to get the data into a format that suits *R*. This part is not too pretty, but it works. The data are read in from a Stata dataset using the `read.dta` function that is included in the *foreign* library. The data are assigned (`<-`) to the object `cola`. The `attach(cola)` statement is not necessary, but including it will enable you to call each of the variables in the object `cola` by name. For example, `cola$price` refers to the variable named `price` in the object named `cola`. Once the data object `cola` is attached, you can simply use `price` to refer to the variable without prefixing it with the object to which it belongs (i.e., `cola$`).

The data in the original Stata dataset are arranged

```
> cola[1:12,]
```

```
obs id choice price feature display
1   1   0   1.79   0   0
2   1   0   1.79   0   0
3   1   1   1.79   0   0
4   2   0   1.79   0   0
5   2   0   1.79   0   0
6   2   1   0.89   1   1
7   3   0   1.41   0   0
8   3   0   0.84   0   1
9   3   1   0.89   1   0
10  4   0   1.79   0   0
```

The MCMCpack routine in *R* wants to see it as

```
id  bev.choice pepsi.price sevenup.price  coke.price
 1       3          1.79           1.79        1.79
 2       3          1.79           1.79        0.89
 3       3          1.41           0.84        0.89
 4       3          1.79           1.79        1.33
```

where each line represents an individual, recording his choice of beverage and each of the three prices he faces. The goal then is to reorganize the original dataset so that the relevant information for each individual, which is contained in 3 lines, is condensed into a single row. To simplify the example, I dropped the variables not being used.

Most of the program below is devoted to getting the data into the proper format. The line

```
pepsi.price <- cola$price[seq(1,nrow(cola),by=3)]
```

creates an object called `pepsi.price`. The new object consists of every third observation in `cola$price`, starting with observation 1. The square brackets [] are used to take advantage of *R*'s powerful indexing ability. The function `seq(1,nrow(cola),by=3)` creates a seqence of numbers that start at 1, increment by 3, and extends until the last row of `cola` i.e., [1 3 6 9 . . . 5466]. When used inside the square brackets, these numbers constitute an index of the object's elements that you want to grab. In this case the object is `cola$price`. The `sevenup.price` and `coke.price` lines do the same thing, except their sequences start at 2 and 3, respectively.

The next task is to recode the alternatives to a single variable that takes the value of 1, 2 or 3 depending on a person's choice. For this I used the same technique.

```
pepsi <- cola$choice[seq(1,nrow(cola),by=3)]
sevenup <- 2*cola$choice[seq(2,nrow(cola),by=3)]
coke <- 3*cola$choice[seq(3,nrow(cola),by=3)]
```

The first variable, `pepsi`, takes every third observation of `cola$choice` starting at the first row. The variable will contain a one if the person chooses Pepsi and a zero otherwise since this is how the variable `choice` is coded in the Stata file. The next variable for Sevenup starts at 2 and the sequence again increments by 3. Since Seven-up codes as a 2 the ones and zeros generated by the sequence get multiplied by 2 (to become 2 or 0). Coke is coded as a 3 and its sequence of ones and zeros is multiplied by 3. The three variables are combined into a new one called bev.choice that takes the value of 1,2, or 3 depending on a person's choice of Pepsi, Seven-up, or Coke.

Once the data are arranged, load the MCMCpack library and use MCMCmnl to estimate the model. In the conditional logit model uses choice specific variables. For MCMCmnl choice-specific covariates have to be entered using a special syntax: `choicevar(cvar,"var","choice")` where `cvar` is the name of a variable in data, `var` is the name of the new variable to be created, and `choice` is the level of `bev.choice` that `cvar` corresponds to.

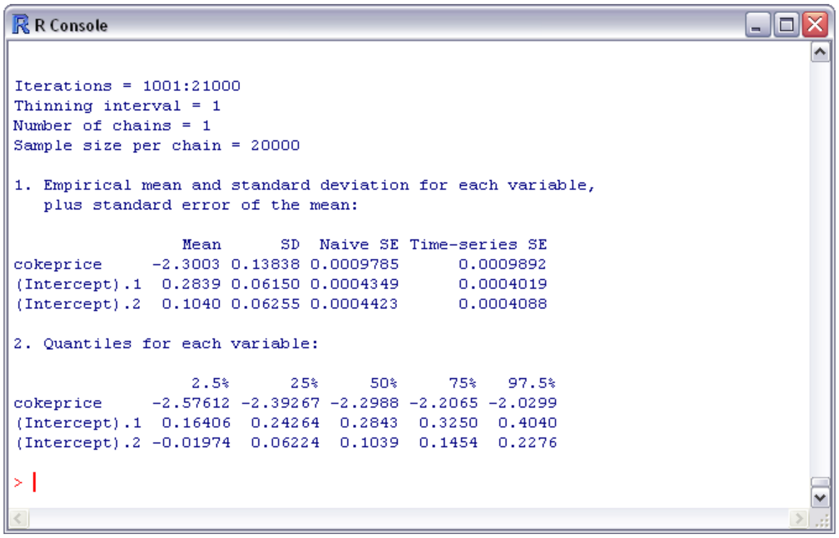
```
library(foreign)
cola <- read.dta("c:/Data/Stata/cola.dta")

attach(cola)  # optional

pepsi.price <- cola$price[seq(1,nrow(cola),by=3)]
sevenup.price <- cola$price[seq(2,nrow(cola),by=3)]
coke.price <- cola$price[seq(3,nrow(cola),by=3)]
```

```
pepsi <- cola$choice[seq(1,nrow(cola),by=3)]
sevenup <- 2*cola$choice[seq(2,nrow(cola),by=3)]
coke <- 3*cola$choice[seq(3,nrow(cola),by=3)]

library(MCMCpack)
posterior <- MCMCmnl(bev.choice ~
               choicevar(coke.price, "cokeprice", "3") +
               choicevar(pepsi.price, "cokeprice", "1") +
               choicevar(sevenup.price, "cokeprice", "2"),
               mcmc=20000, baseline="3")
summary(posterior)
```

In this example, we specified that we want to normalize the conditional logit on the coke choice; this is done using the `baseline="3"` option in MCMCmnl.

The results appear in Figure 16.6.

Figure 16.6: Conditional logit results from the MCMCoprobit estimator in *R*



```
Iterations = 1001:21000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 20000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

                 Mean      SD  Naive SE Time-series SE
cokeprice     -2.3003 0.13838 0.0009785      0.0009892
(Intercept).1  0.2839 0.06150 0.0004349      0.0004019
(Intercept).2  0.1040 0.06255 0.0004423      0.0004088

2. Quantiles for each variable:

                  2.5%      25%     50%     75%   97.5%
cokeprice     -2.57612 -2.39267 -2.2988 -2.2065 -2.0299
(Intercept).1  0.16406  0.24264  0.2843  0.3250  0.4040
(Intercept).2 -0.01974  0.06224  0.1039  0.1454  0.2276

> 
```

16.9.3 Ordered Probit

MCMCpack can also be use to estimate the ordered probit model. It is very easy and the results you get using the Markov chain Monte Carlo simulation method are **very** similar to those from maximizing the likelihood. In principle the maximum likelihood and the simulation estimator used by MCMCpack are asymptotically equivalent.[3] The difference between MCMCpack and Stata's MLE results occurs because the sample sizes for the datasets used is small.

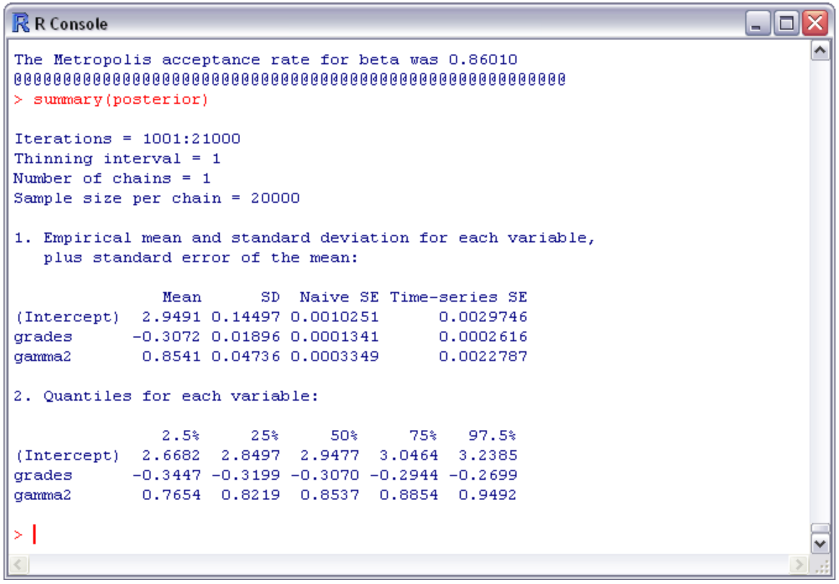
```
library(foreign) nels <- read.dta("C:/userdata/nels_small.dta")
attach(nels)

library(MCMCpack)
  posterior <- MCMCoprobit(psechoice ~ grades, mcmc=20000)
  summary(posterior)
```

The first line loads the `foreign` package into into your *R* library. This package allows you to read in Stata's datasets. The second line creates the data object called `nels`. The attach(nels) statement allows you to refer to the variables in nels directly by their names.

The next line loads `MCMCpack` into *R*. Then the ordered probit estimator (`MCMCoprobit`) is called. The first argument of `MCMCoprobit` is the dependent variable `psechoice`, followed by a ∼, and then the independent variable `grades`. The last argument tells *R* how many simulated values to compute, in this case 20,000. The results of the simulation are stored in the object called `posterior`. The mean or median of this distribution is used as your point estimate (vis-a-vis the MLE). The last line of the program requests the summary statistics from the simulated values of the parameters. The results appear in Figure 16.7, where the MLEs are highlighted in red. One important difference between MCMCpack and the MLE is in how the results are reported. The model as specified in your textbook contains no intercept and 2 thresholds. To include a separate intercept would cause the model to be perfectly collinear. In MCMCpack, the default model includes an intercept and hence can contain only one threshold.

[3] Of course, if you decide to use more information in your prior then they can be substantially different.

Figure 16.7: Ordered probit results from the MCMCoprobit estimator in *R*



```
The Metropolis acceptance rate for beta was 0.86010
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
> summary(posterior)

Iterations = 1001:21000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 20000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

               Mean      SD  Naive SE Time-series SE
(Intercept)  2.9491 0.14497 0.0010251      0.0029746
grades      -0.3072 0.01896 0.0001341      0.0002616
gamma2       0.8541 0.04736 0.0003349      0.0022787

2. Quantiles for each variable:

               2.5%     25%     50%     75%   97.5%
(Intercept)  2.6682  2.8497  2.9477  3.0464  3.2385
grades      -0.3447 -0.3199 -0.3070 -0.2944 -0.2699
gamma2       0.7654  0.8219  0.8537  0.8854  0.9492

> 
```

The ‘slope’ coefficient β, which is highlighted in Figure 16.7, is virtually the same as that reported in Hill et al. (2008). The other results are also similar and are interpreted like the ones produced in **gretl**. The intercept in MCMCpack is equal to $-\mu_1$. The second cut-off in *POE*'s no-intercept model is $\mu_2 = -(Intercept - \gamma_2)$, where γ_2 is the single threshold in the MCMCpack specification. The standard errors are comparable and you can see that they are equivalent to 3 or 4 decimal places to those from the MLE.

16.10 Script

```
open c:\userdata\gretl\data\poe\transport.gdt

probit auto const dtime
genr p1 = $coeff(const)+$coeff(dtime)*20
genr dt = dnorm(p1)*$coeff(dtime)
genr p2 = cnorm($coeff(const)+$coeff(dtime)*30)

open c:\userdata\gretl\data\poe\nels_small.gdt
```

```
series y = psechoice - 1
discrete y
probit y const grades

open c:\userdata\gretl\data\poe\olympics.gdt
smpl year = 88 --restrict
genr lpop = log(pop)
genr lgdp = log(gdp)
poisson medaltot const lpop lgdp
genr mft = exp($coeff(const)+$coeff(lpop)*median(lpop) \
                  +$coeff(lgdp)*median(lgdp))*$coeff(lgdp)

open c:\userdata\gretl\data\poe\mroz.gdt
tobit hours const educ exper age kidsl6
genr H_hat = $coeff(const)+$coeff(educ)*mean(educ) \
                          +$coeff(exper)*mean(exper) \
                          +$coeff(age)*mean(age)+$coeff(kidsl6)*1
genr z = cnorm(H_hat/$sigma)
genr pred = z*$coeff(educ)

smpl hours > 0 --restrict
ols hours const educ exper age kidsl6

smpl --full
ols hours const educ exper age kidsl6

#Heckit
open C:\userdata\gretl\data\poe\mroz.gdt

genr kids = (kidsl6+kids618>0)
genr lwage = log(wage)

list X = const educ exper
list W = const mtr age kids educ

probit lfp W
genr ind = $coeff(const) + $coeff(age)*age + \
           $coeff(educ)*educ + $coeff(kids)*kids + $coeff(mtr)*mtr
genr lambda = dnorm(ind)/cnorm(ind)
ols lwage X lambda
```

```
heckit lwage X ; lfp W --two-step

#Monte Carlo
open C:\userdata\gretl\data\poe\tobit.gdt
smpl 1 200
genr xs = 20*uniform()
loop 1000 --progressive
   genr y = -9 + 1*xs  + 4*normal()
   genr yi = y > 0
   genr yc = y*yi
   ols yc const xs --quiet
   genr b1s = $coeff(const)
   genr b2s = $coeff(xs)
   store coeffs.gdt b1s b2s
endloop

open C:\userdata\gretl\data\poe\tobit.gdt
genr xs = 20*uniform()
genr idx = 1
matrix A = zeros(1000,3)
loop 1000
   smpl --full
   genr y = -9 + 1*xs + 4*normal()
   smpl y > 0 --restrict
   ols y const xs --quiet
   genr b1s = $coeff(const)
   genr b2s = $coeff(xs)
   matrix A[idx,1]=idx
   matrix A[idx,2]=b1s
   matrix A[idx,3]=b2s
   genr idx = idx + 1
endloop

A
matrix bb = meanc(A)
bb
```

And the MNL.inp script for multinomial logit.

```
#To get predictions
scalar b22 = theta[1]
scalar b12 = theta[2]
scalar b23 = theta[3]
scalar b13 = theta[4]

#Use the Quantile function to get the 5% and 50% quantiles
scalar q50 = quantile(grades,.5)
scalar q5 = quantile(grades,.05)

scalar q5 = 2.635

#No College probabilities
scalar p1_50 = 1/(1+exp(b12+b22*q50)+exp(b13+b23*q50))
scalar p1_5 = 1/(1+exp(b12+b22*q5)+exp(b13+b23*q5))

#Marginal effects, No College
scalar p2_50 = 1/(1+exp(b12+b22*(q50+1))+exp(b13+b23*(q50+1)))
scalar p2_5 = 1/(1+exp(b12+b22*(q5+1))+exp(b13+b23*(q5+1)))
scalar m1=p2_50-p1_50
scalar m2=p2_5-p1_5

#2 Year college probabilities
scalar p1_50 = exp(b12+b22*q50)/(1+exp(b12+b22*q50)\
                                 +exp(b13+b23*q50))
scalar p1_5 = exp(b12+b22*q5)/(1+exp(b12+b22*q5)+exp(b13+b23*q5))

#Marginal effects, 2 Year College
scalar p2_50 = exp(b12+b22*(q50+1))/(1+exp(b12+b22*(q50+1)) \
                          +exp(b13+b23*(q50+1)))
scalar p2_5 = exp(b12+b22*(q5+1))/(1+exp(b12+b22*(q5+1))\
                          +exp(b13+b23*(q5+1)))
scalar m3=p2_50-p1_50
scalar m4=p2_5-p1_5

#4 Year college probabilities
scalar p1_50 = exp(b13+b23*q50)/(1+exp(b12+b22*q50) \
                           +exp(b13+b23*q50))
scalar p1_5 = exp(b13+b23*q5)/(1+exp(b12+b22*q5)\
                           +exp(b13+b23*q5))
```

```
#Marginal effects, 4 Year college
scalar p2_50 = exp(b13+b23*(q50+1))/(1+exp(b12+b22*(q50+1))\
                                   +exp(b13+b23*(q50+1)))
scalar p2_5 = exp(b13+b23*(q5+1))/(1+exp(b12+b22*(q5+1))\
                            +exp(b13+b23*(q5+1)))
scalar m5=p2_50-p1_50
scalar m6=p2_5-p1_5
```

Appendix A

gretl commands

A.1 Estimation

- ar : Autoregressive estimation
- arima : ARMA model
- corc : Cochrane-Orcutt estimation
- equation : Define equation within a system
- estimate : Estimate system of equations
- garch : GARCH model
- hccm : HCCM estimation
- heckit: Heckit estimation (2-step and MLE)
- hilu : Hildreth-Lu estimation
- hsk : Heteroskedasticity-corrected estimates
- lad : Least Absolute Deviation estimation
- logistic : Logistic regression

- logit : Logit regression
- mle : Maximum likelihood estimation
- mpols : Multiple-precision OLS
- nls : Nonlinear Least Squares
- ols : Ordinary Least Squares
- panel : Panel models
- poisson : Poisson estimation
- probit : Probit model
- pwe : Prais-Winsten estimator
- system : Systems of equations
- tobit : Tobit model
- tsls : Two-Stage Least Squares
- var : Vector Autoregression
- vecm : Vector Error Correction Model
- wls : Weighted Least Squares

A.2 Tests

- addto : Add variables to specified model
- adf : Augmented Dickey-Fuller test
- arch : ARCH test
- chow : Chow test
- coeffsum : Sum of coefficients
- coint : Engle-Granger cointegration test
- coint2 : Johansen cointegration test

- cusum : CUSUM test
- hausman : Panel diagnostics
- kpss : KPSS stationarity test
- leverage : Influential observations
- lmtest : LM tests
- meantest : Difference of means
- omit : Omit variables
- omitfrom : Omit variables from specified model
- qlrtest : Quandt likelihood ratio test
- reset : Ramseys RESET
- restrict : Linear restrictions
- runs : Runs test
- testuhat : Normality of residual
- vartest : Difference of variances
- vif : Variance Inflation Factors

A.3 Transformation

- diff : First differences
- discrete : Mark variables as discrete
- dummify : Create sets of dummies
- lags : Create lags
- ldiff : Log-differences
- logs : Create logs
- multiply : Multiply variables

- rhodiff : Quasi-differencing
- sdiff : Seasonal differencing
- square : Create squares of variables

A.4 Statistics

- corr : Correlation coefficients
- corrgm : Correlogram
- freq : Frequency distribution
- hurst : Hurst exponent
- mahal : Mahalanobis distances
- pca : Principal Components Analysis
- pergm : Periodogram
- spearman : Spearmanss rank correlation
- summary : Descriptive statistics
- xtab : Cross-tabulate variables

A.5 Dataset

- addobs : Add observations
- append : Append data
- data : Import from database
- delete : Delete variables
- genr : Generate a new variable
- import : Import data
- info : Information on data set

- labels : Print labels for variables
- nulldata : Creating a blank dataset
- open : Open a data file
- rename : Rename variables
- setinfo : Edit attributes of variable
- setobs : Set frequency and starting observation
- setmiss : Missing value code
- smpl : Set the sample range
- store : Save data
- transpos : Transpose data
- varlist : Listing of variables

A.6 Graphs

- boxplot : Boxplots
- gnuplot : Create a gnuplot graph
- graph : Create ASCII graph
- plot : ASCII plot
- rmplot : Range-mean plot
- scatters : Multiple pairwise graphs

A.7 Printing

- eqnprint : Print model as equation
- outfile : Direct printing to file
- print : Print data or strings

- printf : Formatted printing
- tabprint : Print model in tabular form Prediction
- fcast : Generate forecasts
- fcasterr : Forecasts with confidence intervals
- fit : Generate fitted values

A.8 Programming

- break : Break from loop
- else
- end : End block of commands
- endif
- endloop : End a command loop
- function : Define a function
- if
- include : Include function definitions
- loop : Start a command loop
- matrix : Define or manipulate matrices
- run : Execute a script
- set : Set program parameters

A.9 Utilities

- criteria : Model selection criteria
- critical : Critical values
- help : Help on commands

- modeltab : The model table
- pvalue : Compute p-values
- quit : Exit the program
- shell : Execute shell commands

Appendix B

Some Basic Probability Concepts

In Appendix B of *POE* you are confronted with some basic concepts about probability. Since the actual values that economic variables take on are not actually known before they are observed, we say that they are *random*. Probability is the theory that helps us to express uncertainty about the possible values of these variables. Each time we observe the outcome of a random variable we obtain an observation. Once observed, its value is known and hence it is no longer random. So, there is a distinction to be made between variables whose values are not yet observed (random variables) and those whose values have been observed (observations). Keep in mind, though, an observation is merely one of many possible values that the variables can take. Another draw will usually result in a different value being observed.

A probability distribution is just a mathematical statement about the possible values that our random variable can take on. The probability distribution tells us the relative frequency (or probability) with which each possible value is observed. In their mathematical form probability distributions can be rather complicated; either because there are too many possible values to describe succinctly, or because the formula that describes them is complex. In any event, it is common to summarize this complexity by concentrating on some simple numerical characteristics that they possess. The numerical characteristics of these mathematical functions are often referred to as *parameters*. Examples are the mean and variance of a probability distribution. The mean of a probability distribution describes the average value of the random variable over *all* of its possible realizations. Conceptually, there are an infinite number of realizations therefore parameters are not known to us. As econometricians, our goal is to try to estimate these parameters using a finite amount of information available to us.

A statistic is used to gain information about the unknown parameter(s). We collect a number of realizations (called a sample) and then estimate the unknown parameters using a *statistic*. Whereas as a parameter is an unknown numerical characteristic of a probability distribution, a statistic is an *observable* numerical characteristic of a sample. Since the value of the statistic will be different for each sample drawn, it too is a random variable and will be characterized by a probability distribution, which is called the sampling distribution.

Expected values are used to summarize various numerical characteristics of a probability distributions. For instance, suppose X is a random variable that can take on the values 0,1,2,3 and these values occur with probability 1/6, 1/3, 1/3, and 1/6, respectively. The average value or mean of the probability distribution, designated μ, is obtained *analytically* using its expected value.

$$\mu = E[X] = \sum xf(x) = 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{6} = \frac{3}{2} \tag{B.1}$$

So, μ is a parameter. Its value can be obtained mathematically if we know the probability density function of the random variable, X. If this probability distribution is known, then there is no reason to take samples or to study statistics! We can ascertain the mean, or average value, of a random variable without every firing up our calculator. Of course, in the real world we only know that the value of X is not known before drawing it and we don't know what the actual probabilities are that make up the density function, $f(x)$. In order to Figure out what the value of μ is, we have to resort to different methods. In this case, we try to infer what it is by drawing a sample and estimating it using a statistic.

One of the ways we bridge the mathematical world of probability theory with the observable world of statistics is through the concept of a *population*. A statistical population is the collection of individuals that you are interested in studying. Since it is normally too expensive to collect information on everyone of interest, the econometrician collects information on a *subset* of this population–in other words, he takes a *sample.*

The population in statistics has an analogue in probability theory. In probability theory one must specify the set of all possible values that the random variable can be. In the example above, a random variable is said to take on 0,1,2, or 3. This set must be complete in the sense that the variable cannot take on any other value. In statistics, the population plays a similar role. It consists of the set that is relevant to the purpose of your inquiry and that is possible to observe. Thus it is common to refer to parameters as describing characteristics of populations. Statistics are the analogues to these and describe characteristics of the sample.

This roundabout discussion leads to an important point. We often use the words mean, variance, covariance, correlation rather casually in econometrics, but their meanings are quire different depending on whether we are refereing to random variables or merely a realization from one particular sample. When referring to the analytic concepts of mean, variance, covariance, and correlation we are specifically talking about characteristics of random variables whose behavior is characterized by a probability distribution; the expected values can only be ascertained through complete knowledge of the probability distribution functions. It is common to refer to them in this sense as population mean, population variance, and so on.

In statistics we attempt to estimate these (population) parameters using samples and explicit formulae. For instance, we might use the average value of a sample to estimate the average value of the population (or probability distribution).

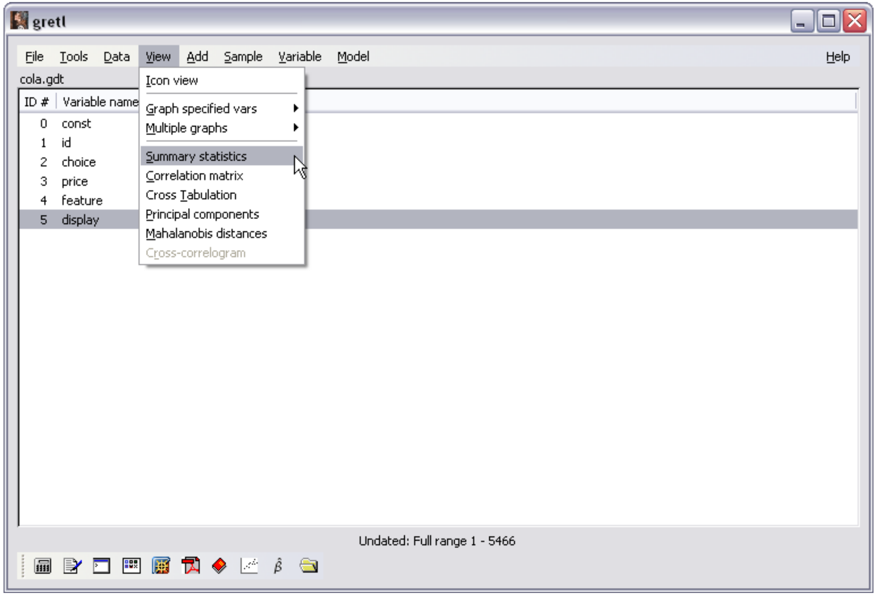
	Probability Distribution	Sample
mean	$E[X] = \mu$	$\frac{1}{n}\sum x_i = \bar{x}$
variance	$E[X-\mu]^2 = \sigma^2$	$\frac{1}{n-1}\sum(x_i - \bar{x})^2 = s_x^2$

When you are asked to obtain the mean or variance of random variables, make sure you know whether the person asking wants the characteristics of the probability distribution or of the sample. The former requires knowledge of the probability distribution and the later requires a sample.

In **gretl** you are given the facility to obtain sample means, variances, covariances and correlations. You are also given the ability to compute tail probabilities using the normal, t-, F and chisquare distributions. First we'll examine how to get summary statistics.

Summary statistics usually refers to some basic measures of the numerical characteristics of your sample. In **gretl** , summary statistics can be obtained in at least two different ways. Once your data are loaded into the program, highlight the desired variables with your cursor and select `View>Summary statistics` from the pull-down menu. Which leads to the

Figure B.1: Choosing summary statistics from the pull-down menu

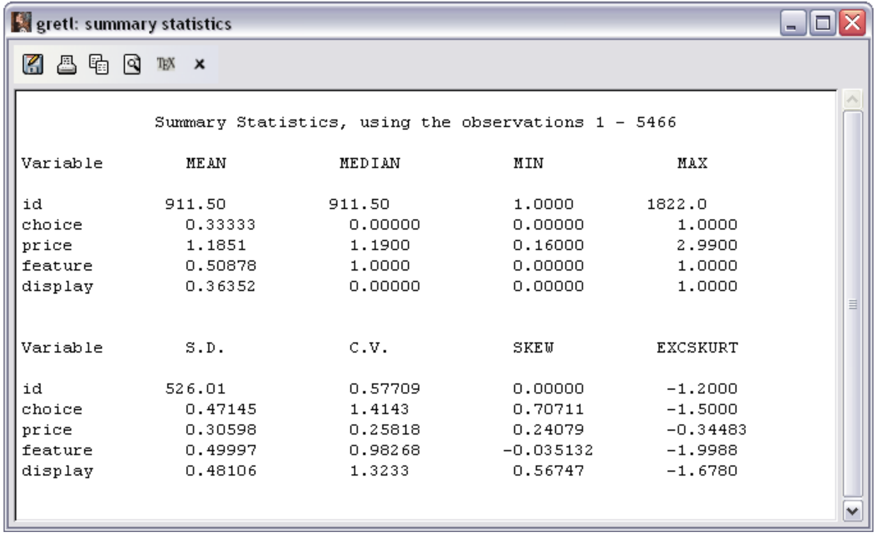


ID #	Variable name
0	const
1	id
2	choice
3	price
4	feature
5	display

Undated: Full range 1 - 5466

output in Figure B.2. The other way to get summary statistics is from the console or script. Recall, **gretl** is really just a language and the GUI is a way of accessing that language. So, to speed things up you can do this. Load the dataset and open up a console window. Then type `summary`. This produces summary statistics for all variables in memory. If you just want summary statistics for a subset, then simply add the variable names after `summary`, i.e., `summary x` gives you the summary statistics for the variable `x`.

Figure B.2: Choosing summary statistics from the pull-down menu yields these results.



```
             Summary Statistics, using the observations 1 - 5466

Variable         MEAN           MEDIAN           MIN            MAX

id             911.50         911.50           1.0000       1822.0
choice           0.33333        0.00000        0.00000         1.0000
price            1.1851         1.1900         0.16000         2.9900
feature          0.50878        1.0000         0.00000         1.0000
display          0.36352        0.00000        0.00000         1.0000


Variable         S.D.            C.V.            SKEW          EXCSKURT

id             526.01           0.57709        0.00000        -1.2000
choice           0.47145        1.4143         0.70711        -1.5000
price            0.30598        0.25818        0.24079        -0.34483
feature          0.49997        0.98268       -0.035132       -1.9988
display          0.48106        1.3233         0.56747        -1.6780
```

Gretl computes the sample mean, median, minimum, maximum, standard deviation (S.D.), coefficient of variation (C.V.), skewness and excess kurtosis for each variable in the data set. You may recall from your introductory statistics courses that there are an equal number of observations in your sample that are larger and smaller in value than the median. The standard deviation is the square root of your sample variance. The coefficient of variation is simply the standard deviation divided by the sample mean. Large values of the C.V. indicate that your mean is not very precisely measured. Skewness is a measure of the degree of symmetry of a distribution. If the left tail (tail at small end of the the distribution) extends over a relatively larger range of the variable than the right tail, the distribution is negatively skewed. If the right tail covers a larger range of values then it is positively skewed. Normal and t-distributions are symmetric and have zero skewness. The χ^2_n is positively skewed. Excess kurtosis refers to the fourth sample moment about the mean of the distribution. 'Excess' refers to the kurtosis of the normal distribution, which is equal to three. Therefor if this number reported by **gretl** is positive, then the kurtosis is greater than that of the normal; this means that it is more peaked around the mean than the normal. If excess kurtosis is negative, then the distribution is flatter than the normal.

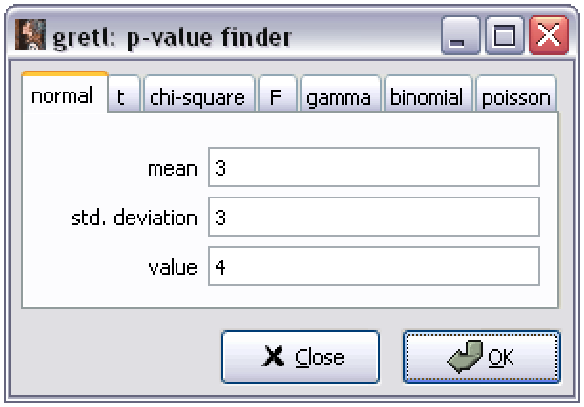
Sample Statistic	Formula
Mean	$\sum x_i/n = \bar{x}$
Variance	$\frac{1}{n-1}\sum(x_i-\bar{x})^2 = s_x^2$
Standard Deviation	$s = \sqrt{s^2}$
Coefficient of Variation	$s/\bar{x}$
Skewness	$\frac{1}{n-1}\sum(x_i-\bar{x})^3/s^3$
Excess Kurtosis	$\frac{1}{n-1}\sum(x_i-\bar{x})^4/s^4 - 3$

You can also use **gretl** to obtain tail probabilities for various distributions. For example if $X \sim N(3,9)$ then $P(X \geq 4)$ is

$$P[X \geq 4] = P[Z \geq (4-3)/\sqrt{9}] = P[Z \geq 0.334] \doteq 0.3694 \qquad \text{(B.2)}$$

To obtain this probability, you can use the `Tools>P-value finder` from the pull-down menu. Then, give **gretl** the value of X, the mean of the distribution and its standard deviation using the dialog box shown in Figure B.3. The result appears in Figure B.4. You can use a script to do this as well. First, convert 4 to a standard normal. That means, subtract its mean, 3, and divide by its standard error, $\sqrt{9}$. The result is a scalar so, open a script window and type:

Figure B.3: Dialog box for finding right hand side tail areas of various probability distributions.

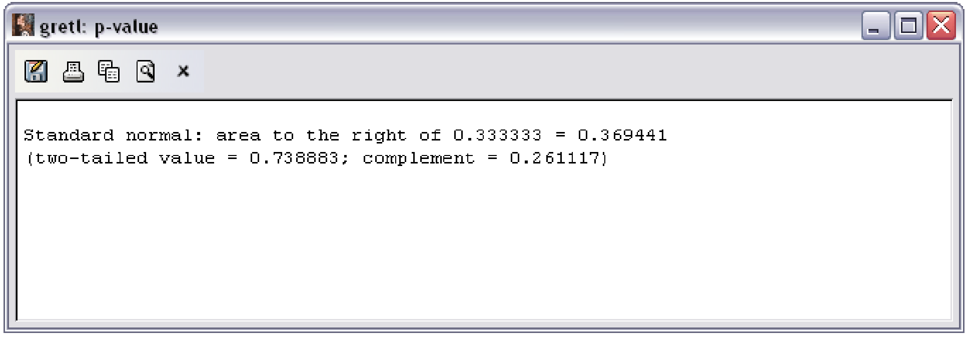


mean 3

std. deviation 3

value 4

Figure B.4: Results from the p value finder of $P[X \geq 4]$ where $X \sim N(3, 9)$. Note, the area in the tail of this distribution to the right of 4 is .369441.



```
Standard normal: area to the right of 0.333333 = 0.369441
(two-tailed value = 0.738883; complement = 0.261117)
```

```
scalar z1 = (4-3)/sqrt(9)
```

Then use the `cdf` function to compute the tail probability of `z1`. For the normal cdf this is

```
scalar c1 = 1-cdf(z,z1)
```

The first argument of the `cdf` function, z, identifies the probability distribution and the second, $z1$, the number to which you want to integrate. So in this case you are integrating a normal cdf from minus infinity to z1=.334. You want the other tail (remember, you want the probability that Z is greater than 4) so subtract this value from 1.

In your book you are given another example $X \sim N(3, 9)$ then find $P(4 \leq X \leq 6)$ is

$$P[4 \leq X \leq 6] = P[0.334 \leq Z \leq 1] = P[Z \leq 1] - P[Z \leq .33] \tag{B.3}$$

Take advantage of the fact that $P[Z \leq z] = 1 - P[Z > z]$ to obtain use the pvalue finder to obtain:

$$(1 - 0.1587) - (1 - 0.3694) = (0.3694 - 0.1587) = 0.2107 \tag{B.4}$$

Note, this value differs slightly from the one given in your book due to rounding error that occurs from using the normal probability table. When using the table, the $P[Z \leq .334]$ was truncated to $P[Z \leq .33]$; this is because your tables are only taken out to two decimal places and a practical decision was made by the authors of your book to forgo interpolation (contrary to what your Intro to Statistics professor may have told you, it is hardly ever worth the effort to interpolate when you have to do it manually). **Gretl**, on the other hand computes this probability out to machine precision as $P[Z \leq \frac{1}{3}]$. Hence, a discrepancy occurs. Rest assured though that these results are, aside from rounding error, the same.

Using the `cdf` function makes this simple and accurate. The script is

```
scalar z1 = (4-3)/sqrt(9)
scalar z2 = (6-3)/sqrt(9)
scalar c1 = cdf(z,z1)
scalar c2 = cdf(z,z2)
scalar area = c2-c1
```

Appendix C

Some Statistical Concepts

The hip data are used to illustrate computations for some simple statistics in your text.

C.1 Summary Statistics

Using a script or operating from the console, open the hip data, `hip.gdt`, and issue the summary command. This yields the results shown in Table C.1. This gives you the mean, median, minimum, maximum, standard deviation, coefficient of variation, skewness and excess kurtosis of your variable(s). Once the data are loaded, you can use **gretl**'s language to generate these as well. For instance, `genr hip_bar = mean(hip)` yields the mean of the variable `hip`. To obtain the sample variance use `genr s2hat = sum((hip-mean(hip)^2)/($nobs-1)`. The script below can be used to compute other summary statistics as discussed in your text.

```
open c:\userdata\gretl\data\poe\hip.gdt
summary
genr hip_bar = mean(hip)
genr s2hat = sum((hip-mean(hip))^2)/($nobs-1)
genr varYbar = s2hat/$nobs
```

Table C.1: Summary statistics from the hip data

```
? open c:\userdata\gretl\data\poe\hip.gdt

Read datafile c:\userdata\gretl\data\poe\hip.gdt periodicity: 1,
maxobs: 50, observations range: 1-50

Listing 2 variables:
  0) const    1) hip

? summary

              Summary Statistics, using the observations 1 - 50
                for the variable 'hip' (50 valid observations)

  Mean                    17.158
  Median                  17.085
  Minimum                 13.530
  Maximum                 20.400
  Standard deviation       1.8070
  C.V.                     0.10531
  Skewness                -0.013825
  Ex. kurtosis            -0.66847

?
```

```
genr sdYbar = sqrt(varYbar)
genr sig_tild = sqrt(sum((hip-mean(hip))^2)/($nobs))
genr mu3 = sum((hip-mean(hip))^3)/($nobs)
genr mu4 = sum((hip-mean(hip))^4)/($nobs)
```

Then, to estimate skewness, $S = \tilde{\mu}^3/\tilde{\sigma}^3$, and kurtosis, $K = \tilde{\mu}^4/\tilde{\sigma}^4$:

```
genr skew = mu3/sig_tild^3
genr kurt = mu4/sig_tild^4
```

Note, in **gretl**'s built in `summary` command, the *excess* kurtosis is reported. The normal distribution has a theoretical kurtosis equal to 3 and the excess is measured relative to that. Hence, Excess K $= \tilde{\mu}^4/\tilde{\sigma}^4 - 3$

If hip size in inches is normally distributed, $Y \sim N(\mu, \sigma^2)$. Based on our estimates, $Y \sim N(17.158, 3.265)$. The percentage of customers having hips greater than 18 inches can be estimated.

$$P(Y > 18) = P\left(\frac{Y - \mu}{\sigma} > \frac{18 - \mu}{\sigma}\right) \tag{C.1}$$

Replacing μ and σ by their estimates yields

```
genr zs = (18 - mean(hip))/sqrt(s2hat)
pvalue z zs
```

The last line actually computes the p-value associated with z-score. So, the `pvalue` command requests that a p-value be returned, the second argument (`z`) indicates the distribution to be used (in this case, z indicates the normal), and the final argument (`zs`) is the statistic itself, which is computed in the previous line.

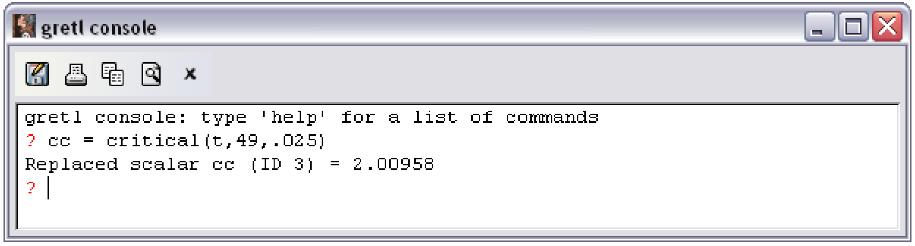
C.2 Interval Estimation

Estimating a confidence interval using the hip data is also easy to do in **gretl**. Since the true variance, σ^2, is not known, the t-distribution is used to compute the interval. The interval is

$$\bar{y} \pm t_c \frac{\hat{\sigma}}{\sqrt{N}} \tag{C.2}$$

where t_c is the desired critical value from the student-t distribution. In our case, $N = 50$ and the desired degrees of freedom for the t-distribution is $N - 1 = 49$. The **gretl** command `critical(t,49,.025` can be used to return the 0.025 critical value from the t_{49} distribution shown in Figure C.1 The computation is

Figure C.1: Obtaining critical values from the t distribution using the console



```
gretl console: type 'help' for a list of commands
? cc = critical(t,49,.025)
Replaced scalar cc (ID 3) = 2.00958
? 
```

```
open c:\userdata\gretl\data\poe\hip.gdt
genr s2hat = sum((hip-mean(hip))^2)/($nobs-1)
genr varYbar = s2hat/$nobs
genr sdYbar = sqrt(varYbar)
genr lb = mean(hip) - 2.01*sdYbar
genr ub = mean(hip) + 2.01*sdYbar
```

which indicates that the interval [16.64,17.67] works 95% of the time. Note these numbers differ slightly from those in your book because we used 2.01 as our critical value. Hill et al. carry their critical value out to more decimal places and hence the difference. You can use **gretl**'s internal functions to improve accuracy. Replace 2.01 with `critical(t,$nobs-1,0.025)` and see what happens!

```
genr lb = mean(hip) - critical(t,$nobs-1,0.025)*sdYbar
genr ub = mean(hip) + critical(t,$nobs-1,0.025)*sdYbar
```

C.3 Hypothesis Tests

Hypothesis tests are based on the same principles and use the same information that is used in the computation of confidence intervals. The first test is on the null hypothesis that hip size does not exceed 16.5 inches against the alternative that it does. Formally, $H_0 : \mu = 16.5$ against the alternative $H_a : \mu > 16.5$. The test statistic is computed based on the sample average, $\bar{Y}$ and is

$$t = \frac{\bar{Y} - 16.5}{\hat{\sigma}/\sqrt{N}} \sim t_{N-1} \tag{C.3}$$

if the null hypothesis is true. Choosing the significance level, $\alpha = .05$, the right-hand side critical value for the t_{49} is 1.677. The average hip size is 17.1582 with standard deviation 1.807 so the test statistic is

$$t = \frac{17.1582 - 16.5}{1.807/\sqrt{50}} = 2.576 \tag{C.4}$$

The **gretl** code to produce this is:

```
open c:\userdata\gretl\data\poe\hip.gdt
genr s2hat = sum((hip-mean(hip))^2)/($nobs-1)
genr varYbar = s2hat/$nobs
genr sdYbar = sqrt(varYbar)
genr tstat = (mean(hip)-16.5)/(sdYbar)
scalar c = critical(t,49,0.025)
pvalue t 49 tstat
```

The `scalar c = critical(t,49,0.025)` statement can be used to get the $\alpha = 0.025$ critical value for the t distribution with 49 degrees of freedom. The next line, `pvalue t 49 tstat`, returns the p-value from the t distribution with 49 degrees of freedom for the computed statistic, `tstat`.

The two-tailed test is of the hypothesis, $H_0 : \mu = 17$ against the alternative, $H_a : \mu \neq 17$.

$$t = \frac{\bar{Y} - 17}{\hat{\sigma}/\sqrt{N}} \sim t_{N-1} \tag{C.5}$$

if the null hypothesis is true. Choosing the significance level, $\alpha = .05$, the two sided critical value is ± 2.01. Hence, you will reject the null hypothesis if $t < -2.01$ or if $t > 2.01$. The statistic is computed

$$t = \frac{17.1582 - 17}{1.807/\sqrt{50}} = .6191 \tag{C.6}$$

and you cannot reject the null hypothesis. The **gretl** code is:

```
genr tstat = (mean(hip)-17)/(sdYbar)
scalar c = critical(t,49,0.025)
pvalue t 49 tstat
```

C.4 Testing for Normality

Your book discusses the Jarque-Bera test for normality which is computed using the skewness and kurtosis of the least squares residuals. To compute the Jarque-Bera statistic, you'll first need to obtain the summary statistics from your data series.

From **gretl** script

```
open c:\userdata\gretl\data\poe\hip.gdt
summary
```

You could also use the point and click method to get the summary statistics. This is accomplished from the output window of your regression. Simply highlight the hip series and then choose `Data>Summary statistics>selected variables` from the pull-down menu. This yields the results in Table C.1.

One thing to note, **gretl** reports excess kurtosis rather than kurtosis. The excess kurtosis is measured relative to that of the normal distribution which has kurtosis of three. Hence, your computation is

$$JB = \frac{N}{6}\left(\text{Skewness}^2 + \frac{(\text{Excess Kurtosis})^2}{4}\right) \tag{C.7}$$

Which is

$$JB = \frac{50}{6}\left(-0.0138^2 + \frac{-0.66847^2}{4}\right) = .9325 \tag{C.8}$$

Using the results in section C.1 for the computation of skewness and kurtosis, the **gretl** code is:

```
open c:\userdata\gretl\data\poe\hip.gdt
genr sig_tild = sqrt(sum((hip-mean(hip))^2)/($nobs))
genr mu3 = sum((hip-mean(hip))^3)/($nobs)
genr mu4 = sum((hip-mean(hip))^4)/($nobs)

genr skew = mu3/sig_tild^3
genr kurt = mu4/sig_tild^4

genr JB = ($nobs/6)*(skew^2+(kurt-3)^2/4)
pvalue X 2 JB
```

Appendix D

Using *R* with **gretl**

Another feature of **gretl** that makes it extremely powerful is its ability to work with another free program called *R*. *R* is actually a programming language for which many statistical procedures have been written. Although **gretl** is reasonably powerful, there are still many things that it won't do. The ability to export **gretl** data into *R* makes it possible to do some sophisticated analysis with relative ease.

Quoting from the *R* web site

> *R* is a language and environment for statistical computing and graphics. It is a GNU project which is similar to the S language and environment which was developed at Bell Laboratories (formerly AT&T, now Lucent Technologies) by John Chambers and colleagues. *R* can be considered as a different implementation of S. There are some important differences, but much code written for S runs unaltered under *R*.
>
> *R* provides a wide variety of statistical (linear and nonlinear modeling, classical statistical tests, time-series analysis, classification, clustering, ...) and graphical techniques, and is highly extensible. The S language is often the vehicle of choice for research in statistical methodology, and *R* provides an Open Source route to participation in that activity.

> One of R's strengths is the ease with which well-designed publication-quality plots can be produced, including mathematical symbols and formulae where needed. Great care has been taken over the defaults for the minor design choices in graphics, but the user retains full control.
>
> *R* is available as Free Software under the terms of the Free Software Foundation's GNU General Public License in source code form. It compiles and runs on a wide variety of UNIX platforms and similar systems (including FreeBSD and Linux), Windows and MacOS.

R can be downloaded from `http://www.r-project.org/` which is referred to as CRAN or the comprehensive *R* archive network. To install R, you'll need to download it and follow the instructions given at the CRAN web site. Also, there is an appendix in the **gretl** manual about using *R* that you may find useful. The remainder of this brief appendix assumes that you have R installed and linked to **gretl** through the programs tab in the `File>Preferences>General` pull down menu. Make sure that the 'Command to launch GNR R' box points to the `RGui.exe` file associated with your installation of *R*.

To illustrate, open the *food.gdt* data in **gretl**.

```
open c:\userdata\gretl\data\poe\food.gdt
```

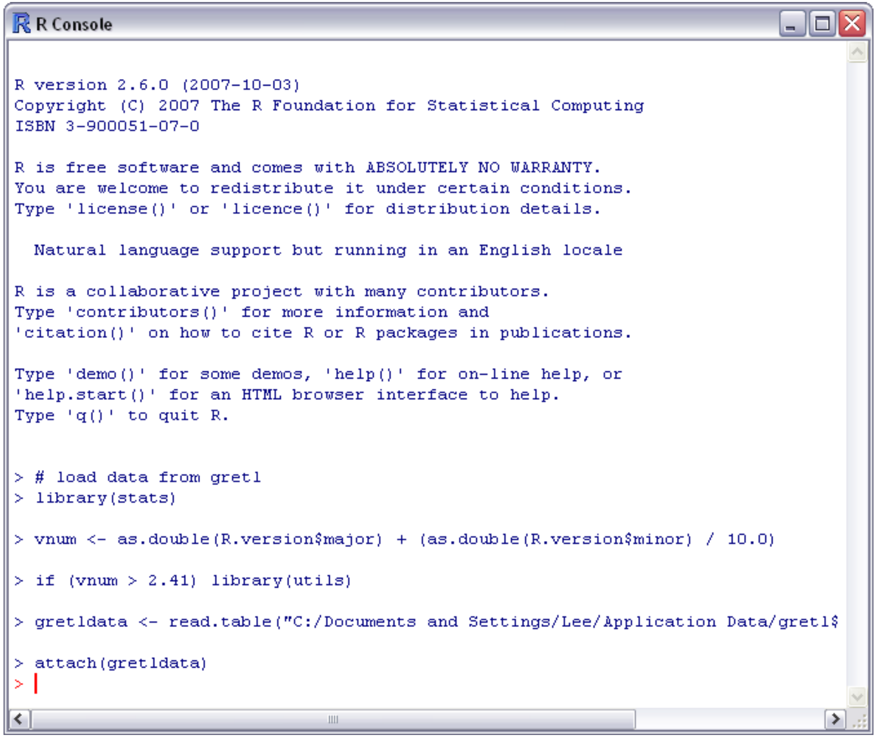
Now, select `Tools>start GNU R` from the pull-down menu. The current **gretl** data set, in this case *food.gdt*, will be transported into *R*'s required format. You'll see the *R* console which is shown in Figure D.1.

To load the data in properly, type the following at the command prompt in *R*.

```
gretldata <-
    read.table("C:/userdata/myfiles/Rdata.tmp", header = TRUE )
```

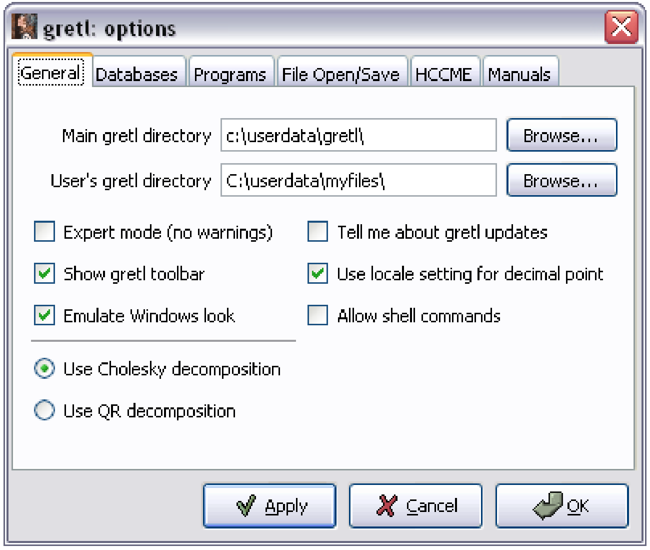
This assumes that you have set **gretl**'s user directory to `C:\userdata\myfiles` using the dialog box shown in Figure (D.2). `Tools>Preferences>General`

Figure D.1: The *R* console when called from **Gretl**



```
R version 2.6.0 (2007-10-03)
Copyright (C) 2007 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.


> # load data from gretl
> library(stats)

> vnum <- as.double(R.version$major) + (as.double(R.version$minor) / 10.0)

> if (vnum > 2.41) library(utils)

> gretldata <- read.table("C:/Documents and Settings/Lee/Application Data/gretl$

> attach(gretldata)
> 
```

Figure D.2: Use this dialog to set the default location for **gretl** files to be written and read.



Main gretl directory: c:\userdata\gretl\

User's gretl directory: C:\userdata\myfiles\

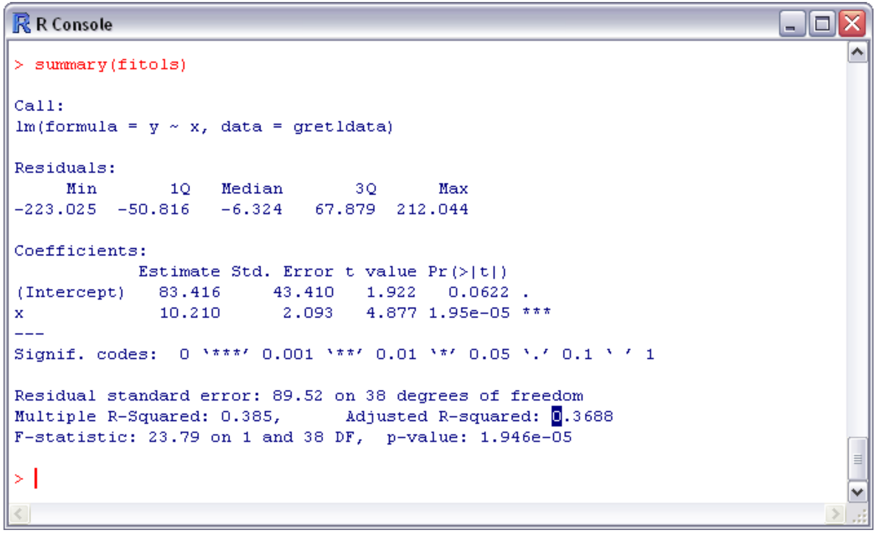
□ Expert mode (no warnings)
☑ Show gretl toolbar
☑ Emulate Windows look
□ Tell me about gretl updates
☑ Use locale setting for decimal point
□ Allow shell commands

● Use Cholesky decomposition
○ Use QR decomposition

The addition of `Header = TRUE` to the code that **gretl** writes for you ensures that the variable names, which are included on the first row of the Rdata.tmp, get read into *R* properly. Then, to run the regression in *R*.

```
fitols <- lm(y˜x,data=gretldata)
```

Figure D.3: The `lm(y x,data=gretldata)` command estimates a linear regression model with y as the dependent variable and x as an independent variable. *R* automatically includes an intercept. To print the results to the screen, you have to use the `summary(anov)` command.



```
> summary(fitols)

Call:
lm(formula = y ~ x, data = gretldata)

Residuals:
     Min       1Q   Median       3Q      Max
-223.025  -50.816   -6.324   67.879  212.044

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   83.416     43.410   1.922   0.0622 .
x             10.210      2.093   4.877 1.95e-05 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 89.52 on 38 degrees of freedom
Multiple R-Squared: 0.385,      Adjusted R-squared: 0.3688
F-statistic: 23.79 on 1 and 38 DF,  p-value: 1.946e-05

> |
```

Before going further, let me comment on this terse piece of computer code. First, in *R* the symbol `<-` is used as the assignment operator; it assigns whatever is on the right hand side (`lm(y∼x,data=gretldata)`) to the name you specify on the left (`fitols`). it can be reversed `->` if you want to call the object to its right what is computed on its left. Also, *R* does not bother to print results unless you ask for them. This is handier than you might think, since most programs produce a lot more output than you actually want and must be coerced into printing less. The `lm` command stands for 'linear model' and in this example it contains two arguments within the parentheses. The first is your simple regression model. The dependent variable is y and the independent variable x. They are separated by the symbol which substitutes in this case for an equals sign. The other argument points to the data set that contains these two variables. This data set, pulled into *R* from

gretl, is by default called `gretldata`. There are other options for the `lm` command, and you can consult the substantial pdf manual to learn about them. In any event, you'll notice that when you enter this line and press the return key (which executes this line) *R* responds by issuing a command prompt, and no results! To print the results from your regression, you issue the command:

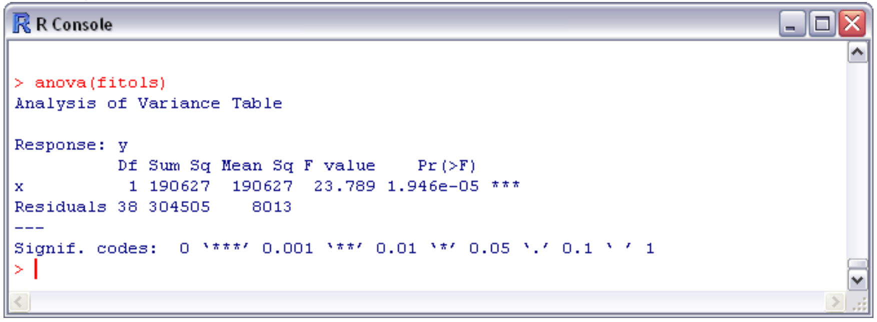
```
summary.lm(fitols)
```

which yields the output shown in Figure D.4. Then, to obtain the ANOVA table for this regression

```
anova(fitols)
```

This gives the result in Figure D.4. It's that simple! One thing to note about how *R* reports analysis of variance. It reports the explained variation (190627) in the top line and the unexplained variation in y (304505) below. It does not report total variation. To obtain the total, you just have to add the explained to the unexplained variation together (190627+304505=495132).

Figure D.4: The `anova(olsfit)` command asks *R* to print the anova table for the regression results stored in olsfit.



```
> anova(fitols)
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x          1 190627  190627  23.789 1.946e-05 ***
Residuals 38 304505    8013
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
```

To do multiple regression in *R*, you have to put each of your independent variables (other than the intercept) into a matrix. A matrix is a rectangular array (which means it contains numbers arranged in rows and columns). You can think of a matrix as the rows and columns of numbers that appear in a spreadsheet program like MS Excel. Each row contains an observation on each of your independent variables; each column contains all of the observations on a particular variable. For instance suppose you have two variables, $x1$ and $x2$, each having 5 observations. These can be combined horizontally into the matrix, X. Computer programmers sometimes refer to this operation as *horizontal concatenation*. Concatenation essentially means that you connect or link objects in a series or chain; to concatenate horizontally means that you are binding one or more columns of numbers together.

The function in R that binds columns of numbers together is `cbind`. So, to horizontally concatenate $x1$ and $x2$ use the command

```
X <- cbind(x1,x2)
```

which takes

$$x1 = \begin{pmatrix} 2 \\ 1 \\ 5 \\ 2 \\ 7 \end{pmatrix}, \quad x2 = \begin{pmatrix} 4 \\ 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}, \quad \text{and yields } X = \begin{pmatrix} 2 & 4 \\ 1 & 2 \\ 5 & 1 \\ 2 & 3 \\ 7 & 1 \end{pmatrix}.$$

Then the regression is estimated using

```
fitols <- lm(y~X)
```

There is one more thing to mention about R that is very important and this example illustrates it vividly. R is case sensitive. That means that two objects x and X can mean two totally different things to R. Consequently, you have to be careful when defining and calling objects in R to get to distinguish lower from upper case letters.

D.1 Packages

The following section is taken with very minor changes from Venables et al. (2006).

All *R* functions and datasets are stored in packages. Only when a package is loaded are its contents available. This is done both for efficiency (the full list would take more memory and would take longer to search than a subset), and to aid package developers, who are protected from name clashes with other code. The process of developing packages is described in section Creating *R* packages in *Writing R Extensions*. Here, we will describe them from a users point of view. To see which packages are installed at your site, issue the command `library()` with no arguments. To load a particular package (e.g., the MCMCpack package containing functions for estimating models in Chapter 16) use the `library` command:

```
> library(MCMCpack)
```

If a package is not installed you can do so if you are connected to the Internet using the `install.packages()` function. To update a package that is already installed use the `update.packages()` functions. Both of these are available through the `Packages` menu in the Windows GUI.

To see which packages are currently loaded, use

```
> search()
```

to display the search list.

To see a list of all available help topics in an installed package, use

```
> help.start()
```

to start the HTML help system, and then navigate to the package listing in the Reference section.

D.2 Stata Datasets

With R you can read in datasets in many different formats. Your textbook includes a dataset written in Stata's format and R can both read and write to this format. To read and write Stata's .dta files, you'll have to load the `foreign` package using the library command:

```
library(foreign)
```

Then, type

```
nels <- read.dta("c:/DATA/Stata/nels_small.dta")
```

and the dataset will be read directly into *R*. There are two things to note, though. First, the slashes in the filename are backwards from the Windows convention. Second, you need to point to the file in your directory structure and enclose the path/filename in double quotes. *R* looks for the the file where you've directed it and, provided it finds it, reads it into memory. It places the variable names from Stata into the object. Then, to retrieve a variable from the object you create (called in this example, `data`, use the syntax

```
pse <- nels$psechoice
```

Now, you have created a new object called `pse` that contains the variable retrieved from the `nels` object called `psechoice`. This seems awkward at first, but believe it or not, it becomes pretty intuitive after a short time.

The command

```
attach(nels)
```

will take each of the columns of `nels` and allow you to refer to them by their variable names. So, instead of referring to `nels$psechoice` you can directly ask for `psechoice` without using the `nels$` prefix. For complex programs, using `attach()` may lead to unexpected results. If in doubt, it is probably a good idea to forgo this option. If you do decide to use it, you can later undo it using `detach(nels)`.

D.3 Final Thoughts

A very brief, but useful document can be found at `http://cran.r-project.org/doc/contrib/Farnsworth-EconometricsInR.pdf`. This is a guide written by Grant Farnsworth about using R in econometrics. He gives some alternatives to using MCMCpack for the models discussed in Chapter 16.

Appendix E

Errata

- page 49. Syntax for pvalue(t,$df,t2) is fixed. This applies the script in chapter 3 as well. Thanks to Greg Coleman.

Appendix F

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